Paradigm for Natural Frequency of an Un-Cracked Simply Supported Beam and its Application to Single-Edged and Multi-Edged Cracked Beam

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Abstract

In this research paper, a theoretical method of analysis of the first natural frequency of an un-cracked simply supported beam in bending mode is presented. The formula of a paradigm is used to determine the natural frequency of an un-cracked beam. The converged natural frequency formula of a paradigm is then extended to a single edged and multi-edged cracked simply supported beam to evaluate their natural frequency. The main attraction of the proposed method is that it gives one more significant way to the researchers to determine the natural frequency of cracked beams. The limited fatigue strength, defects like corrosion, corrosion-erosion, and corrosion fatigue in the beam are the main causes of formation of edged cracks in beams. Hence the evaluation of natural frequency of cracked beams and its use in the inverse problem is of utmost importance to do the condition monitoring of the structures by the vibration methods.

Keywords: paradigm; natural frequency; simply supported beam; ANSYS; modal analysis; zero frequency deflection; condition monitoring

1. Introduction

In turbo machinery, elements like beam, shaft is most practical and always remains associated with the defects like cracks and notches. These defects appear on the structural elements are mainly due to the limited fatigue strength of the material. The elements which are subjected to fatigue loading undergo sudden, complete and catastrophic failure. Hence, for avoiding such failure, it is required to do the periodic condition monitoring of the structures.

Khalkar and Ramachandran [1] presented the paradigm for the natural frequency of un-cracked cantilever beam; afterwards they extended the formula of un-cracked beam to cracked cantilever beam. They provided one additional way to the researchers for the evaluation of a bending natural frequency of an un-cracked and cracked cantilever beam. The main advantage of the presented paradigm is that it gives valid results for the natural frequencies at any arbitrary chosen locations. It is observed that [1], when the crack depth is kept constant and crack location is varied from the cantilevered end, then stiffness of the beam is increased. Vigneshwaran and Behera [2] studied the dynamic characteristics of the beam with multiple breathing cracks. For developing theoretical expressions for evaluation of natural frequencies and mode shapes, a systematic approach has been adopted in this study. For performing the dynamic analysis, an elastic simply supported beam with two breathing cracks is considered. The concept of influence coefficient is used to determine the cracked beam stiffness. The influence coefficients are calculated by using strain energy release rate and castigiliano's theorem. Eigen value approach is also used to evaluate the stiffness and natural frequencies for the multiple cracked beams. It is seen that the presence of cracks causes to change the stiffness and natural frequency of the simply supported beam. Zhong and Oyadiji [3] considered the polynomial function for the obtaining the transverse deflection of the cracked beam. The polynomial function represents the effect of a crack, to the polynomial function which represents the response of the intact beam. Approximate closed-form analytical expressions are derived for the natural frequencies of a random mode of transverse vibration of a cracked simply supported beam by means of a roving mass using the Rayleigh's method. Due to the roving of the mass along the cracked beam its natural frequencies get changed. So the roving mass can provide additional spatial information for damage detection of the beam. Teidj et al. [4] used an explicit analytical model for assessing the effect of a crack on beam strain energy, the beam first resonance frequencies were evaluated as function of a single crack defect characteristics. They used the fracture mechanics approach for obtaining the crack equations. Naik and Maiti [5] presented the vibration based crack detection technique for circumferential cracks in empty straight horizontal pipes in different orientations. Rotational spring is used to modelled the crack. The stiffness of the cracked beam is determined by experimental and vibration method. Thereafter, sensitivity of the vibration method for prediction of crack location on variations in experimental data has been examined. Dirr and Schmalhorst studied [6] the propagating crack that causes the slender uniform round shaft to shake about its major axis. Experiments using a stationary shaft are performed as well. For measuring the crack depth and the actual shape of the cracked cross section, beach marks are used. Singh and Tiwari [7] have studied and presented a two stage identification methodology, which identify the number of cracks and crack parameters. In the first stage, a multi-crack detection and its localization algorithm have developed. In the later stage of the algorithm, the size and the accurate location of cracks are obtained by using multi-objective genetic algorithms. Turgut and Mesut [8] analyzed have studied the Timoshenko beams having different boundary conditions. To study the free vibration characteristics of Timoshenko beams, a Lagrange equation is used. The first eight natural frequencies of Timoshenko beam are considered and tabulated for different thickness-to-length ratio of a beam. From this study, it is seen that shown tabulated results are useful to designers. Papadopoulos [9] investigated the torsional vibrations of a rotor with a transverse crack. For the modeling of the crack a local flexibility matrix is considered. The local flexibility matrix is calculated analytically and measured experimentally. The graphs between the first three natural frequencies and crack parameters are plotted. Khalkar and Ramachandran [10] have studied the EN 8 and EN 47 cracked cantilever beam for the top side and bottom side transversed cracks. Through this research study they found that natural frequency of free vibration is not the function of the crack depth for the same configurations.Khalkar and Ramachandran [11] have studied the steel cracked cantilever beam for the single sided and double sided v-shaped cracks. Through this research study [11], it is observed that the obtained natural frequency of free vibration for the rectangular shape cracked case and v shape cracked case is almostsame for the same configuration. Khalkar and Ramachandran [12] have studied the EN 47 cracked cantilever beam for the transverse and oblique cracks. Through this research study they have [12] found that the natural frequency more abruptly changes for the case of transverse cracked beam than the case of oblique cracked beam for the same configuration.

They have also found that the natural frequency of an un-cracked cantilever beam is almost similar to the natural frequency of a cracked beam which carries crack almost towards the free end. Khalkar and Ramachandran [13] have studied the spring steel cantilever beams for various crack geometries i.e V-shaped, U-shaped and rectangular shaped by vibration analysis. Through this research study they have found that the effects of crack geometries on the stiffness have a minor effect. They have also found that the free vibration based method can effectively predict the crack location and depth in the structure irrespective to the crack geometries.

From the literature survey, it has been found that there is no existence of an integration based approach paradigm for a simply supported beam to evaluate its natural frequency. None of the researchers has worked on the paradigm of a simply supported beam. In this study, the derived formula of a paradigm of a simply supported beam is applied to the cracked simply supported beam for natural frequency. This paradigm gives the valid results for the natural frequency of an un-cracked and cracked simply supported beam. Static (zero frequency deflection) analysis is carried out by ANSYS software on un-crackedand cracked models to get the stiffness parameter of un-cracked case and various cracked cases, afterward the value of obtained stiffness of any cracked case is substituted in the paradigm formula to get its natural frequency in the bending mode. To validate the results of natural frequency, modal analysis numerical experiments are conducted on the same models by using ANSYS simulation.

2. Theory

The schematic diagram of a simply supported beam subjected to zero frequency point load is as shown in Figure 1. By comparing deflection curve of Figure 5 and Figure 6, it is observed that the deflection curve of a simply supported beam at zero frequency is approximately similar to the curve obtained during vibration with first bending natural frequency.

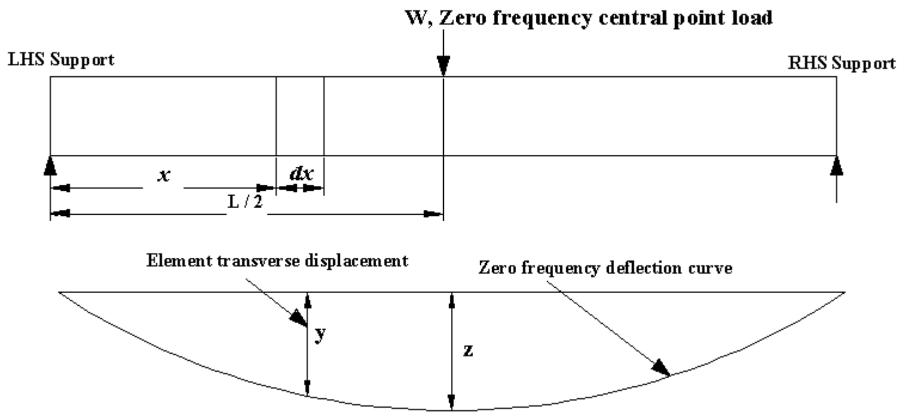

Figure 1. A schematic diagram of a simply supported beam subjected to zero frequency central point load

Let ρ' and L be the mass of the beam per unit length and length of the beam respectively, z is the displacement of midpoint of simply supported beam.

Mass of simply supported beam, $m = \rho' \cdot L$.

Therefore, the mass of the element is $\rho' \cdot dx$.

Consider a small element dx at a distance x from the left hand support of a simply supported beam as shown in Figure 1.

$$Mx = -W\ [(L/2) - x]$$
$$EI\frac{d^2y}{dx^2} = -M_x = W\ [(L/2) - x] \tag{1}$$

Integrate Equation (1)

$$EI\frac{dy}{dx} = W\left(\frac{Lx}{2} - \frac{x^2}{2}\right) + C_1 \tag{2}$$

when $x = 0, \frac{dy}{dx} = 0$ substitute first boundary conditions in Equation (1) then $C_1 = 0$

$$EI\frac{dy}{dx} = \frac{W}{2}(Lx - x^2) \tag{3}$$

Again Integrate Equation (3)

$$EIy = \frac{W}{2}\left(\frac{Lx^2}{2} - \frac{x^3}{3}\right) + C_2 \tag{4}$$

when $x = 0$, $y = 0$ substitute the second boundary conditions in Equation (4) $C_2 = 0$

$$EIy = \frac{W}{2}\left(\frac{Lx^2}{2} - \frac{x^3}{3}\right) \tag{5}$$

We have the deflection at the midpoint of simply supported beam, $z = \frac{WL^3}{48EI}$.

Substituting value of W in Equation (5)

$$y = \frac{24}{L^3}\left(\frac{Lx^2}{2} - \frac{x^3}{3}\right) z \tag{6}$$

Equation (6) gives displacement of the element in the transverse direction

$$y' = \frac{24}{L^3}\left(\frac{Lx^2}{2} - \frac{x^3}{3}\right) z' \tag{7}$$

Equation (7) represents the velocity of the element.

Kinetic energy of the element

$$\frac{1}{2}\rho' dx (y')^2 \tag{8}$$

Total K. E. of beam

$$\frac{1}{2}\rho' \int_0^{L/2} \left[\frac{24}{L^3}\left(\frac{Lx^2}{2} - \frac{x^3}{3}\right) z'\right]^2 dx \tag{9}$$

Total K. E. of beam

$$\frac{1}{2}Kz^2 \tag{10}$$

Apply energy method i.e. Total kinetic energy + Total potential energy = Constant, to un-cracked simply supported beam. After solving it, Equation (11) gets converged this represents the natural frequency of an un-cracked simply supported beam.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{K}{0.47143\ m}} \tag{11}$$

where, 0.47143*m* and *K* are the effective mass and stiffness of an un-cracked simply supported beam respectively.

Equation (11) is also applied to the cracked beam in this research study to get the natural frequency of any cracked case of interest. Initially, the static analysis of a cracked beam is carried out by using ANSYS software to get the zero frequency deflection. To get the zero frequency deflection, an elastic limit point load is applied on the beam, i.e. 100 N, on its middle point. Then the stiffness of cracked case of interest is computed by using the conventional formula (Elastic limit point load / zero frequency deflection). The calculated stiffness is then substituted in Equation (11) to get the natural frequency of any cracked case of simply supported beam.

3. Simulated crack configurations

In this study, natural vibration of an edge-cracked simply supported beam is studied. Fifteen cracked specimens are considered in this case study to find out the natural frequency of such cracked specimen by proposed theoretical method and by ANSYS as well.

Specimens of EN 8 materials are used to study the effect of cracks on natural frequency. EN 8 material is tested in ELCA Lab , Pune, India, to get the material properties i.e. Modulus of Elasticity (*E*) is 2.104 x 10^{11} N/m^2, ρ = 7820 kg/m^3.

Geometric properties: The length and cross sectional area of the beam are 0.36 m and 0.02x 0.02 m^2 respectively.

The value of Poisson's ratio is assumed as 0.3, i.e. cracked specimensare of EN 8 (spring steel) material

The main case is divided into three cases: case 1, case 2 and case 3. Details of each case are given below.

Case 1: In this case nine specimens are considered and each specimen carries one transverse crack as shown in Figure 2. This case is again subdivided into 3 sub cases, in the first sub case, 100 mm crack location is selected from the left hand support of the beam and at this location crack depth is varied from 5 mm to 15 mm by an interval of 5 mm. The second and the third sub cases are similar with the first sub case, the only difference is that instead of 100 mm crack location, 200 mm and 300 mm crack location is chosen for the second and third sub case respectively.

Case 2: In this case three specimens are considered and each specimen carries two transverse cracks as shown in Figure 3. In this case, 100 mm and 200 mm crack locations are chosen for the first and the second crack from the left hand support of the beam and at these locations cracks depths are varied from 5 mm to 15 mm by an interval of 5 mm.

Case 3: In this case three specimens are considered and each specimen carries three transverse cracks as shown in Figure 4. In this case, 100 mm, 200 mm and 300 mm crack locations are chosen for the first, second and the third crack from left hand support of the beam and at these locations cracks depths are varied from 5 mm to 15 mm by an interval of 5 mm.

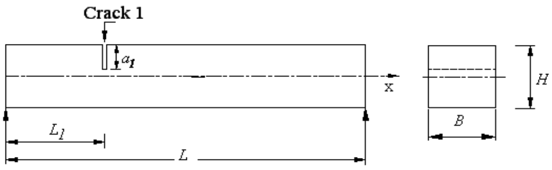

Figure 2. A schematic diagram of a cracked simply supported beam with single crack

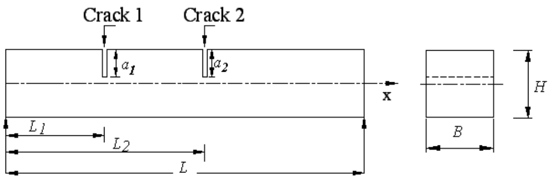

Figure 3. A schematic diagram of a cracked simply supported beam with two cracks

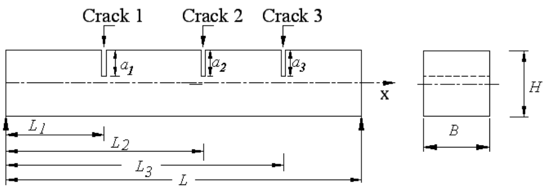

Figure 4. A schematic diagram of a cracked simply supported beam with three cracks

4. Finite element modelling and analysis

ANSYS [14] finite element program is used to determine natural frequencies and zero frequency deflections of cracked beams. A solid 186 structural solid element is selected for modelling the cracked simply supported beam model. Finite element boundary conditions are applied on both the extreme ends of simply supported beam along Y-direction. Static and modal analyses are carried out on each specimen to get zero frequency deflection and natural frequency. In static analysis 100 N loads is applied at the center of simply supported beam. Finite Element Analysis (FEA) plot of zero frequency deflection and natural frequency are shown in the Figure 5 and Figure 6 respectively. In this research study an open edge cracks are considered and 0.5 mm crack width is chosen for all the cracked cases.

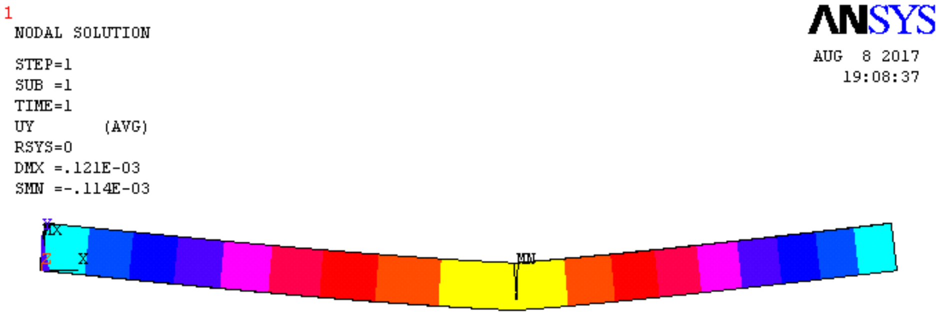

Figure 5. Zero frequency deflection plot of a cracked simply supported beam; L_1/L = 0.556; a_1 = 15 mm

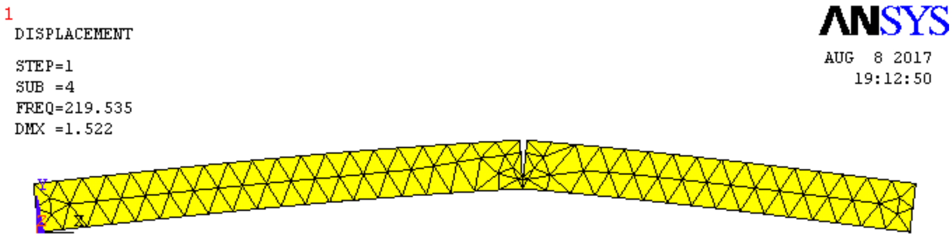

Figure 6. Natural frequency plot of a cracked simply supported beam; L_1/L = 0.556; a_1 = 15 mm

5. Results and discussion

By Finite Element Analysis, the natural frequencies for the various cracked cases are found to confirm the results obtained for the natural frequencies by a proposed theoretical method. The zero frequency deflection and stiffness results for single-edge cracked specimens are presented in Table 1 and 2. Table 3-5 presented the natural frequency results obtained by proposed theoretical method and by FEA analysis for single-edged cracked cases. Table 6 and 7presented the natural frequency results for the two-edged and three-edged cracked cases respectively.

Table 1. Zero frequency deflection of a simply supported beam with single-edged crack, m

Crack location Crack depth mm	L_1 = 100 mm	L_1 = 200 mm	L_1 = 300 mm
a_1 = 5	0.359e-4	0.376e-4	0.354e-4
a_2 = 10	0.402e-4	0.495e-4	0.370e-4
a_3 = 15	0.668e-4	0.114e-3	0.451e-4

Table 2. Stiffness of a simply supported beam with single-edged crack, N/m

Crack location Crack depth mm	L_1 = 100 mm	L_1 = 200 mm	L_1 = 300 mm
a_1 = 5	2785515.32	2659574.46	2824858.75
a_2 = 10	2487562.19	2020202.02	2702702.7
a_3 = 15	1497005.98	877192.98	2217294.9

Table 3. Comparison of first bending natural frequency of a simply supported beam with single-edged crack for 100 mm crack location, Hz

Crack depth mm	Methods	L_1 = 100 mm	
		Natural frequency Hz	% Deviation
a_1 = 5	Proposed method	364.73	2.697
	ANSYS	354.89	
a_2 = 10	Proposed method	344.52	3.941
	ANSYS	330.94	
a_3 = 15	Proposedmethod	267.26	6.095
	ANSYS	250.97	

Table 4. Comparison of first bending natural frequency of a simply supported beam with single-edged crack for 200 mm crack location, Hz

Crack depth mm	Methods	L_1 = 200 mm	
		Natural frequency Hz	% Deviation
a_1 = 5	Proposed method	356.23	1.482
	ANSYS	350.95	
a_2 = 10	Proposed method	310.47	-1.288
	ANSYS	314.47	
a_3 = 15	Proposed method	204.58	-7.312
	ANSYS	219.54	

Table 5. Comparison of first bending natural frequency of a simply supported beam with single-edged crack for 300 mm crack location, Hz

Crack depth mm	Methods	L_1 = 300 mm	
		Natural frequency Hz	% Deviation
a_1 = 5	Proposed method	367.13	2.369
	ANSYS	358.43	
a_2 = 10	Proposed method	359.26	3.462
	ANSYS	346.82	
a_3 = 15	Proposed method	325.41	7.78
	ANSYS	300.09	

Table 6. Comparison of first bending natural frequency of a simply supported beam with double-edged crack, Hz

Crack depth mm	Methods	L_1 =100 mm, L_2 = 200 mm	
		Natural frequency Hz	% Deviation
$a_1 = a_2 = 5$	Proposed method	351.28	1.708
	ANSYS	345.28	
$a_1 = a_2 = 10$	Proposed method	296.56	0.977
	ANSYS	293.66	
$a_1 = a_2 = 15$	Proposed method	183.39	-1.041
	ANSYS	185.30	

Table 7. Comparison of first bending natural frequency of a simply supported beam with three-edged crack, Hz

Crack depth mm	Methods	L_1 = 100 mm, L_2 = 200 mm, L_3 = 300 mm	
		Natural frequency Hz	% Deviation
$a_1 = a_2 = a_3 = 5$	Proposed method	349.94	2.026
	ANSYS	342.85	
$a_1 = a_2 = a_3 = 10$	Proposed method	292.03	1.664
	ANSYS	287.17	
$a_1 = a_2 = a_3 = 15$	Proposed method	180.24	-0.066
	ANSYS	180.36	

From Table 3 - Table 7, it is observed that the error for the natural frequency between the proposed method and the FEA analysis is less than 8% for all the cracked cases i.e. 1-edged, 2-edged and3-edged cracked cases.

From Figures 7-9, it is observed that value of natural frequencies obtained by proposed theoretical method and FEA analysis gives good agreement. For all the cracked cases, the results of natural frequencies obtained by the proposed theoretical method are valid. It is also found that for 2-edged and 3-edgedcracked specimens, the results obtained by a proposed theoretical method for the natural frequencies are more accurate than those of 1-edged cracked specimens. The main advantage of the proposed mathematical model is that it gives outstanding results at all sections of the beam.

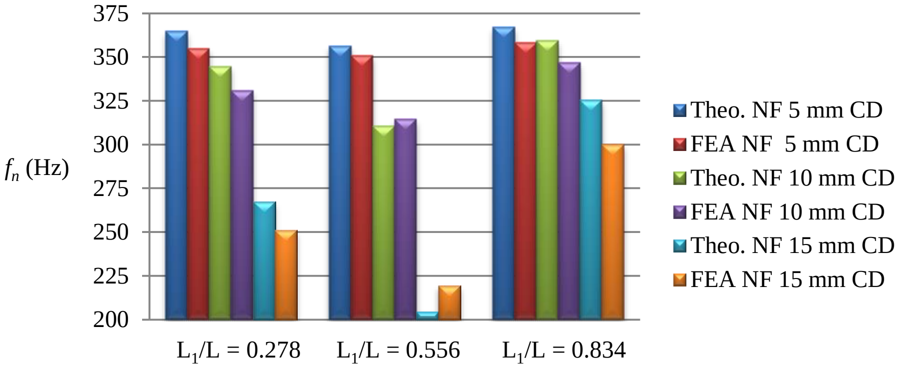

Figure 7. Comparison of first bending natural frequency of a simply supported beam with single-edged crack for 100 mm, 200 mm and 300 mm crack location

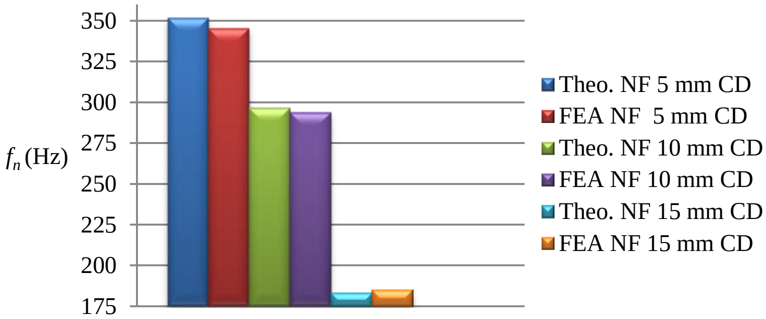

Figure 8. Comparison of first bending natural frequency of a simply supported beam with two-edged cracks: L_1/L = 0.278; L_2/L = 0.556

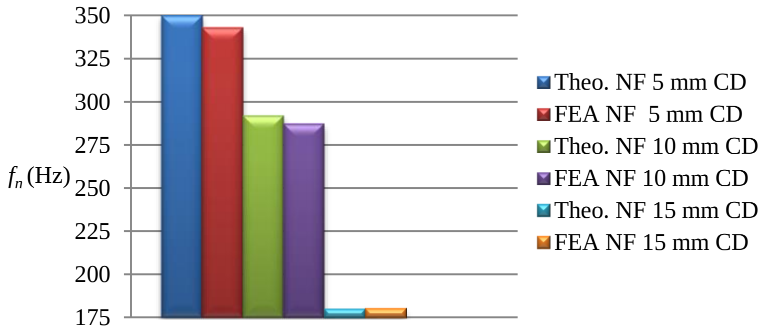

Figure 9. Comparison of first bending natural frequency of a simply supported beam with three-edged cracks: L_1/L = 0.278; L_2/L= 0.556; L_3/L = 0.834

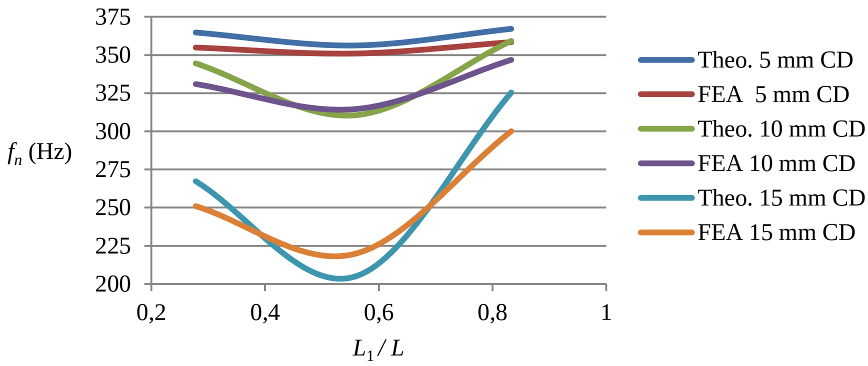

Figure 10. Variation of natural frequency with crack location ratio for single-edged cracked cases

From Figure 10, it is found that as the location of the crack increases from the left hand support of the simply supported beam by keeping the crack depth constant then natural frequency decreases up to midpoint of the beam, and again it increases for all the crack locations which are present between the midpoint of the beam and the right hand support of the beam. It means that the presence of crack at the midpoint of the beam produces largest effect of damping as compared to the presence of other cracks on other locations on the beam. At midpoint of the beam the effect of damping remains largest, and it is mainly due to presence of largest bending moment at the midpoint of the beam. The effect of bending moment remain less for all the cracked cases in which cracks remain present between midpoint of the beam and right hand support of the beam or between the midpoint of the beam and the left hand supportof the beam. Hence the magnitude natural frequency is found to be on higher side for such cracked cases.

6. Conclusion

A. The converge formula of the proposed theoretical method of an un-cracked simply supported beam can be extended to 1-edged or 2-edged or 3-edged cracked simply supported beam as it gives valid results for the natural frequency.
B. The proposed theoretical method gives one more significant way to the researchers to compute the natural frequency of cracked beam.
C. Natural frequencies obtained by the proposed theoretical method and ANSYS simulation are in good agreement which shows the versatility of the proposed method.
D. When location of the crack is increased from left hand support to the midpoint of a simply supported beam by keeping the crack depth constant, then the natural frequency decreased.
E. When location of the crack is increased from midpoint of the beam to the right hand support of a simply supported beam; then the natural frequency again increased.

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