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Received July 17, 1992 (2977)
Revised version February 15, 1993

STUDIA MATHEMATICA 105 (3) (1993)

A remark on disjointness results for stable processes

by

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In a recent paper M. Hernández and C. Houdré [3] have applied the Fourier analysis methods to prove disjointness results for some classes of stable stochastic processes. However, these methods forced the authors to restrict the range of the index of stability to $1 \le \alpha < 2$, instead of $0 < \alpha < 2$.

In this note we would like to show how the disjointness results, going back to the pioneering work of K. Urbanik [5], can be easily understood in the more general setup of symmetric infinitely divisible (ID) stationary processes, including stationary symmetric α-stable ($S\alpha S$) processes for all $0 < \alpha < 2$. Here we will employ some basic facts concerning the hierarchy of ergodic properties for stationary ID processes [2]. In the Gaussian case the moving averages form a subclass of harmonizable processes; however, for non-Gaussian ID processes we have

PROPOSITION. *In the class of symmetric non-Gaussian ID stationary processes a nondegenerate moving average process is never harmonizable.*

Proof. All symmetric ID moving averages are stationary and mixing [2]. Therefore they are ergodic. Harmonizable processes, i.e., the Fourier transforms of independently scattered random measures are stationary iff the random measure is rotation invariant. In sharp contrast with the Gaussian case, symmetric non-Gaussian ID harmonizable processes are not ergodic (cf. [4] and [1] for the $S\alpha S$ case). It follows that both classes are disjoint.

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1991 *Mathematics Subject Classification*: Primary 60G10.

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Received December 15, 1992 (3038)

STUDIA MATHEMATICA 105 (3) (1993)

Weak invertibility and strong spectrum

by

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Abstract. A notion of weak invertibility in a unital associative algebra $\mathcal{A}$ and a corresponding notion of strong spectrum of an element of $\mathcal{A}$ is defined. It is shown that many relationships between the Jacobson radical, the group of invertibles and the spectrum have analogues relating the strong radical, the set of weakly invertible elements and the strong spectrum. The nonunital case is also discussed. A characterization is given of all (submultiplicative) norms on $\mathcal{A}$ in which every modular maximal ideal $M \subseteq \mathcal{A}$ is closed.

1. Introduction. Let $\mathcal{A}$ be a unital associative algebra over the field of complex numbers and let G, $S = \mathcal{A} \setminus G$, and $\mathrm{Rad}(\mathcal{A})$ denote the group of invertibles, the set of singular elements of $\mathcal{A}$ and the Jacobson radical of $\mathcal{A}$ respectively. For an element $a \in \mathcal{A}$, let $\mathrm{Sp}(a)$ denote the spectrum of a in $\mathcal{A}$, that is, the set of scalars λ such that $\lambda - a \in S$ and let $\varrho(a) = \sup\{|\lambda| : \lambda \in \mathrm{Sp}(a)\}$ be the spectral radius of a in the algebra $\mathcal{A}$.

For a subset $F \subseteq \mathcal{A}$, let $P(F)$ denote the perturbation class of F in $\mathcal{A}$, that is, the set of all elements $a \in \mathcal{A}$ such that $a + F \subseteq F$.

In reasonable algebras (such as for example all Banach algebras) the Jacobson radical admits several characterizations in terms of invertibility and spectrum [3, Theorem 2.5] and [4]:

(a) $\mathrm{Rad}(\mathcal{A})$ is the perturbation class of the group G of invertibles.
(b) $\mathrm{Rad}(\mathcal{A}) = \{r \in \mathcal{A} : \mathrm{Sp}(a + r) = \mathrm{Sp}(a), \text{ for all } a \in \mathcal{A}\}$.
(c) $\mathrm{Rad}(\mathcal{A})$ is the largest ideal in $\mathcal{A}$ on which the spectral radius is identically zero.

If the algebra $\mathcal{A}$ carries a submultiplicative norm, then all primitive ideals in $\mathcal{A}$, and hence the Jacobson radical of $\mathcal{A}$, are closed, whenever the group G of invertibles of $\mathcal{A}$ is open in this norm. We will henceforth assume all norms under consideration to be submultiplicative. Following [4] we call a norm on $\mathcal{A}$ *spectral* if the group of invertibles of $\mathcal{A}$ is open in the corresponding topology. The term *Q-norm* is also employed by several

1991 *Mathematics Subject Classification*: Primary 46H05, 16A21.
Key words and phrases: radical, norm, spectrum.