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On the vanishing of Iwasawa invariants of absolutely abelian p-extensions

by

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1. Introduction. Let p be a prime number and $\mathbb{Z}_p$ the ring of p-adic integers. Let k be a finite extension of the rational number field $\mathbb{Q}$, k_∞ a $\mathbb{Z}_p$-extension of k, k_n the nth layer of k_∞/k, and A_n the p-Sylow subgroup of the ideal class group of k_n. Iwasawa proved the following well-known theorem about the order $\#A_n$ of A_n:

Theorem A (Iwasawa). *Let k_∞/k be a $\mathbb{Z}_p$-extension and A_n the p-Sylow subgroup of the ideal class group of k_n, where k_n is the nth layer of k_∞/k. Then there exist integers $\lambda = \lambda(k_\infty/k) \geq 0$, $\mu = \mu(k_\infty/k) \geq 0$, $\nu = \nu(k_\infty/k)$, and $n_0 \geq 0$ such that*

$$\#A_n = p^{\lambda n + \mu p^n + \nu}$$

for all $n \geq n_0$, where $\#A_n$ is the order of A_n.

These integers $\lambda = \lambda(k_\infty/k)$, $\mu = \mu(k_\infty/k)$ and $\nu = \nu(k_\infty/k)$ are called *Iwasawa invariants* of k_∞/k for p. If k_∞ is the cyclotomic $\mathbb{Z}_p$-extension of k, then we denote λ (resp. μ and ν) by $\lambda_p(k)$ (resp. $\mu_p(k)$ and $\nu_p(k)$).

Ferrero and Washington proved $\mu_p(k) = 0$ for any abelian extension field k of $\mathbb{Q}$. On the other hand, Greenberg [4] conjectured that if k is a totally real, then $\lambda_p(k) = \mu_p(k) = 0$. We call this conjecture *Greenberg's conjecture.*

In this paper, we determine all absolutely abelian p-extensions k with $\lambda_p(k) = \mu_p(k) = \nu_p(k) = 0$ for an odd prime p, by using the results of G. Cornell and M. Rosen [1].

2. Main theorem. Throughout this section, we fix an odd prime number p. Let k be an abelian p-extension of $\mathbb{Q}$ and m_k its *conductor*, i.e. m_k is the minimum positive integer with $k \subseteq \mathbb{Q}(\zeta_{m_k})$, where ζ_{m_k} is a primitive m_kth root of unity. Then it follows easily that $m_k = p^a p_1 \ldots p_t$, where a is a non-negative integer and $p_1, \ldots, p_t$ are distinct primes which are congruent

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to 1 modulo p. We denote by k_G the genus field of $k/\mathbb{Q}$. So k_G is the maximal unramified abelian extension of k such that $k_G/\mathbb{Q}$ is an abelian extension. In general, if $k/\mathbb{Q}$ is an abelian extension of odd degree, then it has been shown by Leopoldt that

$$[k_G : k] = \frac{e_1 \ldots e_t}{[k : \mathbb{Q}]},$$

where $e_1, \ldots, e_t$ are the ramification indices of the primes which ramify in $k/\mathbb{Q}$. Hence, in our case, k_G is also an abelian p-extension of $\mathbb{Q}$. Now, let x and y be integers. We denote by $\left(\frac{x}{y}\right)_p$ the pth power residue symbol. Namely, $\left(\frac{x}{y}\right)_p = 1$ means that x is the pth power of some integer modulo y.

Theorem 1. *Let k be an abelian p-extension of $\mathbb{Q}$, and $m_k = p^a p_1 \ldots p_t$ the prime decomposition of its conductor, where the primes $p_1, \ldots, p_t$ are distinct. If*

$$\lambda_p(k) = \mu_p(k) = \nu_p(k) = 0, \tag{1}$$

then $t \le 2$. Conversely, assume that $t \le 2$.

- *If $t = 0$, then the condition* (1) *holds.*
- *If $t = 1$, then the condition* (1) *holds if and only if $k_G \subseteq k_\infty$ and*

$$\left(\frac{p}{p_1}\right)_p \ne 1 \quad \text{or} \quad p_1 \not\equiv 1 \pmod{p^2}. \tag{2}$$

- *If $t = 2$, then the condition* (1) *holds if and only if $k_G \subseteq k_\infty$, and for $(i,j) = (1,2)$ or $(2,1)$,*

$$\left(\frac{p}{p_i}\right)_p \ne 1, \quad \left(\frac{p_i}{p_j}\right)_p \ne 1, \quad p_j \not\equiv 1 \pmod{p^2}, \tag{3}$$

and there exist $x, y, z \in \mathbb{F}_p$ such that

$$\left(\frac{p_j p^x}{p_i}\right)_p = 1, \quad \left(\frac{p p_i^y}{p_j}\right)_p = 1, \quad p_i p_j^z \equiv 1 \pmod{p^2}, \quad xyz \ne -1 \text{ in } \mathbb{F}_p. \tag{4}$$

In the case $t = 2$, the conditions in Theorem 1 are complicated. So we will give an example. We consider the case $p = 3$, $p_1 = 7$ and $p_2 = 19$. We denote by $k(7)$ (resp. $k(19)$) the subfield of $\mathbb{Q}(\zeta_7)$ (resp. $\mathbb{Q}(\zeta_{19})$) with degree 3 over $\mathbb{Q}$. As for the condition $k_G \subseteq k_\infty$, we consider the following field F: There exists a field F such that $k(7) \subsetneq F \subsetneq k(7)k(19)\mathbb{Q}_1$ and $F \ne k(7)k(19), k(7)\mathbb{Q}_1$, where $\mathbb{Q}_1$ is the first layer of the cyclotomic $\mathbb{Z}_3$-extension of $\mathbb{Q}$. Then $m_F = 3 \cdot 7 \cdot 19$ and $k(7)k(19)\mathbb{Q}_1/F$ is a non-trivial unramified extension. Since $k(7)k(19)\mathbb{Q}_1/\mathbb{Q}$ is abelian, $F \subsetneq k(7)k(19)\mathbb{Q}_1 \subseteq F_G$. But, for $F_1 = k(7)k(19)\mathbb{Q}_1$, it follows easily that $F_1 = F_{1,G}$. Hence $F_G \subseteq F_{1,G} = F_1 \subseteq F_\infty$. So, F satisfies the first condition of Theorem 1 (in the case of $t = 2$).

If we consider only the case where p is unramified in k, i.e. $a = 0$, then the statement $k_G \subseteq k_\infty$ can be simplified to $k = k_G$ because $k_1 = k\mathbb{Q}_1$. This restriction is not very strong: In general, for an absolutely abelian p-extension field k, there exists an absolutely abelian extension field k' such that p is unramified in k' and $k_\infty = k'_\infty$. For the above field F, $F' = k(7)k(19)$ satisfies $F_\infty = F'_\infty$ (in fact we have $F_1 = F'_1$) and 3 is unramified in F'.

We continue to examine the above example. If we put $(i, j) = (1, 2)$, then $p_j = 19 \equiv 1 \pmod{3^2}$, so the condition (3) is not satisfied. But if we put $(i, j) = (2, 1)$, then we can verify that $p_i = 19$ and $p_j = 7$ satisfy the conditions (3) and (4). Therefore F satisfies $\lambda_p(F) = \mu_p(F) = \nu_p(F) = 0$.

Also, if K is the maximal subfield of $\mathbb{Q}(\zeta_{7\cdot19})$ which is a 3-extension of $\mathbb{Q}$, then K satisfies the conditions of Theorem 1. (Note that, in general, if k is the maximal subfield of $\mathbb{Q}(\zeta_m)$ ($m = p^a p_1 \ldots p_t$ as above) which is an abelian p-extension of $\mathbb{Q}$, then it follows that $k = k_G$.) Therefore we have

$$\lambda_p(K) = \mu_p(K) = \nu_p(K) = 0.$$

As for the Greenberg conjecture, we can also get the following: In general, it is known that if $L \subseteq M$ then $\lambda_p(L) \le \lambda_p(M)$ and $\mu_p(L) \le \mu_p(M)$ for number fields L, M. Hence for any subfield k of $\mathbb{Q}(\zeta_{7\cdot19})$ which is a 3-extension of $\mathbb{Q}$, i.e. $k \subseteq K$, we have $\lambda_p(k) = \mu_p(k) = 0$. This consideration is generalized as follows:

COROLLARY 2. *Let $m = p^a p_1 \ldots p_t$ satisfy the condition either* (2) (*in the case $t = 1$*) *or* (3) *and* (4) (*in the case $t = 2$*) *of Theorem* 1. *Then for any subfield k of $\mathbb{Q}(\zeta_m)$ which is a p-extension of $\mathbb{Q}$, the Greenberg conjecture for k and p is valid.*

3. The results of G. Cornell and M. Rosen. In this section, we recall some results of [1]. Let p be an odd prime number and $K/\mathbb{Q}$ an abelian p-extension. Then the genus field K_G of $K/\mathbb{Q}$ is also an abelian p-extension of $\mathbb{Q}$. If p does not divide the class number h_K of K, then K does not have any non-trivial unramified abelian p-extension by class field theory, hence $K_G = K$. In the following we will assume $K_G = K$. Furthermore, we introduce the central p-class field K_C of K, i.e. K_C is the maximal p-extension of K such that K_C/K is an unramified abelian p-extension, $K_C/\mathbb{Q}$ is Galois and $\mathrm{Gal}(K_C/K)$ is in the center of $\mathrm{Gal}(K_C/\mathbb{Q})$. Since a p-group must have a lower central series that terminates in the identity, one sees that $p \nmid h_K$ if and only if $K_C = K$. We can reduce our problem to the case where $\mathrm{Gal}(K/\mathbb{Q})$ is an elementary abelian p-group by the following result:

LEMMA 3 ([1], Theorem 1). *Let $K/\mathbb{Q}$ be an abelian p-extension with $K_G = K$. Let k be the maximal intermediate extension between $\mathbb{Q}$ and K*

such that $\mathrm{Gal}(k/\mathbb{Q})$ *is an elementary abelian p-group. Then the p-rank of* $\mathrm{Gal}(K_C/K)$ *is equal to the p-rank of* $\mathrm{Gal}(k_C/k)$.

Moreover, we have the following lemma by Furuta and Tate:

LEMMA 4 ([1], Section 1). *Let K be an absolutely abelian p-extension such that* $\mathrm{Gal}(K/\mathbb{Q})$ *is an elementary abelian p-group and $K_G = K$. Then*

$$\mathrm{Gal}(K_C/K) \simeq \mathrm{Coker}\left(\bigoplus_{i=1}^{n} \bigwedge^2(G_i) \to \bigwedge^2(G)\right),$$

where G_i's are the decomposition groups of the primes ramified in $K/\mathbb{Q}$ and $G = \mathrm{Gal}(K/\mathbb{Q})$.

We assume $\mathrm{Gal}(K/\mathbb{Q}) \simeq (\mathbb{Z}/p\mathbb{Z})^m$. Let $p_1, \ldots, p_t$ be the primes ramifying in K and h_K the class number of K. From genus theory, it follows that if h_K is not divisible by p, then $t = m$. It follows that if $m \geq 4$ then p divides h_K by Lemma 4. So, we assume $t = m$ and $m = 2$ or 3. (If $t = m = 1$, then $p \nmid h_K$, cf. [5].)

LEMMA 5 ([1], Proposition 2). *Suppose $m = 2$ and $p_i \neq p$ for $i = 1, 2$. Then $p \mid h_K$ if and only if $\left(\frac{p_1}{p_2}\right)_p = 1$ and $\left(\frac{p_2}{p_1}\right)_p = 1$.*

Next, we consider the case where p ramifies in $K/\mathbb{Q}$. Suppose $m = 2$ and primes p and p_1 are the only primes ramified in K. Then $K = k(p_1)\mathbb{Q}_1$ and $p_1 \equiv 1 \pmod{p}$, where $k(p_1)$ is the unique subfield of $\mathbb{Q}(\zeta_{p_1})$ which is cyclic over $\mathbb{Q}$ of degree p, and $\mathbb{Q}_1$ is the first layer of the cyclotomic $\mathbb{Z}_p$-extension of $\mathbb{Q}$.

LEMMA 6 ([1], Proposition 3). *Suppose $m = 2$ and primes p and p_1 are the only primes ramified in K. Then $p \mid h_K$ if and only if $\left(\frac{p}{p_1}\right)_p = 1$ and $p_1 \equiv 1 \pmod{p^2}$.*

Next, suppose that $t = m = 3$ and p_1, p_2 and p_3 are all the primes ramified in K. Denote by D_{p_i} the decomposition field of p_i $(i = 1, 2, 3)$ in K. In [1], the following result is given:

LEMMA 7 ([1], Theorem 2). *Suppose $t = m = 3$. Then the following statements are equivalent*:

(a) *h_K is not divisible by p.*
(b) *$[D_{p_1} : \mathbb{Q}] = [D_{p_2} : \mathbb{Q}] = [D_{p_3} : \mathbb{Q}] = p$ and $D_{p_1}D_{p_2}D_{p_3} = K$.*

In the next section, we shall prove Theorem 1 using these results.

4. Proof of Theorem 1. Notations are as in the previous section.

First, suppose $\lambda_p(k) = \mu_p(k) = \nu_p(k) = 0$. Clearly, this condition is equivalent to $A(k_n) = 0$ for any sufficiently large n. Then k_n satisfies $k_n = k_{n,G}$. So it follows easily that $k_G \subseteq k_{n,G} = k_n \subseteq k_\infty$. Since k_n is also an abelian p-extension of $\mathbb{Q}$, we can apply the results of Cornell–Rosen:

Let L be the maximal subfield of k_n such that $\mathrm{Gal}(L/\mathbb{Q})$ is an elementary abelian extension of $\mathbb{Q}$. Since $k_n = k_{n,G}$, $\mathrm{Gal}(k_n/\mathbb{Q})$ is the direct sum of the inertia groups of primes ramified in $k_n/\mathbb{Q}$. Hence it follows that $L = k(p_1)\ldots k(p_t)\mathbb{Q}_1$. By Lemma 3, $A(k_n) = 0$ is equivalent to $p \nmid h_L$. Note that if $t \geq 3$ then we always have $p \mid h_L$ by Lemma 4. Hence we may examine each case, $t = 0$ or 1 or 2.

If $t = 0$ then $L = \mathbb{Q}_1$, hence it is well known that $A(L) = A(\mathbb{Q}_1) = 0$ (cf. [5]).

If $t = 1$ then $L = k(p_1)\mathbb{Q}_1$. By Lemma 6, we get the statement of Theorem 1.

In the following, we assume $t = 2$. In this case, $L = k(p_1)k(p_2)\mathbb{Q}_1$. Let G_p, G_{p_i} $(i = 1, 2)$ be the decomposition groups for p, p_i in $\mathrm{Gal}(L/\mathbb{Q})$ and D_p, D_{p_i} the fixed field of G_p, G_{p_i}, respectively. We note that $D_p \subset k(p_1)k(p_2)$, $D_{p_1} \subset k(p_2)\mathbb{Q}_1$ and $D_{p_2} \subset k(p_1)\mathbb{Q}_1$.

Now, $p \nmid h_L$ shows $[D_p : \mathbb{Q}] = [D_{p_1} : \mathbb{Q}] = [D_{p_2} : \mathbb{Q}] = p$ and $D_pD_{p_1}D_{p_2} = L$ by Lemma 7. Here, we assume that either $\left(\frac{p}{p_1}\right)_p = 1$ or $\left(\frac{p_1}{p_2}\right)_p = 1$ or $p_2 \equiv 1 \pmod{p^2}$, and either $\left(\frac{p}{p_2}\right)_p = 1$ or $\left(\frac{p_2}{p_1}\right)_p = 1$ or $p_1 \equiv 1 \pmod{p^2}$. This is equivalent to

$$(5)\quad D_p = k(p_i) \text{ or } D_{p_i} = k(p_j) \text{ or } D_{p_j} = \mathbb{Q}_1 \quad \text{for } (i,j) = (1,2) \text{ and } (2,1),$$

because $[D_p : \mathbb{Q}] = [D_{p_1} : \mathbb{Q}] = [D_{p_2} : \mathbb{Q}] = p$.

If $D_p = k(p_1)$, then $D_{p_2} \neq k(p_1)$ because $D_pD_{p_1}D_{p_2} = L$. Hence by (5) (put $(i,j) = (2,1)$), we have $D_{p_1} = \mathbb{Q}_1$. Then $D_{p_2} \subseteq k(p_1)\mathbb{Q}_1 = D_pD_{p_1}$, a contradiction $D_pD_{p_1}D_{p_2} = L$. In the same way, if $D_p = k(p_2)$, then $D_{p_1} \neq k(p_2)$ and we have $D_{p_2} = \mathbb{Q}_1$ by (5), a contradiction. Thus, it follows that the assumption (5) contradicts $D_pD_{p_1}D_{p_2} = L$. Therefore, for $(i,j) = (1,2)$ or $(2,1)$, $\left(\frac{p}{p_i}\right)_p \neq 1$, $\left(\frac{p_i}{p_j}\right)_p \neq 1$, and $p_j \not\equiv 1 \pmod{p^2}$.

Without loss of generality, we may assume $(i,j) = (1,2)$. Since $\left(\frac{p}{p_1}\right)_p \neq 1$, p is inert in $k(p_1)$. Hence $\sigma = \left(\frac{k(p_1)/\mathbb{Q}}{p}\right) \neq 1$, where $\left(\frac{k(p_1)/\mathbb{Q}}{p}\right)$ is the Artin symbol, and σ generates $\mathrm{Gal}(k(p_1)/\mathbb{Q})$: $\langle\sigma\rangle = \mathrm{Gal}(k(p_1)/\mathbb{Q})$. We often regard $\langle\sigma\rangle = \mathrm{Gal}(k(p_1)k(p_2)/k(p_2))$ or $\mathrm{Gal}(L/k(p_2)\mathbb{Q}_1)$ in the natural way. Similarly, we put $\tau = \left(\frac{k(p_2)/\mathbb{Q}}{p_1}\right)$ and $\eta = \left(\frac{\mathbb{Q}_1/\mathbb{Q}}{p_2}\right)$. Then $\langle\tau\rangle = \mathrm{Gal}(k(p_2)/\mathbb{Q})$ and $\langle\eta\rangle = \mathrm{Gal}(\mathbb{Q}_1/\mathbb{Q})$.

Since $\left(\frac{p}{p_1}\right)_p \neq 1$, there exists $x \in \mathbb{F}_p$ such that $\left(\frac{p_2p^x}{p_1}\right)_p = 1$. Then

$$\left(\frac{p_2p^x}{p_1}\right)_p = 1 \Leftrightarrow \left(\frac{k(p_1)/\mathbb{Q}}{p_2p^x}\right) = \left(\frac{k(p_1)/\mathbb{Q}}{p_2}\right)\left(\frac{k(p_1)/\mathbb{Q}}{p}\right)^x = 1.$$

Therefore $\left(\frac{k(p_1)/\mathbb{Q}}{p_2}\right) = \sigma^{-x}$. Similarly, there exist $y, z \in \mathbb{F}_p$ such that $\left(\frac{pp_1^y}{p_2}\right)_p = 1$ and $p_1p_2^z \equiv 1 \pmod{p^2}$. Hence $\left(\frac{k(p_2)/\mathbb{Q}}{p}\right) = \tau^{-y}$ and $\left(\frac{\mathbb{Q}_1/\mathbb{Q}}{p_1}\right) = \eta^{-z}$.

Since $\left(\frac{k(p_1)k(p_2)/\mathbb{Q}}{p}\right) = \left(\frac{k(p_1)/\mathbb{Q}}{p}\right)\left(\frac{k(p_2)/\mathbb{Q}}{p}\right) = \sigma\tau^{-y}$, D_p is the fixed field of $\langle\sigma\tau^{-y}\rangle$ in $k(p_1)k(p_2)$. Therefore, when we consider G_p in $\mathrm{Gal}(L/\mathbb{Q})$,

$$G_p = \langle\eta, \sigma\tau^{-y}\rangle.$$

And similarly,

$$G_{p_1} = \langle\sigma, \tau\eta^{-z}\rangle \quad \text{and} \quad G_{p_2} = \langle\tau, \eta\sigma^{-x}\rangle,$$

in $\mathrm{Gal}(L/\mathbb{Q})$. By a direct computation, $G_p \cap G_{p_1} = \langle\sigma\tau^{-y}\eta^{yz}\rangle$. Hence,

$$\begin{aligned} G_p \cap G_{p_1} \cap G_{p_2} &= \langle\sigma\tau^{-y}\eta^{yz}\rangle \cap \langle\tau, \eta\sigma^{-x}\rangle \\ &= \begin{cases} \{1\} & \text{if } xyz \neq -1, \\ \langle\sigma\tau^{-y}\eta^{yz}\rangle & \text{if } xyz = -1. \end{cases} \end{aligned}$$

But our assumption $D_pD_{p_1}D_{p_2} = L$ implies $G_p \cap G_{p_1} \cap G_{p_2} = \{1\}$. Hence $xyz \neq -1$.

Conversely, we assume k satisfies the conditions of Theorem 1 in the case of $t = 2$. Since $k_G = k_\infty$, it follows easily that $L = k(p_1)k(p_2)\mathbb{Q}_1$ is the maximal intermediate extension between $\mathbb{Q}$ and k_n (for a sufficiently large n) such that $\mathrm{Gal}(L/\mathbb{Q})$ is an elementary abelian p-group. Without loss of generality, we may assume $(i, j) = (1, 2)$. Since $\mathrm{Gal}(k(p_1)k(p_2)/\mathbb{Q}) \simeq (\mathbb{Z}/p\mathbb{Z})^2$ and p is unramified in $k(p_1)k(p_2)$, p must decompose in $k(p_1)k(p_2)$. But the condition $\left(\frac{p}{p_1}\right)_p \neq 1$ implies that p is inert in $k(p_1) \subset k(p_1)k(p_2)$, hence we obtain $[D_p : \mathbb{Q}] = p$. Similarly, $\left(\frac{p_1}{p_2}\right)_p \neq 1$ and $p_2 \not\equiv 1 \pmod{p^2}$ imply $[D_{p_1} : \mathbb{Q}] = [D_{p_2} : \mathbb{Q}] = p$. Therefore, as in the above computation of G_p, G_{p_i}, we have $D_pD_{p_1}D_{p_2} = L$ by $xyz \neq -1$. ∎

5. Remarks. The condition of Theorem 1 in [6] means $xyz = 0$ which is a special case of $xyz \neq -1$. Hence, our Corollary 2 contains some known results and there exist infinitely many fields satisfying the conditions of Theorem 1 (cf. [6]).

If $K = k(p_1)k(p_2)$ satisfies the conditions of Theorem 1, then $\lambda_p(k) = \mu_p(k) = 0$ for any field $k \subseteq K$ with $[k : \mathbb{Q}] = p$. This is a result of Fukuda [2]. The case $xyz = -1$ is a more difficult case. But we have some results:

PROPOSITION 8. *Notations are as in Section* 3. *Assume that* $\left(\frac{p}{p_1}\right)_p \neq 1$, $\left(\frac{p_1}{p_2}\right)_p \neq 1$, *and* $p_2 \not\equiv 1 \pmod{p^2}$. *Then* $\lambda_p(k) = \mu_p(k) = 0$ *for the decomposition field* k *of* p *in* $k(p_1)k(p_2)$.

Proof. We apply a result of [3]:

LEMMA 9 ([3]). *Let* k *be a cyclic extension of* $\mathbb{Q}$ *of degree* p. *Then the following conditions are equivalent*:

(a) $\lambda_p(k) = \mu_p(k) = 0$.

(b) *For any prime ideal w of k_∞ which is prime to p and ramified in $k_\infty/\mathbb{Q}_\infty$, the order of the ideal class of w is prime to p.*

If $xyz \neq -1$ then $\lambda_p(k) = \mu_p(k) = 0$ by Corollary 2. So we only consider the case $xyz = -1$. In this case we have $k \neq k(p_i)$ $(i = 1, 2)$. It follows easily that $A(k)$, the p-part of the ideal class group of k, is cyclic of order p, and it is generated by products of primes of k above p. On the other hand, for $i = 1, 2$, the prime $\mathfrak{p}_i$ of k above p_i generates $A(k)$, and is inert in k_∞/k. Since the primes of k above p is principal for some k_n by the natural mapping $A(k) \to A(k_n)$ (cf. [4]), $\mathfrak{p}_i$ are principal in k_∞. ∎

Since all the primes ramified in $k_\infty/\mathbb{Q}_\infty$ are $\mathfrak{p}_1$ and $\mathfrak{p}_2$, which are principal in k_∞, we can apply Lemma 9 and obtain $\lambda_p(k) = \mu_p(k) = 0$.

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