www.ceejme.eu
ww.wsb.pl/wroclaw/ceejme

ISSN electronic version 2353 - 9119
Central and Eastern European Journal
of Management and Economics
Vol. 4, No.2, 105-123, June 2016

A theoretical approach to quantitative downside risk measurement methods

Diana CIBULSKIENÉ, Martynas BRAZAUSKAS

Šiauliai University, Lithuania

Abstract: Evaluating the results of the investment portfolio it is important to take into account not only the expected profitability, but also the risk. Risk measurement is based on the historical data applying various methods. The methods, that take into account the downside volatility, measures risk most effectively. The importance of these methods is emphasized by the empirical research. There are three main downside risk types: downside or asymmetric risk, tail risk, drawdown risk. The paper describes and compares the different risk measurement methodologies and criteria. Market risk measurement methods must meet four basic risk measurement axioms: positive homogeneity, subadditivity, monotonicity, transitional invariance. These axioms represent only a part of evaluating methods for tail risk and drawdown risk. Having conducted empirical studies the scientists have shown that empirical research is becoming more and more popular involving the use of a downside risk measurement methods. This popularity can be explained by the fact that based on the research results the downside risk measurement methodologies help increase the efficiency of investment portfolio.

Keywords: tail risk, downside risk, drawdown risk.
JEL: G10, G20, G31.

1. Introduction

Globalisation and integration of the financial markets promote the growth of the financial markets as well as various financial products. With the growing number of investment funds, the competition among their managers is growing, too. Increasing competition among the investment fund managers forces them to look for more effective methods for risk management and its measurement. Scientists are also working in this area, offering and improving a variety of techniques for risk measurement. In the diversity of these approaches it is necessary to look for the most effective quantitative risk measurement methods.

Correspondence Address: Diana Cibulskiené, Siauliai University, Lithuania. E-mail: diana.cibulskiene@gmail.com.

Markowitz (1952) proposed to assess risk applying a variance. Such risk measurement method is widely used nowadays by the students and researchers as well as fund managers. However, later the same Markowitz (1959) proposed semi-variance for risk measurement. According to him, this is a better approach to risk measurement than a variance, because risk is evaluated taking into account the downside return values. Nevertheless, this method is not widely used for several reasons. First of all, the downside changes are not well-loved by investors. According to the Prospect theory people are more responsive to the downside changes than to positive ones. There is therefore a need to assess the capability of the quantitative downside risk measurement methods.

The object of this article: quantitative downside risk measurement methods.

The aim of this article is to systemize and summarize the quantitative methods for efficient performance evaluation, described in the scientific literature, as well as to evaluate their mutual relations.

The following tasks were formulated:

- To systemize and summarize the quantitative risk measurement methods presented in the scientific literature.
- To perform the analysis on different market risk measurement methods, highlighting their advantages and disadvantages.

2. Tail risk measurement methods

Large price changes in the market appear more often than if the logarithmic returns were described as a normal distribution. This effect can be explained by the fact that the logarithmic return distribution has a fat tail (Leipus, Norvaiša 2004). Real market data is characterized by the fat tails, having higher than usual skewness and kurtosis, so instead of the classical mean-dispersion theory to select portfolio, it is necessary to apply robastic risk metrics such as Value at Risk and Conditional Value at Risk methodology, Mean Absolute Deviation, MiniMax and other models (Kabašinskas 2007).

Skewness coefficient estimates the average data skewness. A negative Skewness coefficient informs about a higher probability of downside returns (Xiong et al., 2014). The Skewness coefficient of the investment portfolio returns is calculated as follows (Cumming et al., 2014):

$$Skew(r) = \frac{\sum_{i=1}^{n}(r_i - \mu)^3}{\sigma_i^3}$$

here r_i - return during the particular period of time, μ – average return during the particular period of time, σ – standard deviation.

Kurtosis coefficient estimates the average data concentration. A positive Kurtosis coefficient informs about the average high return concentration and it may indicate the emerging market bubble. Return distribution sharpness determines the large tails, and this increases the risk of tails. The Kurtosis coefficient of the investment portfolio returns is calculated as follows (Cumming et al., 2014):

$$Kurt(r) = \frac{\sum_{i=1}^{n}(r_i - \mu)^4}{\sigma_i^4}$$

Risk of tail distribution can be evaluated by calculating the Expected Shortfall. The Expected Shortfall is calculated as the average of the biggest shortfalls to a certain level of probability (Acerbi, Tasche 2002, Acerbi et al., 2008):

$$ES = \frac{\sum_{i=1}^{n} r_n}{n}$$

r_n – downside return during the particular period of time, n – number of periods.

Jorion (2007) describes Value at Risk, (VaR) as the largest loss over a particular period of time with a predetermined low probability that the actual loss will be higher. VaR is a corresponding percentile of loss distribution law, i.e. with a certain confidence level, the loss will not be higher than the estimated VaR (Kabašinskas 2007). VaR must be developed not only from a statistical probability calculation, but also on the basis of formulas to calculate the output Value at Risk and making assumptions about the profit or income distribution. To this purpose, according to the historical data the attempts are made to formalize the investment portfolio volatility patterns. Let's say that the profit probability density function is f and the confidence level is 1 - c, then VaR can be calculated as follows (Garbanovas 2010):

$$c = \int_{-VaR}^{\infty} f(r)dr$$

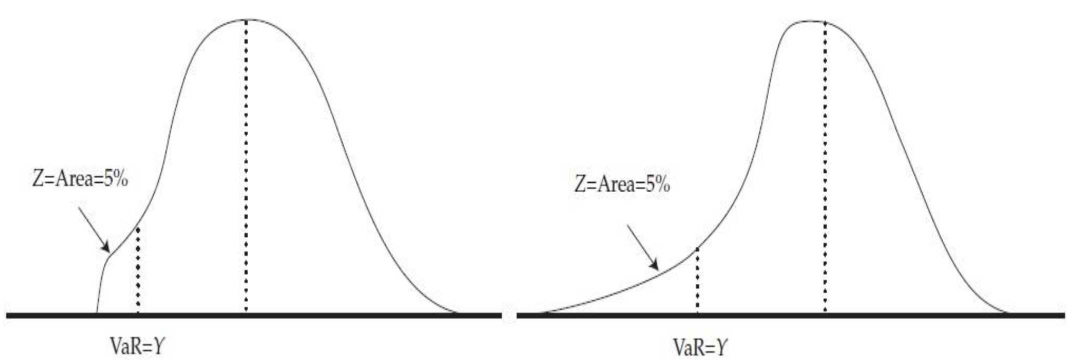
VaR is calculated by three methods: variance-covariance, historical simulation, and Monte Carlo simulation. The variance-covariance method is a parametric method that is used to calculate VaR of the financial instruments, assuming that the factors of the financial instrument market risk and portfolio profit (loss) are distributed according to the normal distribution law (Central Bank of the Republic of Lithuania 2002). Dzikevičius (2005) presents such a VaR calculation formula:

$$VaR = \alpha\sigma\sqrt{t}$$

here α - constant, which defines a one-sided confidence interval of the standard normal distribution, t - time period.

The historical simulation method calculating value at risk uses the data from a previous period. The historical simulation method is simple and easily explained. This method is nonparametric. In the model the probability distributions of the empirical market risk variables are used, so it is not depended on the normal distribution law (Aniūnas et al., 2009). The Monte Carlo simulation is similar to the historical simulation method. In this method the investment portfolio returns and changes of the market variables are generated through the simulations. The Monte Carlo simulation method is effective and flexible, easy to evaluate the nonlinear dependencies of the non-linear financial instrument returns, so it can be applied to various investment portfolios. Applying it the various hypotheses about the changes in the market and their probability distributions are possible (Dzikevičius 2005).

Fig. 1. Distributions with a similar VaR, but having different tales.



Source: Coleman 2011.

The distributions shown in Fig. 2 have the similar Value at Risk, but because of the type of tails (left - thin tail, right - fat tail) may result the different maximum losses. Therefore, in this case the risk is better measured by the Expected Shortfall criteria than by the Value at Risk methodology (Coleman 2011).

Rockafellar and Uryasev (2000) developed a Conditional Value at Risk methodology (CVaR). CVaR is defined as the conditional expected loss provided that the loss will exceed the VaR limit (Kabašinskas 2007). Szego (2002) claims that because of the shortage of the VaR methodology, this method is not suitable for risk measurement. He defines the CVaR and Expected Shortfall as the appropriate risk measurement criterion. The risk criteria should meet four basic properties:

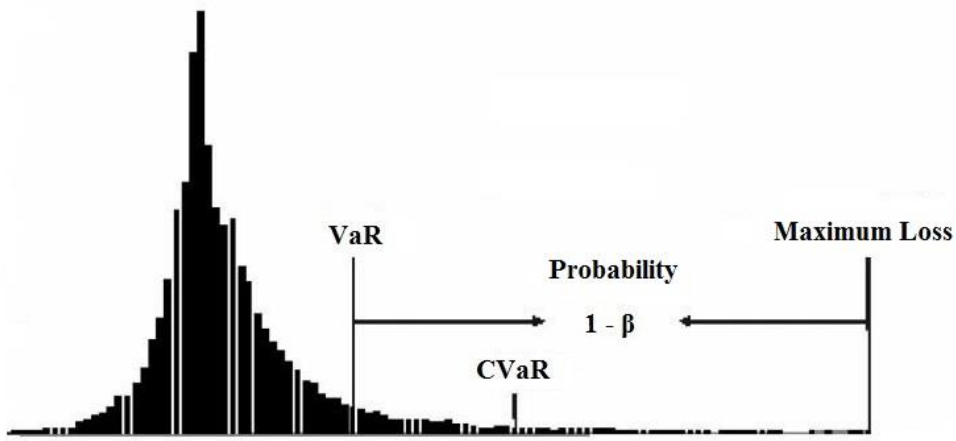
- positive homogeneity: $\rho\ (\lambda x) = \lambda\rho\ (x)$, where $\lambda \geq 0$, capital requirements do not depend on the currency in which the risk is measured;
- subadditivity: $\rho\ (x + y) \leq \rho\ (x) + \rho\ (y)$, a diversified investment portfolio risk can not be larger than the risk of different assets;
- monotonicity: If $x \leq y$ then $\rho\ (x) \leq \rho\ (y)$. If the risk of one asset is larger than the risk of the other asset, then the capital requirements should be increased;
- transitional invariance: $\rho(x + r_f) = \rho(x) - r_f$, r_f - risk-free interest rate. There are no additional capital requirements for the unspecified additional risk (Panjer 2002).

The risk is not subadditive when measured by the VaR methodology, and the diversified portfolio risk may be higher than the individual securities risk, while the CVaR methodology does not have such a shortage (Xiong, Idzorek 2011). Therefore Artzner et al. (1999) distinguishes the CVaR methodology as a proper risk measurement approach. Calculation experiments have shown that most often the CVaR minimization gives the optimal solution of the VaR model, because VaR never exceeds CVaR value. Therefore the portfolios with a low CVaR will also have a small VaR. Rockafellar and Uryasev (2000) showed that if the return-loss is normal, then CVaR and VaR are equivalent risk measures, and they give the same optimal portfolio. But the CvaR and VaR optimal portfolios may be different for the asymmetric laws. The solution obtained by minimizing VaR may be from the larger quintile areas than VaR (Kabašinskas 2007). The CVaR methodology is widely used not only for the fact that it is similar to VaR, but also because it helps to evaluate the damages that will suffer if the VaR limit is exceeded. CVaR is calculated as the average of the total composite probability distribution, fluctuating in (0, p) range, and tailing quintile, below the VaR values, (Shalit 2014):

$$CVaR(p) = \frac{1}{p}\sum_{i=0}^{k(p)} VaR(q_i)\ q(x_i)$$

$$q(x_i) \leq p$$

Fig. 2. VaR and CVaR position in distribution of losses.



Source: Szego 2002**.**

The CVaR methodology is also known as: Mean Excess Loss, Mean Shortfall, Tail Value at Risk (Kabašinskas 2007).

When the data is located under a normal distribution, the VaR is calculated according to the distribution parameters:

$$VaR = \mu + z_c\sigma$$

here z_c - a critical value of the pth quintile of a standard normal distribution. This value is found in the normal distribution tables, depending on the level of confidence (Chambers, Lu 2011).

If the data is not distributed according to the normal distribution then the Cornish-Fisher improvement is applied, where the VaR calculation methodology includes the kurtosis and skewness measurement (Eling, Schuhmacher 2006).

$$MVaR = \mu + z_{CF}\sigma$$

$$z_{CF} = z_c + \frac{(z_c^2 - 1)Skew}{6} + \frac{(z_c^3 - 3z_c)Kurt}{24} - \frac{(2z_c^3 - 5z_c)Skew^2}{36}$$

The methods measuring the risk of the data distribution tails are characterized by robastic properties, these methods are not sensitive to the data exclusions. This is important because most of the data have fat tails, kurtosis or skewness. The criteria calculating the risk of the data distributions tails measures loss limit within a certain period of time. However, they do not measure the potential worst-case scenarios. Also the application of CVaR or VaR methods to estimate the maximum potential loss may lead to significant decrease of the investment portfolio value. Some of the methods evaluating tail risk correspond to the four main risk measurement axioms: positive homogeneity, subadditivity, monotonicity, and transitional invariance.

3. Drawdown risk measurement methods

Securities drawdown is accounted as losses, which occurs over the investment period. The maximum drawdown from the peak is often used to measure the risk (Eling, Schuhmacher 2006). The maximum drawdown measures the maximum possible loss from the largest market value within a certain period of time (Bacon 2008).

In order to avoid losses the average drawdown must be calculated (Bacon 2008):

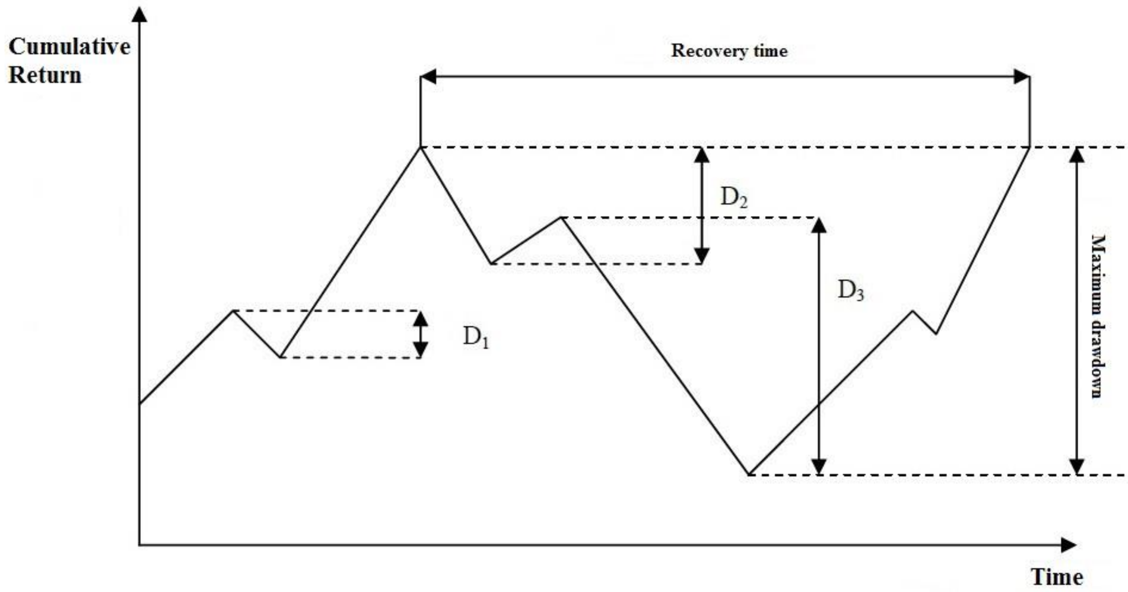
$$\overline{D} = \left| \sum_{j=1}^{j=d} \frac{D_j}{d} \right|$$

here D_j - drawdown over a certain period of time, *d* – a number of drawdown.

Drawdown deviation from the mean is calculated as a standard drawdown deviation (Bacon 2008):

$$DD = \sqrt{\sum_{j=1}^{j=d} \frac{D_j^2}{d}}$$

Fig. 3. Drawdown in the change of portfolio value.



Source: Bacon 2008.

In 1992 Martin and McCann developed the Ulcer index, which shows the maximum loss an investor can expect within a certain period of time. In the 'bear' market the Ulcer index values will be high, and in the 'bull' market the values of this index will be low. Therefore calculating this index, it is important to assess all the market conditions.

$$UI = \sqrt{\sum_{i=1}^{i=n} \frac{D_i^{'2}}{n}}$$

here $D_i^{'}$ - drawdown from the top of the market value over a certain period of time *i* (Bacon 2008).

In order to measure the efficiency of the investment portfolio Young (1991) suggested the Calmar ratio which measures the additional return per maximum unit of drawdown:

$$CR = \frac{\mu - r_f}{D_{\max}}$$

Compared with the original Calmar ratio, a risk-free interest rate is additionally included, which is better assesses the performance of the investment portfolios (Bacon 2008).

In the Sterling ratio a maximum drawdown is replaced by the average of drawdowns. In the original Sterling ratio the highest average of drawdowns plus 10% is used to measure the risk. An added bonus is used, because the highest average of drawdowns will inevitably be less than the maximum drawdown (Kolbadi, Ahmadina 2011).

$$OStR = \frac{\mu}{\overline{D}_{Lar} + 10\%}$$

Bacon (2008) offers to calculate the Sterling ratio using the value of the average drawdowns:

$$StR = \frac{\mu - r_f}{\left| \sum_{j=1}^{j=d} \frac{D_j}{d} \right|}$$

The Sharpe ratio evaluates a variability of both positive and negative volatility. To avoid this lack Burke (1994) proposed to measure the risk applying a square root of the sum of all drawdown squares. To calculate a square helps to measure a large drawdown, when the significance of a low drawdown is negligible.

$$BR = \frac{\mu - r_f}{\sqrt[2]{\sum_{j=1}^{j=d} D_j^2}}$$

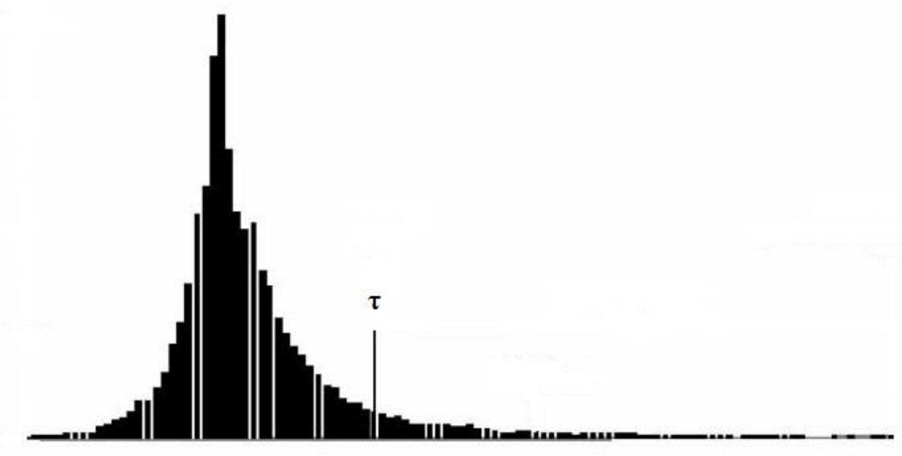
Chekhlov et al. (2005) proposed the Conditional Drawdown (CDD) or Conditional Drawdown at Risk (CDaR) methodology. The CDaR method, as well as the CVaR methodology, is characterized with monotony and transitive invariability. The CDaR methodology satisfies the positive *(ρ (x) ≥ 0)* and subadditive conditions. The value of CDaR is calculated as the average of drawdowns under a certain confidence level.

The methods assessing CDaR are characterized with sensitivity for segregations. Sudden, short-term changes in the market can significantly increase the risk of assets. However, the scientists have proposed the risk measurement methodologies, which comply with the fundamental axioms of risk measurement.

4. Downside risk measurement methods

The idea of downside risk was first proposed by Roy (1952). He described the standard deviation, assessing the downside risk, as a ‘safety first’ rule. This criterion evaluates the probability of the return, lower than the expected (Sing Ong 2000). The measurements of the downside deviations are mainly used for the dynamic asset allocation issues (Kabašinskas 2007). The criteria, evaluating the shortfall risk or downside deviations risk, are asymmetric (Albrecht 2004).

Fig. 4. Position of minimum acceptable return in the loss distribution.



Source: Szego 2002.

To evaluate risk as deviation to the downside, semi-variance is used, what is a better risk measurement than the variance, because the downside changes are evaluated as well (Markowitz 1959). The semi-variance of asset returns is a better risk measurement way for a number of reasons. First of all, investors 'like' more positive than downside volatility. Semi-variance is more useful than variance, when distribution is asymmetric, and it is also useful when distribution is symmetrical. Third, semi-variance provides the information about two dimensions: variance and skewness (Estrada 2007). Investors require higher returns from the shares, which have high downside risk deviations. They do not buy shares, as long as they become cheaper to appropriate level. In the meantime, the shares with high positive deviation risk are overestimated and acquired for high prices (Ishiba et al., 2012). The Prospect theory explains such investors' behavior. The conducted studies have shown that people accept lost two times worse than the same amount of winnings (Kahneman, Tversky 1979). The semi-variance is calculated by the following formula:

$$\sigma_n = \sqrt{\sigma_n^2},$$

here σ_n – standard deviation of the downside return, σ_n^2 – downside return dispersion.

Considering the Prospect theory, which states that investors react to losses much stronger than to profit, Watanabe (2006) proposed the Prospect ratio. The ratio assesses additional return per one unit of downside risk. Incorporated additional factor increases the significance of the lowest returns in the calculation of the investment portfolio performance.

$$PR = \frac{\frac{1}{n}\sum_{i=1}^{i=n}\left(Max(r_i,0) + 2{,}25Min(r_i,0)\right) - \tau}{\sigma_n}$$

here τ – minimum acceptable return rate.

The method of Lower Partial Moments (LPM) measures downside return deviation risk, considering the minimum acceptable return. This method was offered by Bawa (1975) and Jean (1975). When the distribution is even and its data is distributed within $x \in [-\infty; \tau]$, then the LPM is calculated according to the following formula (Unser 2000):

$$LPM_n^{\tau}(r) = \int_{-\infty}^{x} (\tau - r)^n \, dF(r)$$

here r - the results of probability distribution, $F(r)$ – the function of probability distribution density, n - weights, which are assigned as tolerance by an investor.

Sortino and Van der Meer (1991) applied the 'good' and 'bad' variability for risk measurement. In the Sortino ratio a Minimum Acceptable Return (MAR) is used as the point of exclusion. If the achieved return in the investment portfolio is higher than the point of exclusion, it means that the goal has been achieved and the return volatility is considered as a ‘nice volatility’. Less than the minimum acceptable return shows that the goals have not been achieved and it is seen as a ‘bad volatility’ or risk. This criterion is similar to the Sharpe ratio, however the risk-free interest rate is replaced by the MAR, and a standard deviation is replaced by the return lower than the MAR standard deviation or otherwise the standard deviation of downside return (Sourd 2007).

$$S = \frac{E(r_p) - \tau}{\sqrt{\frac{1}{T}\sum_{\tau=0}^{T}(r_{pt} - \tau)^2}}, \; r_{pt} < \tau$$

Sortino and Price (1994) described the downside risk measurement method, in which the utility function is applied in the mean-downside risk model. This criterion is called a Fouse index.

$$Fouse = E(r) - B\delta^2$$

here B – value reflecting the investors' willingness to take risks, δ - calculated downside risks taking into account the minimum expected return.

Keating and Shadwick (2002) claim that the mean-variance model is inaccurate measuring the performance of the investment portfolio efficiency. They proposed the Omega ratio, where a guiding principle is to divide returns into a profitable and unprofitable, according to the minimum expected return and to measure a positive and negative return probability ratio.

$$\Omega(\tau) = \frac{\int_{\tau}^{b} (1 - F(x))dx}{\int_{a}^{\tau} F(x)dx}$$

here a and b – range of returns, F – function of return combined distribution.

This criterion is not suitable for measuring the investment portfolio performance, where returns are distributed according to the normal law. Therefore, in most cases, this method is applicable analysing the portfolio performance of hedge funds (Sourd 2007).

To calculate the Omega ratio in a simpler way is by using the LPM methodology (Eling, Schuhmacher 2006):

$$\Omega = \frac{\mu - \tau}{LPM_1(\tau)} + 1$$

The Omega and Sortino ratios are two of many options to the Kappa ratio. Considering the current situation, the other Kappa ratio options could be more appropriate and effective to help measure the excess return adjusted for downside risk (Kaplan, Knowles 2004):

$$K_n(\tau) = \frac{\mu - \tau}{\sqrt[n]{LPM_n(\tau)}}$$

Considering this formula, the Sortino ratio will be equal to $K_2(\tau)$, the Omega ratio – $K_1(\tau)+1$.

Bawa and Lindenberg (1977) suggested to measure the downside risk applying a negative beta ratio. The downside systemic risk can better explain the investors' expected return (Ang et al., 2006).

$$\beta^{-} = \frac{\operatorname{cov}(r_i, r_m)}{\operatorname{var}(r_m)}$$

where $r_m < \mu_m$.

Generalising the described criteria it is important to note that the asymmetric risk measurement criteria evaluate the risk of downside deviations from the expected values. This is efficient in measuring the risk in the ‘bear’ market conditions. These methods distinguish and assess the investors' ‘disliked‘ downside risk. The usage of the downside deviations for risk measurement helps avoid the problems arising from the application of the symmetrical risk measurement criteria. The risk measurement criteria of the downside deviations are sensitive to distinctness and this can lead to the distorted results of risk measurement.

5. Review of the empirical studies

Tail risk. Xiong et al. (2014) conducted the study, using the U.S. and non-U.S. investment funds, that showed that the standard deviation is characterized by the lack of relation with the criteria measuring a tail risk: Skewness coefficient and Excess Conditional Value at Risk methodology (ECVaR). The research showed that a high volatility is not compensated by a higher return, but a large tail risk is compensated by a higher return. Thus, the risk should not be seen only as volatility, and the investors must take into account the tail risk, as well (Page 2013). Bali et al. (2009) carried out the study in the NYSE, AMEX, NASDAQ stock markets, that showed that there is a strong link between a significant tail risk and expected return.

Kelly and Jiang (2014), analyzing the NYSE, AMEX, NASDAQ stock market companies, found that a tail risk is an effective method to predict stock market returns during a period from one month to five years. During the analysed period of 1963 - 2010 the shares with a small tail risk had lower efficiency than the shares with a large tail risk. Bollerslev and Todorov (2011) analysed the S&P500 future stock market in 1990 - 2008. The empirical research showed that the premium of the most historically high stocks and variance in risk can be explained as the compensation of the tail risk jumps.

The hedges from the tail risk help maximize the return of the investment portfolio, as the insured portfolio encourages to invest into the businesses with a higher growth potential

(Bhansali, Davis 2010). The hedges against a tail risk significantly increase the efficiency of the investment portfolio during a tail risk period. It helps increase the investment portfolio efficiency, even investing in the risky assets (Benson et al., 2013).

Drawdown risk. Zabarankin et al. (2014) proposed to use the CDaR methodology in the CAPM model. The CDaR beta and CDaR alpha ratios help select the financial instruments, as well as to identify the instruments that are needed to hedge against the market downturns. This technique is useful when it is used as the instrument to hedge against the losses. Mendes, Leal (2003), Cherny, Obloj (2013) recommended to use the drawdown risk methodologies as the restrictions on the formation of the optimal investment portfolio.

Downside risk. Estrada (2003, 2004) carried out the study in 1970-2000 on the stock markets in the developed and developing countries, which showed that a semi-variance is a better measure for risk than a variance. The mean-semi-variance method is efficient to seek for the maximum expected return. Later, Estrada (2007) proposed the CAPM model with beta ratio of the negative values. Using the data of the stock markets in the developed and developing countries in 1988-2000, the beta ratio of negative values explained 44 percent of the return volatility in the developed and developing countries and nearly 55 percent of the market returns variability in the developing countries. Moreover, the average return in the analysed countries was more sensitive to the changes of the negative beta ratio values, than to the changes of the same standard beta ratio. The usage of the downside risk helped achieve a higher average expected return in the developing country markets than in developed markets. A higher significance of the negative beta ratio can be explained by the stock return, what was confirmed by Dobrynskaya (2014), with Post, Vliet (2004). Moreover, Dobrynskaya (2014) analysing the currency markets found that the increase in the interest rates in the country determines the increase of the currency downside risk. Jaama, Lam and Isa (2011) analysed the impact of the downside risk on the investment portfolio efficiency in the Kuala Lumpur Stock Exchange in the period of 2004 - 2007. The study showed that the usage of a downside risk is more efficient than the mean-variance model. The most efficient portfolio is compiled using the methodology of the CVaR. This shows that this methodology is a good choice for investors who avoid risk.

Generalising the researchers conducted empirical studies it can be emphasized that the empirical research become more and more popular, involving the usage of the all downside risk measurement methods. This popularity can be explained by the fact that based on the research results the usage of the downside risk measurement methodologies helps increase the efficiency of the investment portfolio. The identification of the downside risk is important in order to hedge the investment portfolio against the possible losses. The need of the investment portfolio hedges is increasing due to the increased market uncertainty.

6. Conclusions

Evaluating the investment portfolio results it is important to take into account not only the expected profitability, but also the risks. Various methods based on historical data are used to measure the risk. The most efficient methods for risk measurement are those, what take into account downside volatility. The significance of these methods is emphasized by the empirical research. There are three main downside risk types: downside or asymmetric risk, tail risk, drawdown risk.

The methods measuring the risk of the data distribution tails are characterized with robastic properties, and these methods are not sensitive to the data exclusions. This is important because most of the data have fat tails, kurtosis or skewness. The criteria calculating the risk of the data distribution tails measures the loss limit within a certain period of time. However, they do not measure the potential worst-case scenarios. Also the application of the CVaR or the VaR methods to estimate the maximum potential loss may lead to significant decrease of the investment portfolio value. Some of the methods evaluating a tail risk correspond to the four main risk measurement axioms: positive homogeneity, subadditivity, monotonicity, and transitional invariance.

The methods measuring conditional loss at risk are characterized with sensitivity to segregations. Sudden, short-term changes in the market can significantly increase the risk of assets. However, scientists have proposed the risk measurement methodologies, which comply with the fundamental axioms of risk measurement.

Asymmetric risk measurement criteria evaluate the risk of downside deviations from the expected values. This is efficient in measuring the risk in the ‘bear’ market conditions. These methods distinguish and assess the investors' ‘disliked’ downside risk. The usage of the downside deviation to measure risk helps avoid the problems arising from the application of the symmetrical risk measurement criteria. The criteria measuring the downside deviation risk are sensitive to segregations and this can lead to distorted risk measurement results.

Bibliography

Acerbi C., Tasche D. (2002), Expected Shortfall: a natural coherent alternative to Value at Risk, “Economic Notes“, vol. 31 no. 2, pp. 379-388.

Acerbi C., Nordio C., Sirtori C. (2008), Expected Shortfall as a Tool for Financial Risk Management, http://arxiv.org/pdf/cond-mat/0102304v1.pdf [30.06.2016].

Albrecht P. (2004), Risk Measure, Encyclopedia of Actuarial Science, John Wiley & Sons, New York.

Ang A., Chen J., Xing Y. (2006), Downside Risk, “Review of Financial Studies“, vol. 19 no. 4, pp. 1191-1239.

Aniūnas P., Nedzveckas J., Krušinskas R. (2009), Variance – Covariance Risk Value Model for Currency Market, “Engineering economics“, vol. 1 no. 61, pp. 18-27.

Artzner P., Delbaen F., Eber J.-M., Heath D. (1999), Coherent Measures of Risk, “Mathematical Finance“, vol. 9 no. 3, pp. 203-228.

Bacon C. (2008), Practical Portfolio Performance, Measurement and Attribution, 2nd ed., John Wiley & Sons, Inc., New Jersey.

Bali T. G., Demirtas K. O., Levy H. (2009), Is There a Relation Between Downside Risk and Expected Stock Return?, “Journal of Financial and Quantitative Analysis“, vol. 44 no. 4, pp. 883-909.

Bawa V. S. (1975), Optimal rules for ordering uncertain prospects, “Journal of Financial Economics“, vol. 2 no. 1, pp. 95-121.

Bawa V. S., Lindenberg E. B. (1977), Capital Market Equilibrium in a Mean-Lower Partial Moment Framework, “Journal of Financial Economics“, vol. 5 no. 2, pp. 189-200.

Benson R., Shapiro R. K., Smith D., Thomas R. (2013), A Comparison of Tail Risk Protection Strategies in the U.S. Market, Alternative Investment Analyst Review, pp. 32-47.

Bhansali V., Davis J. M. (2010), Offensive Risk Management: Can Tail Risk Hedging Be Profitable?, PIMCO Working Paper, Februrary, pp. 20.

Bollerslev T., Todorov V. (2011), Tails, Fears, and Risk Premia, “The Journal of Finance“, vol. LXVI no. 6, pp. 2165-2211.

Burke G. (1994), A sharper Sharpe ratio. The Computerized Trader, “Futures Magazine“, March, p. 56.

Central Bank of the Republic of Lithuania (2002), Nutarimas dėl vidinių rinkos rizikos vertinimo modelių taikymo (Resolution on the aim of the internal market risk measurement model) https://www.lb.lt/valdyb_nutar_151_1 [30.06.2016].

Chambers D., Lun Q. (2011). Value at Risk Estimation: A Review and Extension, Hawaii University International Conferences 2011 Math and Engineering, Honolulu.

Chekhlov A., Uryasev S., Zabarankin M. (2005), Drawdown measure in portfolio optimization, “International Journal of Theoretical and Applied Finance“, vol. 8 no. 1, pp. 13-58.

Cherny V., Obloj J. (2013), Portfolio optimisation under non-linear drawdown constraints in a semimartingale financial model, “Finance and Stochastics“, vol. 17 no. 4, pp 771-800.

Coleman T. S. (2011), A Practical Guide to Risk Management, The Research Foundation of CFA Institute, Charlottesville.

Cumming D., Haß L. H., Schweizer D. (2014), Strategic Asset Allocation and the Role of Alternative Investments, “European Financial Management“, vol. 20 no. 3, pp. 521-547.

Dobrynskaya V. V. (2014), Downside Risk in Stock and Currency Markets, PhD thesis, London.

Dzikevičius A. (2005), Valiutinių pozicijų portfelio rinkos rizikos vertinimo metodų lyginamoji analizė (A comparative analysis of market risk measurement approaches of FX portfolio), Tiltai, no. 2, pp. 1-13.

Estrada J. (2003), Mean-Semivariance Behavior: A Note, “Finance Letters“, no. 1, pp. 9-14.

Estrada J. (2004), Mean-Semivariance Behaviour: An Alternative Behavioural Model, “Journal of Emerging Market Finance“, vol. 3 no. 3, pp. 231-248.

Estrada J. (2007), Mean-semivariance behavior: Downside risk and capital asset pricing, “International Review of Economics and Finance“, no. 16, pp. 169-185.

Eling M., Schuhmacher F. (2006), Does the choice of performance measure influence the evaluation of hedge funds?, “Working papers on Risk Management and Insurance“, no. 29.

Garbanovas G. (2010), Banko vertės ir rizikų portfelio sąveika ir valdymas (Bank value and risk‘s portfolio interdependence and management), Doctoral dissertation, Technika, Vilnius.

Ishibe M., Kakuta Y., Sakamaki S. (2012), Premium for Upside Risk and Downside Risk: decomposition of volatility effect, “Securities Analysts Journal“, p. 12.

Young T. W. (1991), Calmar ratio: A smoother tool, “Futures“, vol. 20 no 1, p. 40.

Jaaman S. H., Lam W. H., Isa Z. (2011), Different downside risk approaches in portfolio optimisation, “Journal of Quality Measurement and Analysis“, vol. 7 no. 1, pp. 77-84.

Jean W. H. (1975), Comparison of Moment and Stochastic Dominance Ranking Methods, “Journal of Financial and Quantitative Analysis“, vol. 10 no. 1, pp. 151-161.

Jorion P. (2007), Financial Risk Manager Handbook, 4th ed., John Wiley & Sons, Inc., New Jersey.

Kabašinskas A. (2007), Finansų rinkų statistinė analizė ir statistinio modeliavimo metodai (Statistical analysis of financial markets and methods of statistical modeling), Doctoral dissertation, Vytautas Magnus University, Vilnius.

Kahneman D., Tversky A. (1979), Prospect Theory: An Analysis of Decision under Risk, “Econometrica“, vol. 47, pp. 263-291.

Kaplan P. D., Knowles J. A. (2004), Kappa: A Generalized Downside Risk-Adjusted Performance Measure, Working Paper, Miscellaneous publication, Morningstar Associates and York Hedge Fund Strategies.

Keating C., Shadwick W. F. (2002) A universal performance measure, “Journal of Performance Measurement“, vol. 6 no. 3, pp. 59–84.

Kelly B., Jiang H. (2014), Tail Risk and Asset Prices, “The Review of Financial Studies“, vol. 27 no. 10, pp. 1-31.

Kolbadi P., Ahmadinia H. (2011), Examining Sharpe, Sortino and Sterling Ratios in Portfolio Management, Evidence from Tehran Stock Exchange, "International Journal of Business and Management", vol. 6 no. 4, pp. 222-236.

Leipus R., Norvaiša R. (2004), Finansų rinkos teorijos taikymas (Application of financial market theories), "Pinigų studija", no. 1, pp. 31-53.

Markowitz H. (1952), Portfolio selection, "The Journal of Finance", vol. 7 no. 1, pp. 77-91.

Markowitz H. (1959), Portfolio Selection Efficient Diversification of Investment, John Wilei & Sons, Inc., New York.

Martin P. G., McCann B. B. (1992), The Investor's Guide to Fidelity Funds: Winning Strategies for Mutual Funds Investing, 2nd ed., Venture Catalyst, Redmond, Washington.

Mendes B. V. M., Leal R. P. C. (2003), Maximum Drawdown: Models and Application, "Journal Of Alternative Investments" vol. 7 no. 4, p. 19.

Page S. (2013), Risk Management beyond Asset Class Diversification, Presentation from the Global Investment Risk Symposium held in Washington, DC, 7-8 March.

Panjer H. H. (2002), Measurement of Risk, Solvency Requirements and Allocation of Capital Within Financial Conglomerates, 27th International Congress of Actuaries, Cancun.

Post T., Vliet P. (2004), Conditional Downside Risk and the CAPM, ERIM Report Series Research in Management.

Rockafellar T. R., Uryasev S. (2000), Optimization of Conditional Value-at-Risk, "Journal of Risk", vol. 2 no. 3, pp. 21-41.

Roy A. D. (1952), Safety First and the Holding of Assets, "Econometrica", vol. 20 no. 3, pp. 431-449.

Shalit H. (2014), Portfolio Risk Management Using the Lorenz Curve, "The Journal of Portfolio Management", vol. 40 no. 3, pp. 152-159.

Sing T. F., Ong S. E. (2000), Asset Allocation in a Downside Risk Framework, "Journal of Real Estate Portfolio Management", vol. 6 no. 3, pp. 213-223.

Sortino F. A., Price L. N. (1994), Performance Measurement in a Downside Risk Framework, "Journal of Investing", vol. 3 no. 3, pp. 59-64.

Sortino F. A., Van der Meer R. (1991), Downside Risk, "Journal of Portfolio Management", vol. 18, pp. 27-31.

Sourd V. L. (2007), Performance Measurement for Traditional Investment Literature Survey, EDHEC Risk and Asset Management Research Centre.

Szego G. (2002), Measures of Risk, "Journal of Banking & Finance", vol. 26, pp. 1253-1272.

Unser M. (2000), Lower partial moments as measures of perceived risk: An experimental study, "Journal of Economic Psychology", vol. 21 no. 3, pp. 253-280.

Watanabe Y. (2007), Is Sharpe Ratio Still Effective?, "The Journal of Performance Measurement", vol. 11 no. 1.

Xiong J. X., Idzorek T. M. (2011), The Impact of Skewness and Fat Tails on the Asset Allocation Decision, "Financial Analyst Journal", vol. 67 no. 2, pp. 23-35.

Xiong J. X., Idzorek T. M., Ibbotson R. G. (2014), Volatility versus Tail Risk: Which One Is Compensated in Equity Funds?, "Journal of Portfolio Management", vol. 40 no. 2, pp. 112-121.

Zabarankin M., Pavlikov K., Uryasev S. (2014), Capital Asset Pricing Model (CAPM) with drawdown measure, "European Journal of Operational Research", no. 234, pp. 508-517.

Teoretyczne podejście do ilościowego pomiaru zagrożenia

Streszczenie:

W ocenie wyników portfela inwestycyjnego ważne jest, aby wziąć pod uwagę nie tylko oczekiwaną rentowność, ale także analizować ryzyko. Pomiar ryzyka na podstawie danych historycznych wymaga zastosowania różnych metod. Metody, które uwzględniają zmienność strat są najskuteczniejszą miarą ryzyka. Znaczenie tych metod jest podkreślane przez różne badania empiryczne. W literaturze wykazano, że coraz bardziej popularne staje się wykorzystanie metod pomiaru zagrożenia. Ta popularność można tłumaczyć faktem, że na podstawie wyników badań metody pomiaru zagrożenia pomagają zwiększyć efektywność portfela inwestycyjnego. Celem artykułu jest zatem przedstawienie oraz porównanie różnych metod miar zagrożenia.

Słowa kluczowe: miary zagrożenia, ryzyko spadku
JEL: G10, G20, G31.