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Asymptotic properties of the iterates of stochastic operators on (AL) Banach lattices

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Abstract. The asymptotic periodicity of stochastic operators on AL Banach lattices is considered.

Let $(E, \| \ \|)$ be a real Banach lattice. We denote by E_+ the cone of positive elements of E. A linear operator $P\colon E \to E$ is said to be *positive* if $Px \in E_+$ for $x \in E_+$ and a *contraction* if $\|Px\| \leqslant \|x\|$ for all $x \in E$. Recently, the asymptotic behaviour of $P^n x$ for such operators have been studied intensively. In particular, if E is $L^1(m)$ and P is a stochastic operator on E (i.e., $P \geqslant 0$ and $\|Pf\| = \|f\|$ for $f \in L^1_+(m)$), then some conditions guaranteeing the regularity of $P^n f$ have been given in [5], [9], [10]. The asymptotic periodicity for an arbitrary nonnegative contraction on Banach lattices was investigated in [1] and [14].

A linear positive contraction P acting on E is said to be *asymptotically stable* if there exists a unique, positive and normalized vector x_* such that for every $x \in E_+$ with $\|x\| = 1$

$$\lim_{n\to\infty} P^n x = x_*.$$

(Clearly, x_* is then P-invariant.)

Recall (see [8] or [12]) that a Banach lattice E is called an AL-*space* if it satisfies the axiom $\|x+y\| = \|x\| + \|y\|$ for all $x, y \in E_+$. If $\|Px\| = \|x\|$ for all positive x from the AL-space E, then P is called a (*generalized*) *stochastic operator* on E. In this paper, the asymptotic behaviour of $P^n x$ (in particular, asymptotic stability) will be investigated, where P is a stochastic operator on a fixed AL-space E.

Remarks 1. It is evident that every $L^1(m)$ is an AL-Banach lattice. Kakutani's result [4] (see also [12], Theorem 8.5) says that the converse holds,

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i.e., for every AL-space E there exists a locally compact space X and a strictly positive Radon measure m on X such that E is isomorphic with $L^1(m)$.

2. Let $(X, \mathscr{B}, m)$ be a standard Lebesgue space and let P be a stochastic operator on $L^1(m)$. It is well known (see [7], p. 115) that there exists a Markov process $\{\zeta_n\}_{n\geq 0}$ with phase space X such that for every measurable $A \in \mathscr{B}$ we have $\int_A P^n f dm = \mathbf{P}_f\ (\zeta_n \in A)$, where $\mathbf{P}_f$ is the probability (on the canonical space) determined by the initial density f. Thus the evolution of the process $\{\zeta_n\}_{n\geq 0}$ is described by the sequence of the iterations $P^n f$, and the asymptotic stability of the operator P means that the distributions of ζ_n converge to some stationary probability (independently of a initial law).

The following proposition gives lattice conditions for the asymptotic stability of stochastic operators. This result seems to be known for σ-finite $L^1(m)$ spaces but for the convenience of the reader we present a short proof here.

PROPOSITION. *Let P be a stochastic operator on an* AL-*space E. If*

(A_ε) *there exists $0 < \varepsilon \leq 1$ such that for every normalized $x_1, x_2 \in E_+$ there exists n such that $\|P^n x_1 \wedge P^n x_2\| \geq \varepsilon$*

and for some positive (nonzero) element $y \in E$ the orbit $\gamma(y) = \{P^n y\colon n \geq 0\}$ is relatively weakly compact, then P is asymptotically stable.

Proof. First, we show that for every positive x_1, x_2 with $\|x_1\| = \|x_2\| = 1$ we have $\lim_{n\to\infty} \|P^n x_1 - P^n x_2\| = 0$.

Let $\alpha_n = \|P^n x_1 \wedge P^n x_2\|$ and $\alpha = \lim_{n\to\infty} \alpha_n$ (clearly, α_n is nondecreasing). If $\alpha < 1$ then there exists a positive n_0 such that $\alpha_n > \alpha - \varepsilon(1-\alpha)$ for $n \geq n_0$. By (A_ε), for some positive m

$$\|P^m(P^n x_1 - P^n x_1 \wedge P^n x_2) \wedge P^m(P^n x_2 - P^n x_1 \wedge P^n x_2)\| \geq \varepsilon(1-\alpha_n).$$

Thus,

$$\begin{aligned}
\|P^{m+n} x_1 \wedge P^{m+n} x_2\| &= \big\|\big(P^m(P^n - P^n x_1 \wedge P^n x_2) \\
&\quad + P^m(P^n x_1 \wedge P^n x_2)\big) \wedge \big(P^m(P^n x_2 - P^n x_1 \wedge P^n x_2) + P^m(P^n x_1 \wedge P^n x_2)\big)\big\| \\
&= \|P^m(P^n x_1 - P^n x_1 \wedge P^n x_2) \wedge P^m(P^n x_2 - P^n x_1 \wedge P^n x_2) \\
&\quad + P^m(P^n x_1 \wedge P^n x_2)\| = \|P^m(P^n x_1 \wedge P^n x_2)\| \\
&\quad + \|P^m(P^n x_1 - P^n x_1 \wedge P^n x_2) \wedge P^m(P^n x_2 - P^n x_1 \wedge P^n x_2)\| \\
&\geq \varepsilon(1-\alpha_n) + \alpha_n > \varepsilon(1-\alpha) + \big(\alpha - \varepsilon(1-\alpha)\big) = \alpha,
\end{aligned}$$

which contradicts $\alpha_n \leqslant \alpha$. Since

$$\|P^n x_1 - P^n x_2\| = \|(P^n x_1 - P^n x_1 \wedge P^n x_2) - (P^n x_2 - P^n x_1 \wedge P^n x_2)\|$$
$$\leqslant \|P^n x_1 - P^n x_1 \wedge P^n x_2\| + \|P^n x_2 - P^n x_1 \wedge P^n x_2\| = 2(1-\alpha_n)$$

and $\alpha_n \to 1$, we have $\|P^n x_1 - P^n x_2\| \to 0$. To end the proof it is enough to prove that there is a P-invariant normalized vector $x_* \in E_+$. In fact, from the above considerations, $0 = \lim_{n\to\infty} \|P^n x - P^n x_*\| = \lim_{n\to\infty} \|P^n x - x_*\|$. The existence of x_* is a consequence of the von Neumann Ergodic Theorem. By this theorem, the weak compactness of $\gamma(y)$ implies the convergence of the Cesàro means $n^{-1}(y + Py + \ldots + P^{n-1} y)$ to a P-invariant vector $\bar{y}$. Clearly, $\bar{y}$ is positive and is normalized by the stochasticity of P.

Remark 3. If P is an asymptotically stable stochastic operator, then there is a linear positive functional $\Lambda \in E^*$ such that $\lim_{n\to\infty} P^n x = \Lambda(x) x_*$ for every $x \in E$. In fact, by the decomposition $x = x^+ - x^-$ we have

$$\lim_{n\to\infty} P^n x = \lim_{n\to\infty} P^n x^+ - \lim_{n\to\infty} P^n x^-$$
$$= \|x^+\| x_* - \|x^-\| x_* = (\|x^+\| - \|x^-\|) x_*.$$

So, $\Lambda(x) = \|x^+\| - \|x^-\|$ is the desired positive linear functional on E.

The following corollary is a generalization of some results from [9].

COROLLARY 1. *Let P be a stochastic operator acting on an* AL-*space E. If there exists $y \in E_+$, $\|y\| < 2$, such that $\lim_{n\to\infty} \|(P^n x - y)^+\| = 0$ for every normalized $x \in E_+$ then P is asymptotically stable.*

Proof. We show that the assumptions of our proposition are fulfilled. The weak compactness of an arbitrary trajectory $\gamma(x)$ ($x \in E_+$, $\|x\| = 1$) is a straightforward consequence of the weak compactness of ordered intervals in AL-spaces (see [12], Corollary, p. 119). We only have to notice that the iterations $P^n x$ are attracted in the norm to the weakly compact interval $[0, y]$. Now we show that condition (A_ε) holds where $0 < \varepsilon < 2 - \|y\|$. Let $x_1, x_2 \geqslant 0$, $\|x_1\| = \|x_2\| = 1$, and let $\delta > 0$ be arbitrary. Since for sufficiently large n

$$\|y\| \geqslant \|(P^n x_1 \wedge y) \vee (P^n x_2 \wedge y)\| = \big\|\big((P^n x_1 \wedge y) - (P^n x_1 \wedge P^n x_2 \wedge y)\big)$$
$$+ \big((P^n x_2 \wedge y) - (P^n x_1 \wedge P^n x_2 \wedge y)\big) + P^n x_1 \wedge P^n x_2 \wedge y\big\|$$
$$= \|P^n x_1 \wedge y\| + \|P^n x_2 \wedge y\| - \|P^n x_1 \wedge P^n x_2 \wedge y\|$$
$$\geqslant 2 - \delta - \|P^n x_1 \wedge P^n x_2\|,$$

we have

$$\|P^n x_1 \wedge P^n x_2\| \geqslant 2 - \|y\| - \delta.$$

The asymptotic stability of positive contractions acting on ordered vector spaces with base was considered in [14]. The following theorem is a generalization of some results from [11].

THEOREM 1. *Let P be a stochastic operator on an AL-space E. If*

(B_ε). *there exists $\varepsilon > 0$ and $m \geqslant 0$ such that for every $x_1, x_2 \in E_+$, $\|x_1\| = \|x_2\| = 1$, we have $\|P^m x_1 \wedge P^m x_2\| \geqslant \varepsilon$,*

then there exists a unique positive normalized $x_* \in E$ and a positive linear functional $\Lambda \in E^*$ such that $\lim_{n\to\infty} P^n = \Lambda \otimes x_*$ in the norm operator topology (in particular, P is quasi-compact).

Proof. First we observe that for every natural k and normalized $x_1, x_2 \in E_+$ we have

$$\|P^{mk}x_1 - P^{mk}x_2\| \leqslant (1-\varepsilon)^k \|x_1 - x_2\|. \tag{1}$$

For $k = 0$, inequality (1) is evident. Now by (B_ε) we get

$$\begin{aligned}
&\|P^m x_1 - P^m x_2\| \\
&= \left\| P^m\left(\frac{x_1 - x_1 \wedge x_2}{1 - \|x_1 \wedge x_2\|}\right) - P^m\left(\frac{x_2 - x_1 \wedge x_2}{1 - \|x_1 \wedge x_2\|}\right)\right\| (1 - \|x_1 \wedge x_2\|) \\
&= \|P^m z_1 - P^m z_2\| (1 - \|x_1 \wedge x_2\|) \\
&= (1 - \|x_1 \wedge x_2\|)\, \|(P^m z_1 - P^m z_1 \wedge P^m z_2) - (P^m z_2 - P^m z_1 \wedge P^m z_2)\| \\
&= 2(1 - \|x_1 \wedge x_2\|)(1 - \|P^m z_1 \wedge P^m z_2\|) \\
&\leqslant 2(1-\varepsilon)(1 - \|x_1 \wedge x_2\|) = (1-\varepsilon)\|x_1 - x_2\|,
\end{aligned}$$

where

$$z_1 = \frac{x_1 - x_1 \wedge x_2}{1 - \|x_1 \wedge x_2\|} \quad \text{and} \quad z_2 = \frac{x_2 - x_1 \wedge x_2}{1 - \|x_1 \wedge x_2\|}.$$

Thus, (1) can be obtained by iterating the last inequality. Let $x \in E_+$, $\|x\| = 1$, be fixed and let k be such that $2(1-\varepsilon)^k < \delta$. Then for every $y \in E_+$, $\|y\| = 1$, and $n \geqslant km$ we have

$$\|P^n y - P^{km} x\| = \|P^{km} P^{n-km} y - P^{km} x\| \leqslant 2(1-\varepsilon)^k < \delta.$$

Since δ can be taken arbitrarily small, the trajectory $\gamma(y) = \{P^n y\colon n \geqslant 0\}$ is relatively norm compact. Let x_* be a normalized positive P-invariant element in E (the von Neumann Ergodic Theorem guarantees the existence of x_*). By (1), for every $y \in E_+$, $\|y\| = 1$, we have $\|P^{km} y - x_*\| \leqslant 2(1-\varepsilon)^k$ and since the sequence $\|P^n y - x\|$ is nonincreasing,

$$\sup_{\|y\| = 1,\, y \in E_+} \|P^n y - x_*\| \leqslant 2(1-\varepsilon)^{[n/m]}.$$

Let $\Lambda(x) = \|x^+\| - \|x^-\|$. By the above inequality we get

$$\|P^n - \Lambda \otimes x_*\| = \sup_{\|y\|=1} \|P^n y - \Lambda(y)x_*\|$$
$$\leqslant \sup_{\|y\|=1} \{\|P^n y^+ - \Lambda(y^+)x_*\| + \|P^n y^- - \Lambda(y^-)x_*\|\},$$
$$2\sup\{\|P^n y - \Lambda(y)x_*\|: y \in E_+, \|y\| = 1\} \leqslant 4(1-\varepsilon)^{[n/m]} \to 0.$$

We now consider the asymptotic periodicity of the iterates of stochastic operators. A stochastic operator P acting on an AL-space E is called *asymptotically periodic* if there exists a finite collection $e_1, \ldots, e_r$ of positive normalized pairwise orthogonal elements of E and $\lambda_1, \ldots, \lambda_r$, positive functionals from E^*, such that $\|P^n x - \sum_{j=1}^{r} \lambda_j(x) e_{\alpha^n(j)}\| \to 0$ as $n \to \infty$ and $Pe_j = e_{\alpha(j)}$, where α is some permutation of the set $(1, 2, \ldots, r\}$. If α is cyclic, then P is called *asymptotically cyclic* and instead of $\alpha(j)$ we will write $j+1$ for $j \in \{1, 2, \ldots, r\}$, and the sum is taken modulo r. The asymptotic periodicity (cyclicity) is a generalization of asymptotic stability, however, the ω-limit sets of trajectories remain still finite. Recall that if a stochastic operator has the so-called weak constrictor (see [5] and [6] for details) then it is asymptotically periodic. For arbitrary Banach latices the asymptotic periodicity of positive contractions has been obtained in [1] (see also [2] and [14]) where strong constrictivity was assumed. The following theorem is a generalization of some results from [2]. We will use results from [3] and [13] concerning the behaviour of P on limit sets. Our definitions and notations agree with [13].

The limit set $\omega(x)$ of the trajectory $\gamma(x) = \{P^n x: n \geqslant 0\}$ is the set $\{y \in E: \exists n_k \nearrow \infty, P^{n_k} x \to y\}$. It is known (see [3]) that if $\omega(x) \neq \emptyset$ then $\omega(x)$ is a P-invariant minimal subset of E ($y \in \omega(x) \Rightarrow \overline{\gamma(y)} = \omega(x)$). Moreover, it can be shown that in this case P is an invertible isometry on $\omega(x)$. We denote by Ω the set of all limit points $\bigcup_{x \in E} \omega(x)$. Now we are in a position to formulate the following:

THEOREM 2. *Let P be a stochastic operator on a real* AL-*space E. If for every $x \in E$ the limit set $\omega(x) \neq \emptyset$ and*

(C) *there exists a natural k such that for each nonzero positive x_1, x_2 there exist positive n, m with $|n-m| \leqslant k$ such that $P^n x_1 \wedge P^m x_2 \neq 0$,*

then P is asymptotically cyclic and the length of the cycle $r \leqslant k+1$.

Proof. Let $x \in \Omega$. By minimality of $\omega(x)$ there is a sequence $\boldsymbol{n} = (n_k)$ such that $P^{n_k} x \to x$. We set $\Omega(\boldsymbol{n}) = \{y \in E: P^{n_k} y \to y\}$. Clearly, $\Omega(\boldsymbol{n})$ is a closed linear (nontrivial) subspace of E. Moreover, it is a sublattice of E. In fact, for every $y \in \Omega(\boldsymbol{n})$ by positivity of P we have $P^{n_j}|y| \geqslant |P^{n_j} y|$. Since $P|_{\Omega(\boldsymbol{n})}$ is an invertible isometry (the synchronous argument works here, see [14] for details), we get

$$\|P^{n_j}|y|\| \geqslant \||P^{n_j} y|\| = \|P^{n_j} y\| = \|y\| = \||y|\| \geqslant \|P^{n_j}|y|\|$$

and thus $P^{n_j}|y| = |P^{n_j}y|$ by the AL axiom. The convergence $P^{n_j}|y| \to |y|$ is now a straightforward consequence of the continuity of the modulus $|\ |$ (see [12], p. 83). Let y, z be positive normalized elements of $\Omega(\boldsymbol{n})$. It is easy to notice that (C) implies $y \wedge P^m z \neq 0$ for some $0 \leqslant m \leqslant k$. Now we define by induction two sequences $\{y_j\}$, $\{z_j\}$ of nonnegative elements of $\Omega(\boldsymbol{n})$. Let $y_0 = z_0 = y \wedge z$ and

$$(2) \qquad y_{j+1} = \Big(y - \sum_{t=0}^{j} y_t\Big) \wedge P^{j+1}\Big(z - \sum_{t=0}^{j} z_t\Big), \qquad z_{j+1} = P^{-(j+1)} y_{j+1},$$

where the inverse P^{-1} is in $\Omega(\boldsymbol{n})$.

Observe that for every $0 \leqslant m < j$ we have

$$(3) \qquad \Big(y - \sum_{t=0}^{j} y_t\Big) \wedge P^m\Big(z - \sum_{t=0}^{j} z_t\Big) = 0.$$

In fact, since $P^m z_m = y_m$, we have

$$\begin{aligned}\Big(y - \sum_{t=0}^{j} y_t\Big) \wedge P^m\Big(z - \sum_{t=0}^{j} z_t\Big) &\leqslant \Big(y - \sum_{t=0}^{m} y_t\Big) \wedge P^m\Big(z - \sum_{t=0}^{m} z_t\Big) \\ &= \Big(y - \sum_{t=0}^{m-1} y_t - y_m\Big) \wedge \Big(P^m\Big(z - \sum_{t=0}^{m-1} z_t\Big) - P^m z_m\Big) \\ &= \Big(y - \sum_{t=0}^{m-1} y_t\Big) \wedge P^m\Big(z - \sum_{t=0}^{m-1} z_t\Big) - y_m = 0.\end{aligned}$$

By assumption (C) and by (3) it follows that $y_{k+1} = y_{k+2} = \ \ = 0$ and $z_{k+1} = z_{k+2} = \ldots = 0$. Therefore, we have proved that for any positive normalized vectors y, $z \in \Omega(\boldsymbol{n})$ there are sequences $(y_j)_{j=0}^{k}$, $(z_j)_{j=0}^{k}$ such that $\sum_{j=0}^{k} y_j = y$, $\sum_{j=0}^{k} z_j = z$ and $P^j z_j = y_j$. Now, let us fix a positive normalized $y \in \Omega(\boldsymbol{n})$. So, for arbitrary positive normalized $z \in \Omega(\boldsymbol{n})$, we have $z = \sum_{j=0}^{k} P^{-j} y_j \leqslant \sum_{j=0}^{k} P^{-j} y$. Since ordered intervals are weakly compact in an AL-space (see [12], p. 119), the unit ball of $\Omega(\boldsymbol{n})$ is weakly compact. In particular, $\Omega(\boldsymbol{n})$ must be finite-dimensional as the reflexive AL-space (see [12], Corollary 2, p. 128).

Let positive normalized pairwise orthogonal $e_1, e_2, \ldots, e_s$ form a base in $\Omega(\boldsymbol{n})$. Since e_t's are extremal in B_1^+ (the nonnegative part of the unit ball of $\Omega(\boldsymbol{n})$) and $P\colon B_1^+ \to B_1^+$ is affine and invertible, there exists a permutation β of the set $\{1, 2, \ldots, s\}$ such that $Pe_t = e_{\beta(t)}$. It follows that for every $y = \sum_{t=1}^{s} \lambda_t(y) e_t$ we have $Py = \sum_{t=1}^{s} \lambda_t(y) e_{\beta(t)}$, where the coordinates λ_t are clearly nonnegative functionals. Consequently, for some $d = d(\boldsymbol{n}) \geqslant 1$ the identity $P^d|_{\Omega(\boldsymbol{n})} = \mathrm{Id}|_{\Omega(\boldsymbol{n})}$ holds. Let $\Omega(\boldsymbol{n})$, $\Omega(\boldsymbol{n}')$ be two sublattices corresponding to x, $x' \in \Omega$, respectively. It is easy to see that $P^d|_{\mathrm{sp}(\Omega(\boldsymbol{n}),\Omega(\boldsymbol{n}'))} = \mathrm{Id}|_{\mathrm{sp}(\Omega(\boldsymbol{n}),\Omega(\boldsymbol{n}'))}$ for

$d = d(n)d(n')$. In particular, we have span $(\Omega(n), \Omega(n')) \subseteq \Omega((jd)_{j=0}^{\infty})$, so $\dim\{\mathrm{span}(\Omega(n), \Omega(n'))\} \leqslant k+1$. It follows that Ω is a finite-dimensional sublattice of E, $\dim\Omega \leqslant k+1$. By $e_1, e_2, \ldots, e_r$ we denote a normalized positive pairwise orthogonal base in Ω. Arguing as before, we can show that there is a permutation α of the set $\{1, 2, \ldots, r\}$ such that $Pe_t = e_{\alpha(t)}$. Our condition (C) implies that it must be one-cyclic and $\alpha(j) = j+1 \pmod r$.

Now, for each $x \in E$, there exist a sequence $n_k \to \infty$ and coefficients $s_1(x), \ldots, s_r(x)$ such that

$$P^{n_k}x \to \sum_{j=1}^{r} s_j(x)e_j. \tag{4}$$

Choosing a subsequence of the form $n_{k_j} = k_j r + p$, the convergence (4) can be rewritten as follows:

$$\begin{aligned}\|P^{n_{k_j}}x - \sum_{j=1}^{r} s_j(x)e_j\| &= \|P^{n_{k_j}}x - \sum_{j=1}^{r} s_j(x)P^{k_j r}e_j\| \\ &= \|P^{n_{k_j}}x - P^{n_{k_j}}(\sum_{j=1}^{r} s_j(x)e_{j-p})\| = \|P^{n_{k_j}}(x - \sum_{j=1}^{r} s_{j+p}(x)e_j)\| \to 0.\end{aligned}$$

Since P is a contraction, for every $x \in E$ there exists a collection of scalars $\lambda_1(x), \ldots, \lambda_r(x)$ such that $\|P^n(x - \sum_{j=1}^{r} \lambda_j(x)e_j)\| \to 0$. To finish the proof we only have to show that $\lambda_j \in E_+^*$. The positivity of λ_j's is a simple consequence of the positivity of P. Let $x, y \in E$ be arbitrary. Since

$$\begin{aligned}0 &= \lim_{n\to\infty} P^n(x+y - \sum_{j=1}^{r} \lambda_j(x+y)e_j) \\ &= \lim_{n\to\infty} \big(P^n(x - \sum_{j=1}^{r} \lambda_j(x)e_j) + P^n(y - \sum_{j=1}^{r} \lambda_j(y)e_j) \\ &\quad + P^n(\sum_{j=1}^{r} (\lambda_j(x) + \lambda_j(y) - \lambda_j(x+y))e_j)\big),\end{aligned}$$

the third component must converge to 0. Therefore,

$$\begin{aligned}&\|\sum_{j=1}^{r} (\lambda_j(x) + \lambda_j(y) - \lambda_j(x+y))e_j\| \\ &\qquad = \lim_{n\to\infty} \|P^n(\sum_{j=1}^{r} (\lambda_j(x) + \lambda_j(y) - \lambda_j(x+y))e_j)\| = 0.\end{aligned}$$

By the linear independence of the vectors e_j's we get the additivity of λ_j's. Clearly, the homogeneity of λ_j is a straightforward consequence of the linearity of P.

COROLLARY 2. *Let P be a stochastic operator on an* AL*-space E. If for every $x \in E$ the limit set $\omega(x) \neq \emptyset$ and for every $0 \neq x_1, x_2 \geqslant 0$ there exists $n \geqslant 0$ such that $P^n x_1 \wedge P^n x_2 \neq 0$, then P is asymptotically stable.*

Proof. It is enough to observe that if in Theorem 2 the parameter k is taken to be 0, then the dimension of Ω is exactly 1, so $r = 1$.

Remark 4. In Theorem 2 above, condition (C) cannot be replaced by the following weaker one:

(C') for every nonzero x_1, $x_2 \in E_+$ there exist $n \geqslant 0$, $m \geqslant 0$ such that $P^n x_1 \wedge P^m x_2 \neq 0$,

even we additionally assume that every trajectory $\gamma(x)$ is relatively norm compact. In fact, let $\tau = \exp(2\pi is)$ for some irrational $s \in R$ and let T be the unit circle. Clearly, $P_\tau f(z) = f(\tau z)$ is a strongly almost periodic stochastic operator on $L^1(T)$. For this it is enough to observe that $\{P^n f\}$ is relatively norm compact in $C(T)$ if f is continuous, and that the imbedding $C(T) \hookrightarrow L^1(T)$ is continuous. Next notice that every power P^k is ergodic, so the space of periodic vectors in $L^1(T)$ contains only constants. Clearly, for every $f \in L^1(T)$ and every $r \in R$ we have $\|P^n(f - r\mathbf{1})\|_{L^1} = \|f - r\mathbf{1}\|_{L^1}$ so, if f is not constant, then it cannot be approximated by periodic vectors. Finally, note that Ω is the whole space $L^1(T)$ and that condition (C') is satisfied. In fact, let f, $g \in L^1_+(T)$ be arbitrary nonzero elements. If $P^n f \wedge P^m f = 0$ for every n, m, then

$$N^{-1} \sum_{j=0}^{N-1} P^j f \perp N^{-1} \sum_{j=0}^{N-1} P^j g,$$

so there would be two orthogonal $\bar{f}$, $\bar{g} \in \mathrm{Fix}(P)$. But this contradicts the ergodicity of P.

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