Influence of the Reinforcing Bar Corrosion Level on the Flexural Crack's Width in the Existing Structure

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Abstract

A study on the influence of the reinforcing bar corrosion level on the flexural crack's width in the existing structure is presented. Parametric studies of the crack width development of corrosion damage were performed using a block model for the reinforced concrete element with corroded bars. The analytic description of the bond-slip law «τ-s» and the establishment of the parametric points of this diagram are decisive in this model. Using the block model, the distribution of the concrete $\varepsilon_{ct}(x)$ and reinforcement $\varepsilon_s(x)$ strains for the different level of corrosion damage, normal crack width was obtained and the effect of the level of corrosion damage was established.

Keywords: existing structure, corrosion, crack width

1 Introduction

As is was stated in [14], there is a growing need for reliable methods of assessing the load-carrying capacity (actual resistance) and the remaining service life of deteriorated existing structures to achieve optimized maintenance. Due to social and economic need of utilizing existing structures, their damage assessment and safety evaluation are of major concern.

The basic international standard, which provides general requirements and procedures for the assessment of existing structures, based on the principals of structural reliability and consequences of failure following ISO 2394 [16], is standard ISO 13822 [5]. It applies to the assessment of any type of existing structure that was originally designed, analysed and specified based on accepted engineering principles and/or rules, as well as structures constructed based on good workmanship, historic experience and accepted professional practice [22].

In accordance with *fib* Model Code 2010 [18] for assessing performance at the serviceability limit state, the influence of bond stiffness on deflection will likely be small compared to the loss of reinforcement and of concrete cross-section, the width of corrosion-induced longitudinal cracks is likely to exceed that of flexural cracks by the time change in flexural crack widths would be observed, and by this stage of deterioration the serviceability limit state of durability will in any case critical. In this paper, the analysis of the results of the influence of the reinforcing bars corrosion level on the flexural crack width is presented.

2 Development of the resistance model for corroded RC-element

The numerical study of the damaged RC-elements with corroded reinforcement bars was utilized based on block model that takes into account the constitutive relationship of material and overcomes the hypothesis of no-slip between corroded rebars and concrete including bond-slip relationship «τ-s» (see Figure 1).

In general case, the static problem if formally solved by following a known set of four equations:

(a) translation (axial) equilibrium of the cross-section:
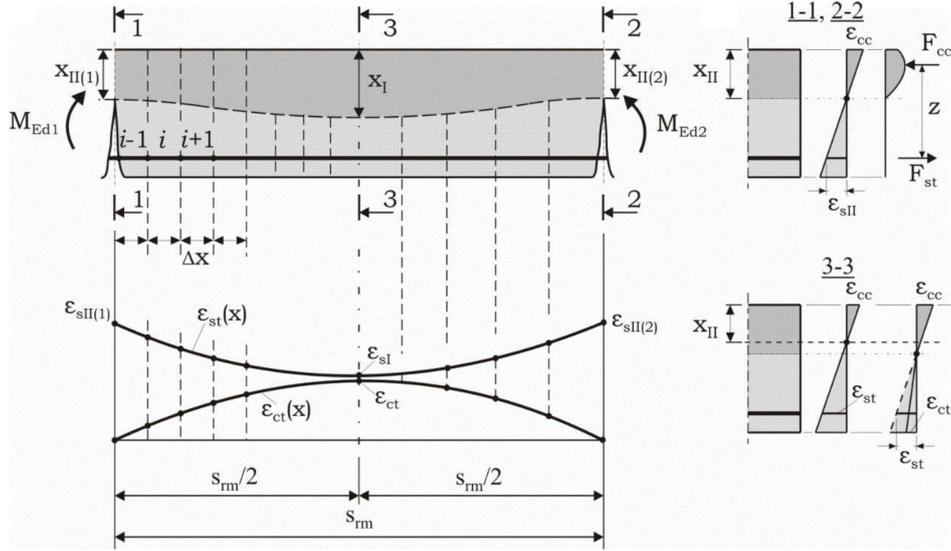
Figure 1. Relative strain distribution

$$\int_{Ac} \sigma_c(x,y)b(y)dy - \int_{Act} \sigma_{ct}(x,y)b(y)dy - A_s\sigma_s(x) = 0 \tag{1}$$

(b) rotational equilibrium about the geometrical axis of the cross-section:

$$\int_{Ac} \sigma_c(x,y)yb(y)dy - \int_{Act} \sigma_{ct}(x,y)yb(y)dy - A_s\sigma_s(x)d_s = M(x) \tag{2}$$

(c) translation (axial) equilibrium of the bar:

$$\frac{d\sigma_s(x)}{dx} - \frac{4}{\phi}\tau(x) = 0 \tag{3}$$

where A_c, A_{ct}, A_s are the area of the concrete in compression, of concrete in tension, of the steel bars, respectively;

ϕ is the bar diameter.

A fourth equation is obtained by developing the definition of the slip in terms of strain, i.e. by the following equation:

$$\frac{ds(x)}{dx} = \varepsilon_s(x) - \varepsilon_{ct}(x) \tag{4}$$

According to the proposed kinematic model [13], in the section at abscissa x (see Figure 1.), the strain $\varepsilon_c(x,y)$ in the generic concrete fibre at the distance y from the section centroid, is immediately expressed as follows:

$$\varepsilon_c(x,y) = \frac{y - d_s - d_c}{d_c}\varepsilon_c(x,d_g) \tag{5}$$

i.e. as a function of the maximum concrete strain in compression $\varepsilon_c(x,d_g)$ and the neutral axis depth d_c (see Figure 1.). The solution of the proposed equations system appears to be complicated due to nonlinearity of some equations and to the dependency of the bond on the distance of the generic section from the crack. Therefore the problem can only be solved numerically. Following [13], it is worth making discretization at finite differences, by deriding the space between two cracks in $(n-1)$ subintervals with small length Δx (see Figure 1.). For solving the problem at the finite differences the iterative procedure based on the «regula falsi» is very useful [13].

In the generic iteration j, the procedure allows calculation of the values of the parameters at the node $(i + 1)$ by the values assumed at the node (i); the expressions at the finite differences of Eq. (3) and (4) are used as follows:

$$\sigma_{s,i+1}^{(j)} = \sigma_{s,i}^{(j)} + \frac{4}{\phi}\tau_{(i)}^{j} \cdot \Delta x \tag{6}$$

$$s_{i+1}^{(j)} = s_i^{(j)} + \Delta x \left(\frac{\varepsilon_{s,i+1}^{(j)} - \varepsilon_{s,i}^{(j)}}{2} - \frac{\varepsilon_{ct,i+1}^{(j)} - \varepsilon_{ct,i}^{(j)}}{2} \right) \tag{7}$$

The solution is based on obtaining the value $\varepsilon_{s,1}^{(j)}$ and $\varepsilon_{c,1}^{(j)}$ and therefore $\sigma_{s,1}^{(j)}$ and $\sigma_{c,1}^{(j)}$, through the constitutive laws by equilibrium in the cracked section, where $\sigma_{ct,1}^{(j)} = 0$.

For the damaged RC-element with corroded steel bar, the proposed equations can be solved only when relationship «τ-s» for corroded bars is known.

3 Bond-slip relation for RC-element with corroded steel bars

3.1 Analytical bond-slip model

The bond-slip relation of concrete and corroded steel rebar could be expressed by a continuous model [18] according to the experimental results. As it was shown in numerous publication [14, 21, 23] the bond-slip relation for corroded steel bar depends on a considerable number of influencing factors including rib geometry (relative rib area), concrete strength, position and orientation bar during casting, state of stress, boundary conditions (environmental), concrete cover, duration of aggressive effects. As it was stated in our analytical study, in general case, the bond-slip curve for confined and unconfined concrete presented in Figure 2 can be considered applicable as an average formulation for a board range of cases. The model in case of the corroded bar could be written by the following expression [23]:

$$\text{OA} - \text{stage} : \frac{\tau}{\tau_0} = 2\sqrt{\frac{s}{s_0}} - \frac{s}{s_0}, 0 < s \leq s_0 \tag{8}$$

$$\text{AE} - \text{stage} : \tau = \tau_0 \frac{(s_u - s)^2 \cdot (2s + s_u - 3s_0)}{(s_u - s_0)^3} + \tau_u \frac{(s - s_0)^2 \cdot (3s_u - 2s - s_0)}{(s_u - s_0)^3}, s_0 < s \leq s_u \tag{9}$$
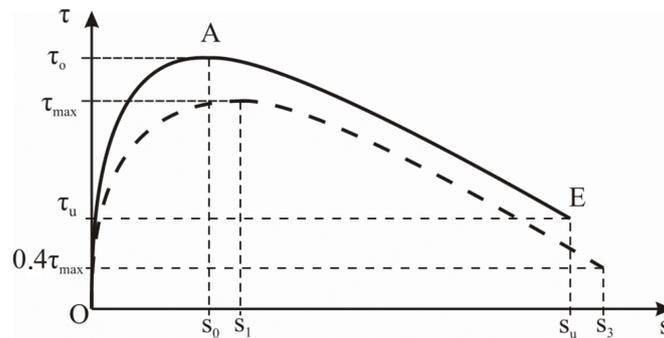
$$\text{EF} - \text{stage} : \tau = \tau_u, s > s_u \tag{10}$$

Figure 2. The bond-slip curve for confined and unconfined concrete [23]

It should be stated, that for a correct description of the relationship «τ-s» in case of the corroded reinforcement bar, it is necessary to determine the values at the parametric points: bond strength and slippages (s_0, s_u).

3.2 Bond strength

Design and assessment code and standard rules are derived on the assumption that the strains in both concrete and reinforcement are the same, that is perfect bond exists between materials. Any degradation mechanism that reduced concrete tensile strength and/or induces cracking around reinforcement is likely to reduce the bond strength.

Corrosion induces longitudinal cracking and, as such, the bond is likely to be affected.

The decreasing of bond strength between concrete and steel bar primary caused by following reasons: (1) the abrasion of rebar's rib; (2) the reduction of frictional force between concrete and rebar by virtue of the flake corrosion products on the surface of rebar; (3) the weakness of concrete active reaction to rebar because of longitudinal cracking while corrosion.

A small amount of corrosion, up to the level required to induce longitudinal cracking, do not cause loss of bond capacity, and may even augment bond strength to a modest degree, particularly where the bar is in a «poor» casting position. A greater level of corrosion, residual bond strength is strongly influenced by the degree of confinement provided by secondary reinforcement in the form of links and by the surrounding structure.

Following to commentary to *fib* Model Code 2010 [18] most data on bond resistance of corroded reinforcement are obtained from the test in which corrosion activity has been accelerated, and corrosion rates are in excess, or well in excess, of those measured in the field exposure. Consequently, experimental data must be interpreted with caution.

The magnitude of the reduction in residual bond strength is highly dependent on the confinement to the bar and is also affected by concrete quality and environment. The values in Table 2. may be taken as indicative only; however, detailed guidance should be sought in the case where residual strength of a corroding structure is of concern.

In case of *fib* Model Code 2010 [18] guidelines (see Table 1.), the equivalent surface crack indicates the width of corrosion-induced longitudinal crack which correlates with the residual strength indicated in typical conditions. It should be appreciated that the residual strength of concrete structures is also affected by cross-section loss of both steel and concrete. From the other side under 6.1.1.3.3 *fib* Model Code 2010 [18], if cracks parallel to the bar axis are present, the bond strength for pull-out failure should be modified by the factor Ω_{cr}:

Ω_{cr}=1,0, where concrete is uncracked parallel to the bar axis;

Ω_{cr}=1-1$w_{cr(l)}$, where the concrete is cracked parallel to the bar axis and $w_{cr(l)}$ is the crack width (in mm).

It should be pointed, that if $w_{cr(l)}$ ≥83 mm, Ω_{cr}=0 and bond strength f_{bd}=0. But it is contradicting to the experimental results presented in numerous publication [9, 10, 14, 22] and the magnitudes listed in Table 1.

Table 1. The magnitude of the reduction in residual bond strength for corroded reinforcement

Corrosion penetration, mm	Equivalent surface crack, mm	Confinement	Residual bond stress (strength) (as% of f_{bd}),bar type	
			Ribbed	Plain
0.05	0.2…0.4	Nolinks	50…70	70…90
0.10	0.4…0.8		40…50	50…60
0.25	1,0…2,0		25…40	30…40
0.05	0.2…0.4	Links	95…100	95…100
0.10	0.4…0.8		70…80	95…100
0.25	1,0…2,0		60…75	90…100

The different expressions utilizing the bond strength assessment of the corroded steel bar proposed by authors were analyzed in our study and summarized in Table 2.

As shown from Table 2. most of the proposed expressions are complex and contains numerous uncertainties and unknown parameters which can be obtained by testing procedure only in every case.

Table 2. The generalized expressions utilizing the bond strength assessment of the corroded steel bar

Author	Expressions to evaluate, $f_{bd,corr}$
D. Coronelli [9]	$f_{bd,corr} = \left| k(x)p^{max}(x) + \tau_b^0(x) + \mu(x) \cdot p_{corr}(x) \right|$
D. Coronelli, P. Gambarova [10]	$f_{bd,corr} = 0.6 \cdot \left(0.5 + \frac{c}{\phi}\right) f_{ct,sp}(1 - \beta x^\mu) + \frac{k \cdot A_{Tr} \cdot f_{yw}}{s \cdot \phi}$
T. El Maaddawyet al [11]	$f_{bd,corr} = (A_1 + A_2 \cdot m_1) \cdot \left(0.55 + 0.24 \cdot \frac{c}{\phi}\right) \cdot \sqrt{f} + 0.191 \cdot \frac{A_{Tr} \cdot f_{yw}}{s \cdot \phi}$
D.V. Valet al [20]	$\frac{f_{bd,corr}}{f_{bond,0}} = \begin{cases} 1 + (k_{1-}x) \cdot \frac{x}{x_{cr}} & x \leq x_{cr} \\ max\left[k_1 - k_2(x - x_{cr}); 0.15\right] & x > x_{cr} \end{cases}$
X. Wang, X.Liu [21]	$f_{bd,corr} = \tau_u(x) = \tau_{crx} + tg\alpha \cdot p_{corr}$
N.S. Ottosen [15]	$f_{bd,corr} = k \cdot \left(0.5 + \frac{c}{\phi}\right) \cdot f_{ct}$
T. El Maaddawyet al [11]	$f_{bd,corr} = R \cdot \left(0.55 + 0.24 \cdot \frac{c}{\phi}\right) \cdot \sqrt{f_{cm}} + 0.191 \cdot \frac{A_{sw} \cdot f_{yw}}{s \cdot \phi}$
J. Rogriguez [17]	$f_{bd,corr} = 0.6 \cdot \left(0.5 + \frac{c}{\phi}\right) \cdot f_{ct,sp} \cdot (1 - \beta x^\mu) + \frac{k \cdot A_{sw} \cdot f_{yw}}{s \cdot \phi}$
fibBulletin 2000 [6]	$\Delta f_{bd,corr} = \lambda \cdot \Delta w_{cr(l)}$
J. Cairns, Y. Du and D. Law [7]	$f_{bd,corr} = \frac{1}{(1 + 0.8 \cdot w_{cr(l)})} \cdot f_{bond,0}$
P. Thoft-Christensen [19]	$f_{bd,corr} = (1 - 0.30 \cdot w_{cr(l)}) \cdot f_{bond,0}$
fibModel Code 2010 [18]	$f_{bd,corr} = (1 - 1.2 \cdot w_{cr(l)}) \cdot f_{bond,0}$
K. Lundgrenet al [14]	$f_{bd,corr} = k_{Asw} \cdot f_{b,conf} + (1 - k_{Asw}) \cdot f_{b,unconf}$

A most appropriate bond strength model the corroded bar was choose based on the statistical evaluation of the model error under EN1990, AnnexD [12] (see Figure 3 and Table 3) compared experimental and theoretical values.

Table 3. Comparison of statistical parameters

Author	The statistical evaluation of the model error	
	b	V_δ, %
J. Cairnset al [7]	1.56	27.4
P. Thoft-Christensen [19]	1.09	29.9

Based on the results of the own studies, the following expression for assessment bond strength of corroded reinforced bar and surrounding concrete was proposed:

$$f_{bd,corr} = \frac{1}{(1 + 0.8 w_{cr(l)})} \cdot f_{bond,0} \tag{11}$$

where $w_{cr(l)}$ is the longitudinal surface crack width (mm) and $f_{bond,0}$ is the bond strength for the uncorroded bar under fib Model Code 2010 [18].

3.3 Corrosion cracks

After corrosion initiation, hydrated rust accumulates around the bar, causing pressure and leading to cover cracking.

To predict the damage caused by corroding reinforcing bars, knowledge of the state of stress in the surrounding concrete is required, and this can be determined to employ a concrete ring or thick-wall cylinder, as it was proposed by most of the researches [4]. The concretering approximates the effect of surrounding concrete, but due to different geometry between the cover ring model and the real cover, the stresses will only approximately correspond to the stress in the real situation. In the last decade numerous model [9, 17, 22] for corrosion cracking assessment was
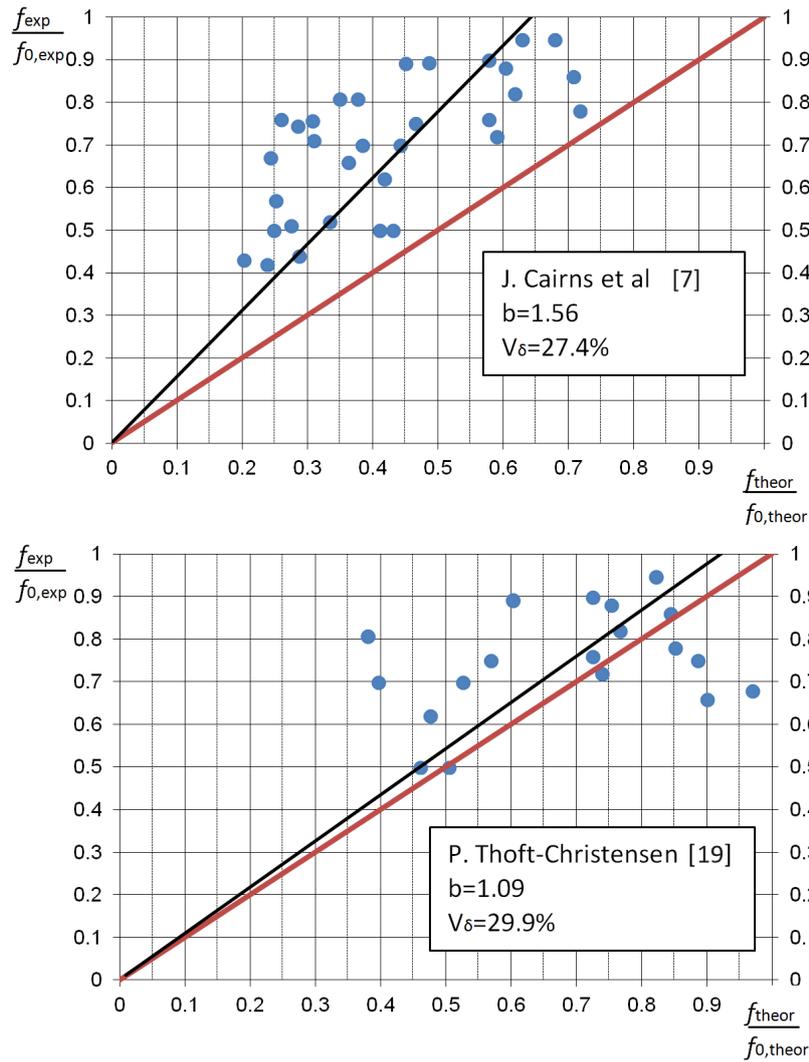
Figure 3. Comparison of experimental and theoretical values

proposed. Based on the results of the own comparative study, the following relations (expression) for calculation of the corrosion crack with opening proposed (by J. Rodriguez et al [17]):

$$w_{cr(l)} = 0.05 + \beta \cdot (x - x_{cr}) \tag{12}$$

where c is the penetration depth for steel bar (um);

x_{cr} is the critical penetration depth initiated longitudinal crack; β is the empirical coefficient.

For calculation of the critical penetration depth C. Alonso et al [3] empirical expression was adopted:

$$x_{cr} = 7.53 + 9.32\frac{c}{\phi} \tag{13}$$

where c is the concrete cover and ϕ is the bar diameter.

Comparison of the theoretical crack width $w_{cr(l)}$ values obtained by the generalized model (12) and (13) with experimental data, obtained by the test are presented in Figure 4.

Taking into account statistical uncertainties evaluated by EN 1990, Annex D [12] (b=0.34; V_δ =50.9%) expression (12) can be rewritten as:
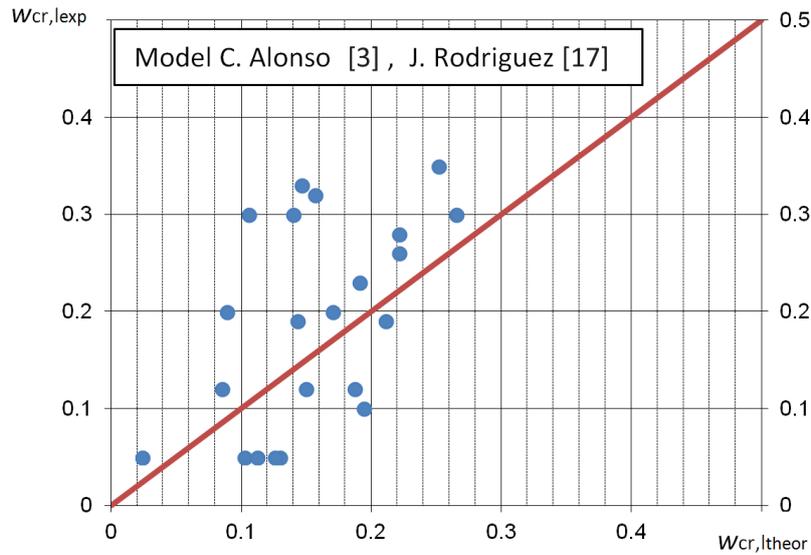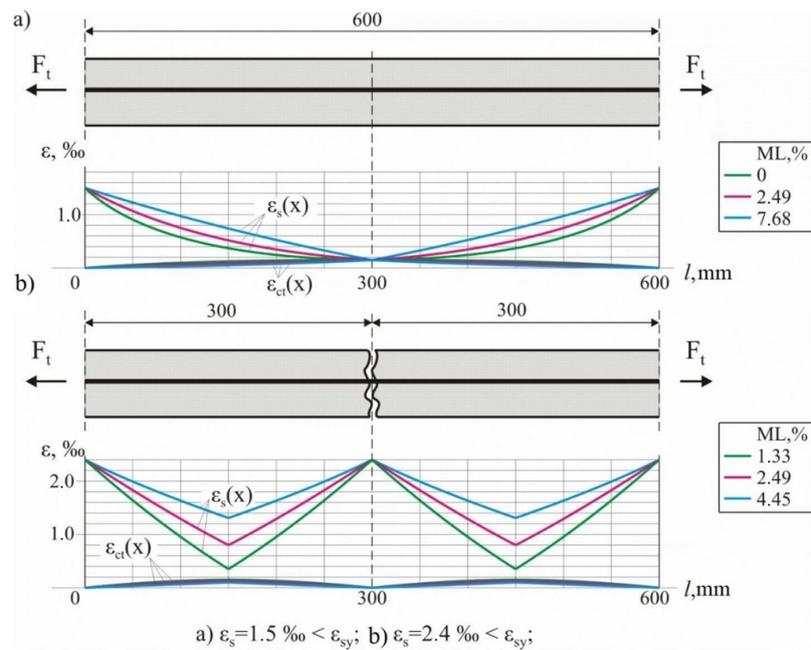
Figure 4. Comparison of experimental and theoretical values

Figure 5. The distribution of the concrete $\varepsilon_{ct}(x)$ and reinforcement $\varepsilon_s(x)$ strains for the different level of corrosion damage and practically constant value (*a)before cracking; b) stabilized cracking*) $\varepsilon_s(0) = 0.15\%$; (exploitation service stage) (example for $f_{ck} = 20MPa, \phi12mm, c/\phi = 3.5$)

$$w_{cr(l)} = k \cdot [0.05 + \beta \cdot (x - x_{cr})] \tag{14}$$

where k is the model empirical coefficient equal to 0.34.

4   Results of the numerical study and brief discussion

Numerical studies of the reinforced concrete beam elements with wide combination of the input parameters (concrete strength, ratios c/ϕ ,levels of the corrosion damage as a mass loss $ML, \%$)utilized with usage of the proposed numerical

model.

An example, of the distribution of the concrete $\varepsilon_s(x)$ and reinforcement $\varepsilon_{ct}(x)$ strains for the different level of corrosion damage and practically constant value $\varepsilon_s(0) = 1.5\%_0$ (exploitationor service stage) and presented in Figure 5. Normal to the axis of elements crack width was calculated as:

$$w_k = 2 \int_0^{S_{rm}/2} [\varepsilon_s(x) - \varepsilon_{ct}(x)] \cdot dx \tag{15}$$

Relationships between normal crack width (w_k) and corrosion damage level $ML, \%)$ are shown in Figure 6 and example of obtained relations between normal crack width w_k and longitudinalcrack width (w_l) for different corrosion damage level $(ML, \%)$ are shown in Figure 7 The red solid and dashed lines indicate the critical penetration depth initiated a longitudinal crack.
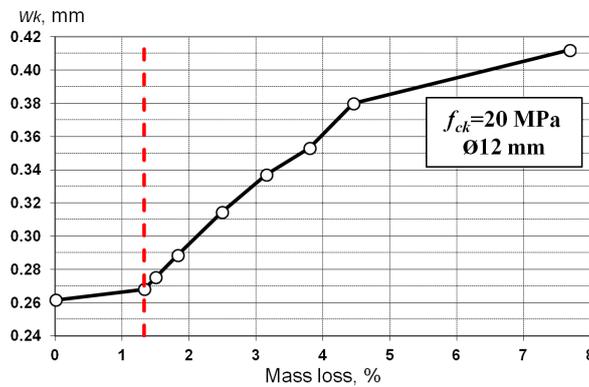

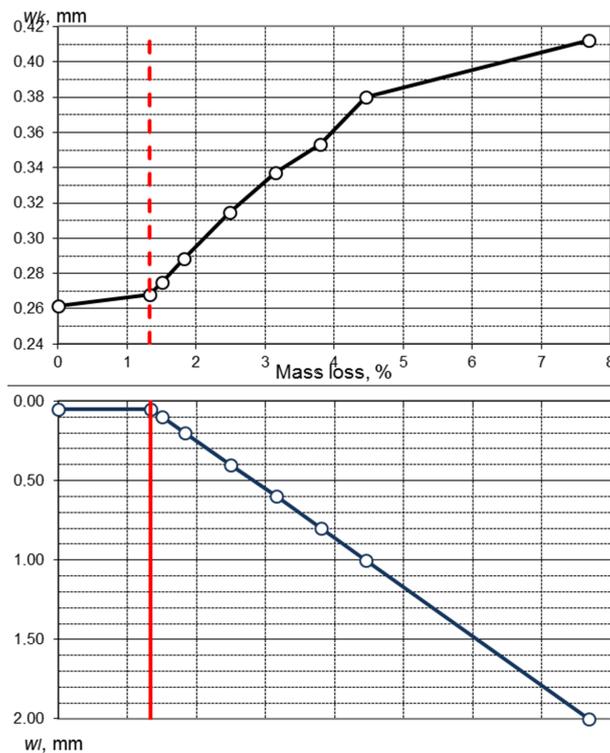Figure 6. Relationships between normal crack width (w_k) and corrosion damage level $(ML, \%)$

Figure 7. Relations between normal crack width (w_k) and longitudinalcrack width (w_l) for different corrosion damage level $(ML, \%)$ (example for $f_{ck} = 20 MPa$, ϕ 12mm, $c/\phi = 3.5$)

5 Conclusion

Performed numerical studies have shown that a significant effect of corrosion damage of the steel reinforcement (as a mass loss ML, %). on the crack's width is observed only after the mass loss is greater than 1.2%. Beyond this point, where is an approximately linear reduction in the bond strength and increasing of the crack opening [1, 2, 8].

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