Si X est localement compact, on a alors les égalités:

$$\mathcal{C}(T\times X)=\mathcal{C}(T,\mathcal{C}_c(X))=\mathcal{E}\mathcal{T}((T;\mathcal{A}),\pi\mathcal{C}_c(X))=\mathcal{E}\mathcal{T}((T;\mathcal{A}),\mathbf{Hom}(X;\mathcal{K}))$$
$$=\mathcal{E}\mathcal{T}(T\times X;\mathcal{A}\otimes\mathcal{K})$$

d'après 4.5 et 3.4.

Bibliographie

[B_1] N. Bourbaki, *Topologie générale*, ch. 2, 4-ème éd., Paris 1965.
[B_2] — *Topologie Générale*, ch. 10, 2-ème éd., Paris 1961.
[B_3] H. Buchwalter, *Topologie et compactologies*, Pub. Dép. Math. Lyon 6 (1969), pp. 1–74.
[B_4] — *Sur le théorème de Glicksberg-Frolík*, C. R. Acad. Sc. Paris, série A, t. 273 (1971), pp. 11–14.
[B_5] — *Produit topologique, produit tensoriel et c-replétion*, Colloque International d'Analyse fonctionnelle de Bordeaux, Bull. Soc. Math. France, Suppl. au numero de Septembre 1972, pp. 51–71.
[BJ] N. Blanchard et M. Jourlin, *La topologie de la convergence bornée sur les algèbres de fonctions continues*, Pub. Dép. Math. Lyon 6 (1969), pp. 85–96.
[BP_1] H. Buchwalter et R. Pupier, *Une caractérisation topologique de la complétion universelle d'un espace complètement régulier*, C. R. Acad. Sc. Paris, série A, 268 (1969), pp. 1534–1536.
[BP_2] — — *Complétion d'un espace uniforme et formes linéaires*, C. R. Acad. Sc. Paris, série A, 273 (1971), pp. 96–98.
[I] J. R. Isbell, *Uniform spaces*, Math. Surveys nb. 12, Providence 1964.
[K] M. Katětov, *On real-valued functions in topological spaces*, Fund. Math. 38 (1951), pp. 85–91.
[L] F. E. J. Linton, *Autonomous categories and duality of functors*, J. Algebra 2 (1965), pp. 315–349.
[Mc] S. MacLane, *Categories for the working mathematician*, 1971.
[N] N. Noble, *A note on z-closed projections*, Proc. A.M.S. 23 (1969), pp. 73–76.
[P_1] R. Pupier, *Méthodes fonctorielles en topologie générale*, Thèse Sciences Mathématiques, Université de Lyon I, février 1971.
[P_2] — *Topological completion of a product*, à paraitre in Revue Roumaine de Mathématiques Pures et Appliquées.
[W] S. Warner, *The topology of compact convergence on continuous functions spaces*, Duke Math. J. 25 (1958), pp. 265–282.

UNIVERSITÉ DE SAINT-ETIENNE
DEPARTEMENT DE MATHEMATIQUES

Reçu par la Rédaction le 15. 9. 1972

Atomic compactness in $\aleph_1$-categorical Horn theories

by

John T. Baldwin (East Lansing, Mich.)

Abstract. Theorem. *If T is an almost strongly minimal $\forall\exists$ Horn theory then every model of T is atomic compact.*

Mycielski [7] introduced the notion of an atomic compact algebra. A structure $\mathcal{A}$ is atomic compact if each class Σ of atomic formulas (possibly with constants naming elements of $\mathcal{A}$) which is finitely satisfiable in $\mathcal{A}$ is satisfiable in $\mathcal{A}$. Atomic compact structures have been intensively investigated e.g. [8, 11]. Taylor's recent paper [10] contains an extensive bibliography on atomic compact structures.

We were struck by the remark in [7] that all divisible Abelian groups were atomic compact. This meant in particular that every model of the complete $\aleph_1$-categorical theory of infinite, torsion-free divisible Abelian groups was atomic compact. We sought sufficient conditions on a complete first order theory T for all its models to be atomic compact.

We found a narrow class of $\aleph_1$-categorical first order theories which satisfy this condition. These are the almost strongly minimal, model complete, Horn theories. Various properties of almost strongly minimal theories are investigated in [1, 2]. A theory T is model complete if every submodel of a model of T is an elementary submodel. A Horn theory is one axiomatized by Horn sentences. We rely on the fact [9] that the class of models of a Horn theory is closed under direct power. Various examples of theories of the type we are considering are given in [5]. This paper assumes familiarity with [11], sections 1 and 2 of [4] and the first section of [1].

We deal with structures $\mathcal{A}$ which may have both relations and function for a first order language L. The logical connectives of L are $\rightarrow$, $\wedge$, $\vee$, $\sim$, $\forall$ and $\exists$. A formula A is existential ($\exists$-formula) if it is equivalent to a formula in prenex normal form all of whose quantifiers are existential. A is an $\forall\exists$ formula if it is equivalent to a formula in prenex normal form whose prefix consists of a string of universal quantifiers followed by a string of existential quantifiers. A McKinsey formula is a conjunction of atomic and negation of atomic formulas (neg-atomic) at most one of which is atomic. A Horn formula is a formula in prenex normal form whose matrix is a conjunction of McKinsey formulas. A primitive formula is a $\exists$-formula whose matrix is a conjunction of atomic and neg-atomic formulas. A formula is positive if it contains no implication or negation symbols. A formula is negative if it is the negation of a positive formula.

Let $\mathcal{A}$ be an L-structure we say a *set* X *is definable in* $\mathcal{A}$ if there is a formula $A \in S_{n+1}(L)$ and elements $a_1, \ldots, a_n \in |\mathcal{A}|$ such that

$$X = \{a \in |\mathcal{A}|\,|\ \mathcal{A} \models A(a, a_1, \ldots, a_n)\} = A(v_0, a_1, \ldots, a_n)(\mathcal{A}).$$

If we have chosen $A \in S_1(L(\mathcal{A}))$ we may write $A(\mathcal{A})$ for the set of members of $|\mathcal{A}|$ satisfying $\mathcal{A}$. We can naturally embed $\mathcal{A}$ in the countable direct power of $\mathcal{A}$ by the diagonal map. For each $a \in |\mathcal{A}|$ we let $\bar{a}$ be the image of a in $\mathcal{A}^\omega$. If $A \in S_1(L(\mathcal{A}))$, $\bar{A}$ is that member of $S_1(L(\mathcal{A}^\omega))$ obtained by substituting $\bar{a}$ for a. In the following lemma we collect some relationships between the cardinality of $A(\mathcal{A})$ and $\bar{A}(\mathcal{A}^\omega)$.

LEMMA 1. i) *If $A \in S_1(L(\mathcal{A}))$ is a Horn formula and $|A(\mathcal{A})| > 1$ then $\bar{A}(\mathcal{A}^\omega)$ is infinite.*

ii) *Let $B \in S_1(L(\mathcal{A}))$ be a formula of the form $\exists v_1, \ldots, \exists v_n B_1 \wedge B_2$ where B_1 is neg-atomic and B_2 is a Horn formula. If*

$$\mathcal{A} \models \exists v_1, \ldots, \exists v_n, \exists v_{n+1}, \exists v_{n+2}\big(v_{n+1} \neq v_{n+2} \wedge B_1(v_{n+1}) \wedge B_2(v_{n+1}) \wedge B_2(v_{n+2})\big)$$

then $\bar{B}(\mathcal{A}^\omega)$ is infinite

iii) *Let $B \in S_1(L(\mathcal{A}))$ be a formula of the form $B_1 \wedge B_2$ where B_1 is the negation of a positive primitive formula and B_2 is a Horn formula. If $B(\mathcal{A}) \neq \emptyset$ and $|B_2(\mathcal{A})| > 1$ then $\bar{B}(\mathcal{A}^\omega)$ is infinite.*

Proof. i) Let $b_1 \neq b_2 \in |\mathcal{A}|$ such that $\mathcal{A} \models A(b_1) \wedge A(b_2)$. Let $f_i \in \mathcal{A}^\omega$ be defined by

$$f_i(j) = \begin{cases} b_1 & \text{for} \quad j < i, \\ b_2 & \text{for} \quad j \geqslant i. \end{cases}$$

Then for each i $\mathcal{A}^\omega \models \bar{A}(f_i)$.

ii) Let $b_1 \neq b_2 \in |\mathcal{A}|$ such that

$$\mathcal{A} \models \exists v_1, \ldots, \exists v_n B_1(b_1) \wedge B_2(b_1) \wedge B_2(b_2).$$

Define f_i as in i). Once again $\mathcal{A}^\omega \models \bar{B}(f_i)$ for each i.

iii) Choose $b_1 \neq b_2 \in |\mathcal{A}|$ such that

$$\mathcal{A} \models B(b_1, a_1, \ldots, a_n) \wedge B_2(b_2, a_1, \ldots, a_n)$$

and proceed as in i) and ii).

For the rest of the paper we deal with an almost strongly minimal theory T. Applying the definition [1] *we fix a principal extension T' of T and a strongly minimal formula D in $L(T')$ such that if $\mathcal{A} \models T'$*, $|\mathcal{A}| = \mathrm{cl}(D(\mathcal{A}))$. We assume $\mathrm{cl}(\emptyset) \cap D(\mathcal{A})$ is infinite for each model $\mathcal{A}$ of T'.

If T is model complete so is T'. Moreover, if T is a model complete Horn theory the class of models of T' is closed under direct power. If T is model complete we may further assume that D is a primitive formula. For, since T is model complete D is equivalent to an existential formula. Thus D is equivalent to a disjunction of primitive formulas. One of the disjuncts say A_i defines an infinite set such that $D \wedge \sim A_i(\mathcal{A})$ is finite since D is minimal. We take A_i as our strongly minimal formula.

We say that the set X is a *strong basis* for the model $\mathcal{A}$ if for each $a \in |\mathcal{A}|$ there is a primitive formula $A_a \in S_{n+1}(L)$ and elements $x_1, \ldots, x_n$ of X such that

$$\mathcal{A} \models A_a(a, x_1, \ldots, x_n) \wedge \exists!\,{}^1 v_0 A_a(v_0, x_1, \ldots, x_n).$$

For the rest of the paper we assume T is an almost strongly minimal Horn theory and T' is chosen as specified above. Note that by Lindström's Theorem [6], T is model complete.

THEOREM 1. *If $\mathcal{A}$ is a model of T' and X is a basis of $D(\mathcal{A})$ then X is a strong basis for $\mathcal{A}$.*

Proof. By [4, Lemma 8] $\mathcal{A}$ is prime over X. For each $a \in |\mathcal{A}|$ let A_a be the element of $S_{n+1}(L)$ such that there exist $x_1, \ldots, x_n \in X$ and $A_a(v_0, x_1, \ldots, x_n)$ generates the principal type in $\mathrm{Th}(\mathcal{A}, X)$ realized by a. Then as T' is model complete and A_a generates a principal type A_a can be chosen to be a primitive formula. Since A_a generates the principal type realized by a in $\mathrm{Th}(\mathcal{A}, x)$ and since $|\mathcal{A}| = \mathrm{cl}(D(\mathcal{A}))$ there exists an integer k such that

$$\mathcal{A} \models A_a(a, x_1, \ldots, x_n) \wedge \exists!^k v_0 A_a(v_0, x_1, \ldots, x_n).$$

If $|A_a(v_0, x_1, \ldots, x_n)(\mathcal{A})| > 1$ or some conjunct of A_a is negative then $A_a(v_0, \bar{x}_1, \ldots, \bar{x}_n)(\mathcal{A}^\omega)$ is infinite. This result follows from Lemma 2 noticing in the first instance that A_a is a Horn formula and in the second that taking the neg-atomic conjunct for B_1 and the remainder of A_a for B_2 that the hypothesis of Lemma 1 (ii) holds (as A_a generates a principal type). Thus X is a strong basis for $\mathcal{A}$.

Note that the choice of A_a is unique up to equivalence.

COROLLARY 1. *Every non-saturated model of an almost strongly minimal $\forall\exists$ Horn theory has at most $\aleph_0$ automorphisms.*

Proof. T is $\aleph_1$-categorical so by Lindstrom's theorem [6] T is model complete. Hence by Theorem 1 if $\mathcal{A} \models T'$ and X is a basis for $D(\mathcal{A})$ X is a strong basis for $\mathcal{A}$. By [4], if $\mathcal{A}$ is unsaturated X is finite. Hence by [3, Theorem 2] $\mathcal{A}$ has at most $\aleph_0$ automorphisms. By [3, Lemma 1] the reduct of $\mathcal{A}$ to a model of T also has at most $\aleph_0$ automorphisms. Since each model of T is saturated if and only if its expansion to a model of T' is, the result is now immediate.

Following [11] we say a substructure $\mathcal{A}$ of a structure $\mathcal{B}$ is a retract of $\mathcal{B}$ if there is a homomorphism f from $\mathcal{B}$ onto $\mathcal{A}$ such that $f|\mathcal{A} = 1_{\mathcal{A}}$.

THEOREM 2. *If $\mathcal{A}$ and $\mathcal{B}$ are models of T' and $\mathcal{A} \precsim \mathcal{B}$ then $\mathcal{A}$ is retract of $\mathcal{B}$.*

Proof. We first show that if $\mathcal{A}$, $\mathcal{B}$ are models of T and $\mathcal{A} \precsim \mathcal{B}$ such that X is a basis for $D(\mathcal{A})$ and $X \cup \{y\}$ is a basis for $D(\mathcal{B})$ then $\mathcal{A}$ is a retract of $\mathcal{B}$. Fix any $z \in \mathrm{cl}(\emptyset) \cap D(\mathcal{A})$. Let $f(x) = x$ if $x \in X$ and let $f(y) = z$. We will show f has a unique extension to a retraction of $\mathcal{B}$ onto $\mathcal{A}$. To extend the domain of f to all of $|\mathcal{B}|$ note that by Theorem 1 and the remark thereafter for each $b \in |\mathcal{B}|$ there is a unique formula A_b, a unique integer n, and a unique set of n elements $x_1, \ldots, x_n$ from $X \cup \{y\}$ such that

$$\mathcal{B} \models A_b(b, x_1, \ldots, x_n) \wedge \exists!^1 v_0 A(v_0, x_1, \ldots, x_n)\,.$$

Moreover y is among $x_1, \ldots, x_n$ if and only if $b \in |\mathcal{B}| - |\mathcal{A}|$. If $b \in |\mathcal{A}|$ let $f(b) = b$. If $b \notin |\mathcal{A}|$ choose $f(b)$ such that

$$\mathcal{A} \models A_b(f(b), x_1, \ldots, x_{n-1}, z)$$

where without loss of generality we assume $y = x_n$. In order to show f is well defined it suffices to show

$$\mathcal{A} \models \exists!^1 v_0 A_b(v_0, x_1, \ldots, x_{n-1}, z)\,.$$

Let $\mathcal{B}(v_0) = \exists v_1 A_b(v_1, x_1, \ldots, x_n, v_0) \wedge D(v_0)$. Since $y \notin \mathrm{cl}(X)$ $B(\mathcal{B})$ and hence $B(\mathcal{A})$ is infinite. Let $C_1(v_0) = \exists v_1 A_b(v_1, x_1, \ldots, x_n, v_0) \wedge D(v_0)$. Since A_b is a positive primitive formula and $D(\mathcal{A})$ is infinite if $C_1(\mathcal{A}) \neq \emptyset$ by Lemma 1 iii) $\overline{C}_1(\mathcal{B}^\omega)$ is infinite. But this contradicts the minimality of $D(\mathcal{B}^\omega)$ as $\mathcal{B}^\omega \precsim \mathcal{B}$ implies $\overline{B}(\mathcal{B}^\omega)$ is infinite. So $C_1(\mathcal{A}) = \emptyset$. The formula $\exists^{>1} v_1 A_b(v_1, x_1, \ldots, x_{n-1}, v_0) \wedge D(v_0)$ abbreviates

$$C(v_0) = \exists v_1 \exists v_2 [v_1 \neq v_2 \wedge A_b(v_1, x_1, \ldots, x_{n-1}, v_0) \wedge A_0(v_2, x_1, \ldots, v_0) \wedge D(v_0)]\,.$$

Since $|D(\mathcal{A})| > 1$ we can apply Lemma 1 ii) to $C(v_0)$ and conclude by a similar argument to that for C_1 that $C(\mathcal{A}) = \emptyset$. Thus

$$\mathcal{A} \models \forall v_0 [D(v_0) \to \exists!^1 v_1 (v_1, x_1, \ldots, x_{n-1}, v_0)]$$

and in particular $\mathcal{A} \models \exists!^1 v_0 A(v_0, x_1, \ldots, x_{n-1}, z)$. Thus f is a map from $|\mathcal{B}|$ onto $|\mathcal{A}| = 1_{\mathcal{A}}$.

It remains to show that f is a homomorphism. That is if $b_1, \ldots, b_n \in |\mathcal{B}|$ and R is an n-ary atomic formula such that $\mathcal{A} \models R(b_1, \ldots, b_n)$ we must show $\mathcal{B} \models R(f(b_1), \ldots, f(b_n))$. Let $A_1, \ldots, A_n$ be the positive primitive formulas generating the principal types of $b_1, \ldots, b_n \in \mathrm{Th}(\mathcal{B}, X \cup \{y\})$. We may assume that A_i and A_j contain distinct bound variables if $i \neq j$ and that the same set $\{x_1, \ldots, x_m\}$ occur in each A_i. If y is in this set let it be x_m.

$$\mathcal{B} \models \forall v_1, \ldots, \forall v_n [\bigwedge_{i=1}^{n} A_i(v_i, x_1, \ldots, x_m) \to R(v_1, \ldots, v_n)]\,.$$

If $y \notin \{x_1, \ldots, x_m\}$, $\mathcal{A} \models R(f(b_1), \ldots, f(b_n))$. If $y \in \{x_1, \ldots, x_m\}$ we want to show $\mathcal{A} \models C(z)$ where

$$C(v_0) = \forall v_1, \ldots, \forall v_n [\bigwedge_{n=1}^{n} A_i(v_i, x_1, \ldots, x_{m-1}, v_0) \to R(v_1, \ldots, v_n)]\,.$$

If

$$B(v_0) = \exists v_1, \ldots, \exists v_n [\bigwedge_{i=1}^{n} A_i(v_i, x_1, \ldots, x_{m-1}, v_0) \wedge R(v_1, \ldots, v_m) \wedge D(v_0)]$$

and $B(\mathcal{A}) \neq \emptyset$ then by Lemma 1 ii) (noticing that $\bigwedge_{i=1}^{n} A_i(v_i, x_1, \ldots, x_{m-1}, v_0) \wedge \wedge D(v_0)$ is equivalent to a Horn formula) $\overline{B}(\mathcal{A}^\omega)$ is infinite. That is, if $\mathcal{A} \models \sim C(z)$ then $\sim \overline{C}(v_0) \wedge D(v_0)(\mathcal{A}^\omega)$ is infinite. On the other hand $\mathcal{B} \models C(y)$ and $y \notin \mathrm{cl}(X)$ so $C(v_0) \wedge D(v_0)(\mathcal{B})$ is infinite. Taking a common elementary extension of $\mathcal{B}$ and $\mathcal{A}_\omega$ this contradicts the strong minimality of D. Thus f is a homomorphism.

It is now easy to establish by induction that if $\mathcal{A}$, $\mathcal{B}$ are models of T' $\mathcal{A} \precsim \mathcal{B}$, and $\mathcal{B}$ has finite dimension over $\mathcal{A}$, i.e. X is a basis for $D(\mathcal{A})$, Y is a basis for $D(\mathcal{B})$ extending X, and $|Y - X|$ is finite, then $\mathcal{A}$ is a retract of $\mathcal{B}$. Moreover the retraction is determined by taking each member of $Y - X$ into a fixed member z of $\mathcal{A}$.

Now let $\mathcal{B}$ be an arbitrary elementary extension of $\mathcal{A}$. Let X be a basis for $D(\mathcal{A})$ and $X \cup \{y_\alpha\}_{\alpha<\lambda}$ be a basis for $D(\mathcal{B})$. Then the map f defined by fixing each member of $\mathcal{A}$ and taking each $\langle y_\alpha \rangle_{\alpha<\lambda}$ to a fixed $z \in |\mathcal{A}|$ is the required retraction. To verify this notice that each finite subset of $\mathcal{B}$ is contained in an elementary extension of $\mathcal{A}$ which has finite dimension over $\mathcal{A}$.

THEOREM 3. *If T is an almost strongly minimal model complete Horn theory then every model of T is atomic compact.*

Proof. It is immediate from Theorem 2 and Theorem 2.3 of [11] which asserts that a structure $\mathcal{A}$ is atomic compact if it is a retract of its every elementary extension, that each model $\mathcal{B}$ of T' is atomic compact. But then so is the reduct of $\mathcal{B}$ to a model of T.

COROLLARY 1. *If T is an almost strongly minimal $\forall\exists$ Horn theory then every model of T is atomic compact.*

Proof. By Lindstrom's Theorem [6] T is model complete and the result follows from Theorem 3.

COROLLARY 2. *If T is a universal Horn theory and T', the theory of the infinite models of T, is complete then every model of T is atomic compact.*

Proof. All finite structures are atomic compact so it suffices to consider the models of T'. By the main theorem of [5] T' is $\aleph_1$-categorical. By the corollary to Theorem 1 of [5], T' is almost strongly minimal. T' is model complete by Lindstrom's theorem and the result follows.

The situation regarding possible strengthenings of the last three results is clarified by noticing that the last example in [4] has the following properties. T is a $\forall\exists$ $\aleph_1$-categorical Horn theory which is not almost strongly minimal but each model of T is atomic compact.

References

[1] J. T. Baldwin, *Almost strongly minimal theories I*, J. Symbolic Logic 37 (1972).

[2] — *Almost strongly minimal theories II* (to appear J. Symbolic Logic).

[3] — *The number of automorphisms of a model of an $\aleph$-categorical theory* (to appear Fund. Math.).

[4] — and A. H. Lachlan, *On strongly minimal sets*, J. Symbolic Logic 36 (1971), pp. 79–96.

[5] — — *On universal Horn theories categorical in some infinite power* (to appear Algebra Universalis).

[6] P. Lindstrom, *On model completeness*, Theoria 30 (1964), pp. 183–196.

[7] J. Mycielski, *Some compactifications of general algebras*, Colloq. Math. 13 (1964), pp. 1–9.

[8] — and C. Ryll-Nardzewski, *Equationally compact algebras II*, Fund. Math. 61 (1968), pp. 271–281.

[9] J. R. Shoenfield, *Mathematical Logic*, 1967.

[10] W. Taylor, *Some constructions of compact algebras*, Annals of Math. Log. 3 (1971), pp. 395–437.

[11] B. Węglorz, *Equationally compact algebras* (I), Fund. Math. 59 (1966), pp. 289–298.

MICHIGAN STATE UNIVERSITY
East Lansing, Michigan

Reçu par la Rédaction le 17. 11. 1972

On limit numbers of real functions

by

Jacek Marek Jędrzejewski (Łódź)

Abstract. In this work is given a general way of introducing of limit numbers for real function of real variable. With every real number is connected some family of sets fulfilling two natural conditions. They assure that for arbitrary function at every point x there exists at least one limit number and the set of all limit numbers at a point x is closed. By adequate adjustment of the family $\mathfrak{B}$ one can get usual limit numbers or approximate limit numbers. The main results of the work are concerned with the questions of: the set of points of $\mathfrak{B}$-asymmetry, connections between the ordinary continuity and $\mathfrak{B}$-continuity and $\mathfrak{B}$ semicontinuity of upper and lower $\mathfrak{B}$-functions of Baire.

Introduction. The aim of this work is to generalize the notion of limit numbers and approximate limit numbers and to find some properties of these generalized limit numbers. To obtain this it will be convenient to use the following definition:

If $f\colon R\to R$ and $x_0\in R$, where R denotes the set of all real numbers, then g is called the *limit number of f at x_0* if and only if, for every $\varepsilon>0$, x_0 is a point of accumulation of the set $\{x\colon |f(x)-g|<\varepsilon\}$.

The starting point of my considerations is the following remark: the family $\mathfrak{B}$ of all sets having x_0 as a point of accumulation have the following properties:

(1) every set including the set from $\mathfrak{B}$ also belongs to $\mathfrak{B}$,

(2) if $E_1\cup E_2\in\mathfrak{B}$, then $E_1\in\mathfrak{B}$ or $E_2\in\mathfrak{B}$,

(3) if $E_1\in\mathfrak{B}$, then for every $t>0$ also $E_1\cap(x_0-t, x_0+t)\in\mathfrak{B}$.

There is a very similar situation in the case of approximate limit numbers. Now $\mathfrak{B}$ is the family of all sets for which x_0 is not a point of dispersion.

The foregoing generalization will depend on making use of rather arbitrary families of sets fulfilling only conditions (1)-(3). These conditions seem to be natural, because the set of limit numbers of an arbitrary functions at every point obtained by means of them is non empty and closed.