Proceedings of the International Conference on Oxide Materials for Electronic Engineering, May 29–June 2, 2017, Lviv

Global Maxima of the Acousto-Optic Effect in $CaWO_4$ Crystals

A.S. Andrushchak[a], O.A. Buryy[a,*], N.M. Demyanyshyn[b], Z.Yu. Hotra[a] and B.G. Mytsyk[b]

[a]Lviv Polytechnic National University, Bandery Str. 12, Lviv, 79000, Ukraine

[b]Karpenko Physico-Mechanical Institute of the NAS of Ukraine, Naukova Str. 5, Lviv, 79060, Ukraine

The global maxima of the acousto-optic interaction are theoretically determined for $CaWO_4$ crystals by extreme surfaces method. As it is shown, the highest value of the acousto-optic figure of merit M_2 is equal to 14.0×10^{-15} s^3/kg and achieved in the case of the diffraction of ordinary or extraordinary light beam on the slow quasi-transversal acoustic wave. At that the incident light wave propagates close to c-axis of the crystal and the corresponding acoustic wave propagates in the direction close to the perpendicular one.

DOI: 10.12693/APhysPolA.133.928

PACS/topics: 78.20.Fm, 78.20.Hp

1. Introduction

Acousto-optic (AO) effect, i.e. the diffraction of the electromagnetic wave on the acoustic one, is widely used in science and technique. Particularly, AO devices allow to control and process of light beam, to characterize of electromagnetic or acoustic waves, etc. [1–3]. The effectiveness of these devices strongly depends on the geometry of interaction, i.e. on the propagation and polarization directions of electromagnetic and acoustic waves. In general, the optimal geometry of AO interaction can be determined by extreme surfaces method [4–6]. Here we use this method for optimization of the AO interaction geometry (the Bragg diffraction) in $CaWO_4$ crystal which is interesting for applications in near-UV acoustooptic filters and Q-switching modulators for high-power solid-state lasers [7]. All calculations were carried out for the electromagnetic wavelength of 633 nm and the acoustic wave frequency of 100 MHz.

2. Basic relations

As it is known, the relative intensity of the diffracted light wave at the known intensity of the acoustic wave I_s is determined by expression [3]:

$$\frac{I_\nu}{I_\mu} = \sin^2\left(\frac{\pi L}{\sqrt{2}\lambda}\sqrt{M_2 I_s}\right), \tag{1}$$

where I_μ, I_ν are the intensities of the incident and diffracted waves correspondingly, L is the length of AO interaction, λ is the wavelength of the light, M_2 is the AO figure of merit,

$$M_2 = \frac{n_\mu^3 n_\nu^3}{\rho V_q^3} p_{ef}^2 \cos\gamma_\nu. \tag{2}$$

Here n_μ, n_ν are the refractive indices of the incident and diffracted light beams correspondingly, ρ is the crystal density, V_q is the phase velocity of the acoustic wave and γ is its drift angle, p_{ef} is the effective elasto-optic coefficient

$$p_{ef} = \boldsymbol{i}_\mu \boldsymbol{i}_\nu \hat{p}\, \boldsymbol{a} \boldsymbol{f}_q, \tag{3}$$

where $\boldsymbol{i}_\mu$ and $\boldsymbol{i}_\nu$ are the unit vectors of the incident and diffracted waves polarizations, $\boldsymbol{a}$ is the acoustic wave normal, $\boldsymbol{f}_q$ is the unit vector of the acoustic wave polarization, $\hat{p}$ is the tensor of elastooptic coefficients. The velocities and the polarizations of the acoustic waves V_q are determined from Christoffel's equation

$$\hat{\boldsymbol{a}} c \boldsymbol{a} \boldsymbol{f}_q = \rho V_q^2 \boldsymbol{f}_q, \tag{4}$$

where $\boldsymbol{c}$ is the tensor of elastic modulus. This equation has got three solutions corresponding to three possible waves propagating in the same direction $\boldsymbol{a}$: quasi-longitudinal (l,QL), quasi-transversal fast (f,QTF) and quasi-transversal slow (s,QTS) ones.

The polarizations of the incident and diffracted waves are easily determined for the propagation directions that do not coincide with the optical axis of the crystal. Under the usual assumption that the AO interaction does not significantly distort the optical indicatrix, the polarizations can be determined in the same way as in the non-distorted case. Particularly, if the propagation direction is defined by the angles (θ, ϕ) of the spherical coordinate system, the directions of the ordinary and extraordinary waves polarizations are determined by the angles (90°, $\phi \pm 90^o$) and ($\theta \pm 90°$, ϕ) correspondingly. However, if the light beam propagates along the optical axis of the crystal ($\theta = 0$), the polarizations are strongly determined by the distortions of the optical indicatrix. Indeed, in the undistorted case the indicatrix cross-section perpendicular to the optical axis is the circle, so all polarization directions corresponding to the angles $\theta = 90°$, $\phi = 0{-}360°$ are possible. Generally, the acousto-optic interaction distorts the cross-section from the circle to the elliptical form and only two mutually perpendicular directions correspond to the light beam polarizations. These directions can be determined as follows.

*corresponding author; e-mail: oburyi@polynet.lviv.ua

As it is known, the changes of the dielectric impermeability tensor due to the acousto-optic interaction is equal to [8]:

$$\hat{\eta} - \hat{\eta}_0 = \hat{p}\hat{\gamma}, \tag{5}$$

where $\hat{\eta}_0$ and $\hat{\eta}$ are the dielectric impermeability tensor in the absence and the presence of the acoustic wave correspondingly, $\hat{\gamma}$ is the tensor of deformations. In the case of the plane wave

$$\hat{\gamma} = \frac{2\pi}{\Lambda} A_0 \boldsymbol{a} \boldsymbol{f}_q \sin\left(\boldsymbol{K}\boldsymbol{r} - \Omega t\right), \tag{6}$$

where A_0, Λ, Ω and $\boldsymbol{K}$ are the amplitude, the wavelength, the frequency, and the wave vector of the acoustic wave, correspondingly [8]. Thus the components of the dielectric impermeability tensor can be written as

$$\eta_{ij} = \hat{\eta}_{0ij} + \Delta\eta_{ij} g\left(\boldsymbol{r}, t\right), \tag{7}$$

where

$$g\left(\boldsymbol{r}, t\right) = \frac{2\pi}{\Lambda} A_0 \sin\left(\boldsymbol{K}\boldsymbol{r} - \Omega t\right)$$

$$\Delta\eta_{ij} = \sum_{k,l=1}^{3} p_{ijkl} a_k f_{ql} (or \Delta\hat{\eta} = \boldsymbol{p}\boldsymbol{a}\boldsymbol{f}_q), \Delta\eta_{ij} = \Delta\eta_{ji}.$$

At the known $\Delta\eta_{ij}$ components the possible refractive indices n of the beam propagating in the direction $\boldsymbol{m} \equiv (m_1, m_2, m_3)$ are determined from the quadratic equation [9]:

$$\begin{vmatrix} \eta_{11} - \chi & \eta_{12} & \eta_{13} & m_1 \\ \eta_{12} & \eta_{22} - \chi & \eta_{23} & m_2 \\ \eta_{13} & \eta_{23} & \eta_{33} - \chi & m_3 \\ m_1 & m_2 & m_3 & 0 \end{vmatrix} = 0, \tag{8}$$

where $\chi = n^{-2}$.

If the light beam propagates along the optical axis, $m_1 = m_2 = 0$, $m_3 = 1$ and Eq. (8) comes to

$$\begin{vmatrix} \eta_{11} - \chi & \eta_{12} \\ \eta_{12} & \eta_{22} - \chi \end{vmatrix} = 0. \tag{9}$$

In the undistorted case $\eta_{12} = \eta_{23} = 0$, $\eta_{11} = \eta_{22} = \eta_0$ for the uniaxial crystal and the solution of (8) is $\chi = \chi_0 = \eta_0$. Denoting $\Delta\chi = \chi - \chi_0 = \chi - \eta_0$ one can write Eq. (9) as

$$\begin{vmatrix} \Delta\eta_{11} g\left(\boldsymbol{r}, t\right) - \Delta\chi & \Delta\eta_{12} g\left(\boldsymbol{r}, t\right) \\ \Delta\eta_{12} g\left(\boldsymbol{r}, t\right) & \Delta\eta_{22} g\left(\boldsymbol{r}, t\right) - \Delta\chi \end{vmatrix} = 0, \tag{10}$$

or

$$\begin{vmatrix} \Delta\eta_{11} - \Delta\chi^* & \Delta\eta_{12} \\ \Delta\eta_{12} & \Delta\eta_{22} - \Delta\chi^* \end{vmatrix} = 0, \tag{11}$$

where $\Delta\chi^* = \frac{\Delta\chi}{g(\boldsymbol{r},t)}$.

The solutions of (11) are

$$\Delta\chi^*_{\pm} = \tag{12}$$

$$0.5\left(\Delta\eta_{11} + \Delta\eta_{22} \pm \sqrt{\left(\Delta\eta_{11} - \Delta\eta_{22}\right)^2 + 4\Delta\eta_{12}^2}\right),$$

and the corresponding refractive indices are equal to

$$n_{\pm} = \left(\eta_0 + \Delta\chi^*_{\pm} g\left(\boldsymbol{r}, t\right)\right)^{-1/2}. \tag{13}$$

The directions of polarization $\boldsymbol{i} \equiv (i_1, i_2, i_3)$ are determined from the equation [9]:

$$\begin{cases} \left(\eta_{11} - \chi\right) i_1 + \eta_{12} i_2 + \eta_{13} i_3 + m_1 \mu = 0, \\ \eta_{12} i_1 + \left(\eta_{22} - \chi\right) i_2 + \eta_{23} i_3 + m_2 \mu = 0, \\ \eta_{13} i_1 + \eta_{23} i_2 + \left(\eta_{33} - \chi\right) i_3 + m_3 \mu = 0, \\ m_1 i_1 + m_2 i_2 + m_3 i_3 = 0, \end{cases} \tag{14}$$

under the condition $i_1^2 + i_2^2 + i_3^2 = 1$, where μ is the Lagrange multiplier. If the light beam propagates along the optical axis, $m_1 = m_2 = 0$, $m_3 = 1$, $i_3 = 0$ and the solutions of (14) are

$$i_{1\pm} = \left(1 + \frac{\left(\Delta\eta_{11} - \Delta\chi^*_{\pm}\right)^2}{\Delta\eta_{12}^2}\right)^{-1}, \tag{15}$$

$$i_{2\pm} = \left(1 + \frac{\left(\Delta\eta_{22} - \Delta\chi^*_{\pm}\right)^2}{\Delta\eta_{12}^2}\right)^{-1}, \tag{16}$$

where $\Delta\chi^*_{\pm}$ is determined by (12). It should be noted that the components of the unit vectors of polarization $\boldsymbol{i}$ do not depend on $\boldsymbol{r}$ or t. Such dependences take place for the refractive indices (see (13)), however, because the average value of $n_{\pm}$ is equal to n_o, in our calculations we assumed that the refractive index for the light wave propagating along the optical axis is still equal to n_o.

So for such a light wave the possible polarizations are determined in three steps: (i) the calculation of the values $\Delta\eta_{ij} = \sum_{k,l=1}^{3} p_{ijkl} a_k f_{ql}$, (ii) the determination of $\Delta\chi^*_{\pm}$ in accordance with (12), (iii) the determination of the vectors $\boldsymbol{i}$ in accordance with (15), (16). It should be emphasized that the analysis of the AO figure of merit for the light waves propagating along the c-axis was not carried out in our previous works [4, 5, 7], where this direction were omitted during calculation.

The parameters of $CaWO_4$ crystal used in our calculations are given in Table I.

Parameters of $CaWO_4$ crystal TABLE I

Parameters	Values	References
Density [kg/m^3]	6150	[10]
Main indices of refraction ($\lambda = 633$ nm)	$n_o = 1.920$; $n_e = 1.936$	[11]
Elastic modulus [GPa]	$C_{11} = 145.9$; $C_{12} = 62.6$; $C_{13} = 39.2$; $C_{16} = -19.2$; $C_{33} = 127.4$; $C_{44} = 33.5$; $C_{66} = 38.7$	[12]
Elastooptic coefficients	$p_{11} = 0.215$; $p_{12} = 0.17$; $p_{13} = 0.24$; $p_{16} = 0.11$; $p_{31} = 0.25$; $p_{33} = 0.21$; $p_{44} = 0.11$; $p_{45} = -0.34$; $p_{61} = 0.025$; $p_{66} = -0.31$	[13]

The elastooptic coefficients of $CaWO_4$ crystal given in this table were calculated from the values of piezooptic coefficients measured by interferometric method [13–15].

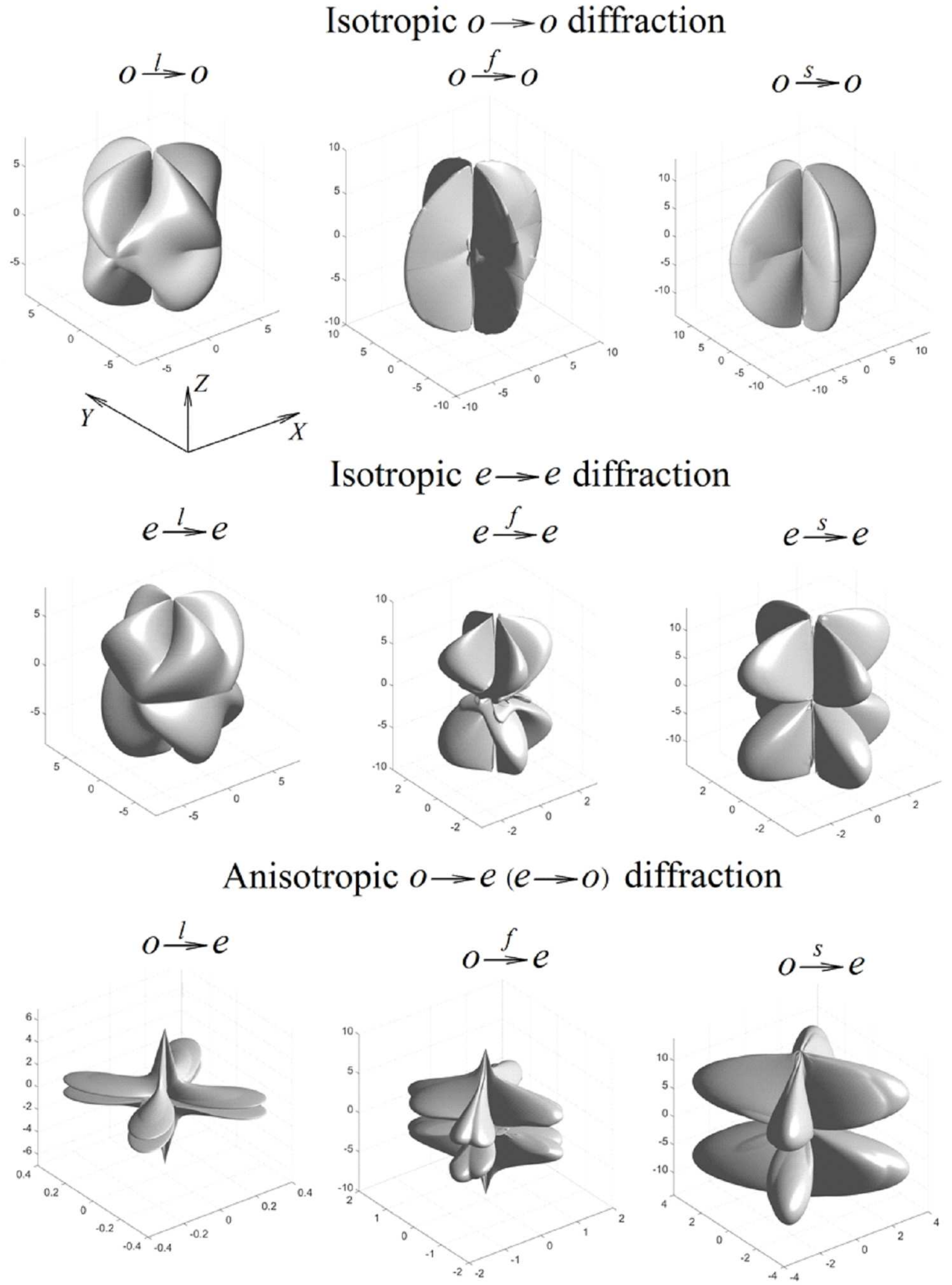

Fig. 1. The extreme surfaces for the different types of diffraction. The units on all axes are 10^{-15} s^3/kg. The letters over the arrows denote the type of acoustic wave.

The method used for optimization consists in analysis of the surfaces constructed in the following way. For each propagation direction of the incident light wave given by the spherical angles θ_μ and ϕ_μ, we calculate the maximal M_2 value obtained after optimizing over the angles θ_a, ϕ_a specifying propagation direction of the acoustic wave.

At that the momentum conservation law

$$\boldsymbol{k}_\nu = \boldsymbol{k}_\mu \pm \boldsymbol{K} \tag{17}$$

where $\boldsymbol{k}_\nu$, $\boldsymbol{k}_\mu$ and $\boldsymbol{K}$ are the wave vectors of the electromagnetic diffracted (ν), electromagnetic incident (μ) and acoustic waves respectively, limit the region where searching of the optimal angles θ_a, ϕ_a must be carried out. The dependences of the maximal M_2 values on angles θ_μ and ϕ_μ are presented by special type ("extreme") surfaces. Full characterization of AO interaction requires construction of 12 extreme surfaces because of four possible types of diffraction depending on polarizations of light beams and three possible types of acoustic wave (l, f or s).

TABLE II

Results of optimization

Type of acoustic wave	Directions of light waves propagation		Acoustic wave			$\lvert p_{eff}\rvert$	$M_{2\,\max}^{extr}$, 10^{-15} s^3/kg
	Incident wave, $\theta_{\mu max}$, $\phi_{\mu max}$ [deg]	Diffracted wave, $\theta_{\nu max}$, $\phi_{\nu max}$ [deg]	Direction of propagation, θ_a, ϕ_a [deg]	Polarization, θ_f, ϕ_f [deg]	Velocity, V_q [m/s]		
Isotropic $o \to o$							
QL	34; 86	34.3; 85.4	110.2; 28.6	106.7; 33.5	4470	0.29	7.8
QTF	61; 88	61; 88.9	90; 178.5	90; 79.1	2336	0.14	9.5
QTS	7; 1	6.8; 353	91.6; 73.7	89.5; 161	1883	0.11	14.0
Isotropic $e \to e$							
QL	36; 83	36.4; 83.4	120.3; 118.7	117.1; 123	4404	0.29	8.3
QTF	1; 87	0.2; 78.8	89.4; 89.1	99.4; 350	2360	0.15	9.3
QTS	1; 1	2; 352	91.4; 344	89.6; 71	1883	0.11	14.0
Anisotropic $o \to e$							
QL	along Z	0.4; 293	90.2; 293	90.2; 293	4512	0.26	6.0
QTF	along Z	0.8; 359	90.4; 359	98.0; 79.4	2355	0.15	9.4
QTS	0.3; 83.1	1; 1	89.6; 163.6	89.9; 70.9	1882	0.11	14.0

3. Results and discussion

The extreme surfaces for AO interaction in $CaWO_4$ crystal are shown in Fig. 1. As well as in [4, 5], the extreme surfaces for $o \to e$ and $e \to o$ diffractions on the same acoustic waves are visually similar at the frequency of 100 MHz, so they are shown for $o \to e$ case only.

The results of optimization are indicated in Table II. As it is shown in [4], the maximal values of M_2 must be the same for the $o \to e$ and $e \to o$ diffraction, so only the results for $o \to e$ case are given in Table II. As it is seen from the table, the maximal value of the AO figure of merit $M_{2\,\max}^{extr}$ achieved in the cases of the diffraction on the slow quasi-transversal wave. The obtained maximal value are the same both for isotropic ($o \to o$ or $e \to e$) or anisotropic ($o \to e$ or $e \to o$) diffraction because the difference between the diffraction types becomes unessential for the beam propagating in the direction close to c-axis. The corresponding optimal directions of acoustic waves are close to the perpendicular one for all types of diffraction. The velocities of these waves are about 1882–1883 m/s and the corresponding effective elastooptic coefficients p_{ef} are equal to 0.11 that corresponds to the value of the elasto-optic coefficient p_{16} of $CaWO_4$ crystal.

As it is followed from the data given in Table II, the highest effective elasto-optic coefficient (corresponding to $M_{2\,\max}^{extr}$) is achieved in the cases of isotropic diffraction ($o \to o$ or $e \to e$) on the quazi-longitudional acoustic wave. The obtained value of $p_{eff} = 0.29$ is higher than the separate values of elasto-optic coefficients of $CaWO_4$ crystal (see Table I). The corresponding value of M_2 is almost twice lower than the highest achievable one however, this case is interesting because of high acoustic wave velocity. Indeed, as it is known [8] the low velocities of the acoustic wave usually correspond to the relatively high attenuation, so it is preferable to ensure the maximum of M_2 due to the high value of the elasto-optic coefficient firstly.

TABLE III

The acousto-optical figure of merit M_2 for different crystals

Crystal	UV absorption edge [nm]	M_2 10^{-15} s^3/kg	Reference
$CaWO_4$	130	14.0 (633 nm)	this work
SiO_2	200	1.56 (630 nm)*	[8]
$LiNbO_3$	400	15.9 (633 nm)	[4]
$LiTaO_3$	300	1.37 (630 nm)*	[16]
$La_3Ga_5SiO_{14}$	324	5.1 (633 nm)	[17]
SrB_4O_7	130	0.63 (633 nm)	[5]
TeO_2	360	34.5 (630 nm)*	[8]
GaP	600	44.6 (630 nm)*	[8]
KH_2PO_4	250	7.1 (632.8 nm)	[18]

*Measured values; the other values indicated in the table are calculated in accordance with (2).

The comparison of the obtained value with the ones of other acousto-optic materials (some of them are indicated in Table III) shows that $CaWO_4$ crystal successfully joins the high enough acousto-optic figure of merit with the high transmittance in the UV spectral region (to 130 nm). The obtained value of M_2 is in order higher than the one of other materials (SiO_2, SrB_4O_7) which are also useful for UV acousto-optic modulators, so calcium tungstate is promising for AO devices function in the near-UV spectral region. Moreover, as it follows from our preliminary estimations, the attenuation of the acoustic waves in $CaWO_4$ crystal are relatively low, that can be the additional advantage of this material.

4. Conclusions

The maxima of the AO interaction are determined for $CaWO_4$ crystal by extreme surfaces method. As it is shown the maximal value of the acousto-optical figure of merit M_2 is equal to 14.0×10^{-15} s^3/kg achieved in the case of diffraction (isotropic or anisotropic) on the slow quasi-transversal wave. At that the incident light wave propagates in the direction close to c-axis of the crystal and the corresponding acoustic wave propagates in the direction close to the perpendicular one with the velocity of about 1882–1883 m/s. The corresponding effective elastooptic coefficient p_{ef} is about 0.11 for all types of diffraction. Nevertheless, the maxima of isotropic diffraction ($o \to o$ or $e \to e$) on the quasi-longitudinal acoustic wave correspond to the value of the effective elasto-optic coefficient $p_{eff} = 0.29$ that is higher than the separate values of elasto-optic coefficients of $CaWO_4$ crystal.

The high values of the acousto-optical figure of merit and the transmittance in the spectral region to 130 nm indicate calcium tungstate crystal as a promising material for AO devices function in the near-UV spectral region.

Acknowledgments

This research was supported by Ministry of Education and Science of Ukraine in the frames of projects "Anisotropy" (registration number is 0116U004136) and "Modulator" (registration number is 0117U004847).

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