thor's[1]), P is regularly accessible from E, and hence $E+P$ is arcwise connected. Hence it follows that $G+P$ is arcwise connected, and therefore, by another theorem of mine[2]) $G+P$ is arcwise connected im kleinen. But since H is connected im kleinen, it follows with the aid of a theorem of R. L. Wilder's[3]) that P is not a limit point of $H-(G+P)$. Hence H is arcwise connected im kleinen at every point of K. It was shown above that H is arcwise connected im kleinen at every point of $H-K$. Hence H is arcwise connected im kleinen at every one of its points. Then since H is connected, it follows by a theorem of the author's[4]) that H is arcwise connected.

I have just shown that every connected subset of each maximal cyclic curve of M is arcwise connected. Therefore, by a theorem of mine[5]), every connected subset of M is arcwise connected.

Theorem 31. *If no maximal cyclic curve of a continuous curve M contains an infinite collection of mutually exclusive simple closed curves, then every connected subset of M is arcwise connected.*

Theorem 31 follows immediately from Theorems 7 and 30.

Theorem 32. *If the ramification points of each maximal cyclic curve of a continuous curve M are finite in number, then every connected subset of M is arcwise connected.*

Theorem 32 is an immediate consequence of Theorems 18 and 32.

Problem. *If the ramification points of each maximal cyclic curve of a continuous curve M are countable in number, then is every connected subset of M arcwise connected?*

[1]) Loc. cit. Theorem 15.

[2]) G. T. Whyburn, *Concerning certain types of continuous curves*, loc. cit. Theorem 5.

[3]) *A Theorem on connected point sets which are connected im kleinen*, Bull. Amer. Math Soc., vol. 32 (1926), pp 338—340.

[4]) G. T. Whyburn, *Concerning the complementary domains of continua*, loc. cit., Theorem 12.

[5]) G. T. Whyburn, *Concerning the structure of a continuous curve*, loc. cit., Theorem 33. This theorem is as follows: *In order that every connected subset of a continuous curve M should be arcwise connected it is necessary and sufficient that every connected subset of each maximal cyclic curve of M should be arcwise connected.*

University of Texas, Nov. 14, 1927.

A separation theorem.

By

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In my paper *Concerning the separation of point sets by curves*[1]) it is stated that if T is a totally disconnected closed subset of the boundary of a simply connected domain D and there exists a continuum K containing T and such that $K-T$ is a subset of D then there exists a simple closed curve J containing T and enclosing $K-T$ and such that $J-T$ is a subset of D. That this proposition does not hold true, in the form in which it is stated, even for the case where T is a single point on the outer boundary of D, may be seen with the aid of the following example.

Example. Let T, A, B and C denote the points (0, 0), (2, 0), (2, 2) and (0, 2) respectively. For each positive integer n, let F_n denote the point $(1, 1/n)$ and let T_n denote the point whose abscissa is $1/n$ and whose ordinate is $(2n+1)/(n^2+n)$. Let M denote the continuum composed of the straight line intervals TA, AB, BC and CT together with all the straight line intervals of the sequence TF_1, TF_2, TF_3,... Let K denote the sum of all the intervals of the sequence TT_1, TT_2, TT_3,... Let D denote the bounded complementary domain of the continuum M. There exists no simple closed curve J containing T and enclosing $K-T$ and such that $J-T$ is a subset of D.

The following modification of the proposition in question holds true and suffices as a substitute in some of the applications in which the use of that proposition may seem to be indicated.

[1]) Proc. Nat. Acad. Sc., vol. 11 (1925), pp. 469—476.

Theorem. *If, in space of two dimensions, T is a totally disconnected closed subset of the boundary of a simply connected bounded domain D and K is a continuum containing T and such that $K-T$ is a subset of D, and L is a maximal connected subset of $K-T$, then there exists a simple closed curve J containing a part of T and enclosing L and such that $J-J.T$ is a subset of $D-(K-T)$.*

Proof. [1]). Let S denote the set of all points of the plane and let H denote the boundary of D. Since T is closed and bounded there exists a sequence of bounded point sets $D_1, D_2, D_3, \ldots$ such that (a) for every n, the set $S-D_n$ is closed and D'_{n+1} is a subset of D_n, (b) T is the set of all points common to the point sets $D_1, D_2, D_3, \ldots$, (c) not all points of K belong to D'_1. For each n let K_n denote the set of all those points of K which do not belong to D'_n. There exists a finite set G_1 of circular domains, all of diameter less than 1, such that (a) every point of K'_1 belongs to some domain of the set G_1, (b) no point of D'_2+H is in, or on the boundary of, any domain of G_1. There exists a finite set G_2 of circular domains, all of diameter less than $1/2$, such that (a) every point of K'_2-K_1 is in some domain of G_2, (b) no point of D'_3+H is in, or on the boundary of, any domain of G_2. There exists a finite set G_3 of circular domains, all of diameter less than $1/3$, such that (a) every point of K'_3-K_2 is in some domain of G_3, (b) no point of D'_4+H or of $S-D_1$ is in, or on the boundary of, any domain of G_3. This process may be continued. Thus there exists an infinite sequence $G_1, G_2, G_3, \ldots$ such that (a) for every n, G_n is a finite set of circular domains, all of diameter less than $1/n$, (b) for every n, G_{n+2} covers $K'_{n+2}-K_{n+1}$ but no point of $D'_{n+3}+H$ or of $S-D_n$ is in, or on the boundary of, any domain of G_{n+2}. Let Z denote the set obtained by adding together the domains of all the sets $G_1, G_2, G_3, \ldots$ and let R denote the greatest connected subset of Z that contains L. The continuum R' is connected im kleinen. For suppose first that P is a point of R' not belonging to T. There exists a circle C with center at P and neither containing nor enclosing any point of T. There exists a positive integer m such that if $n>m$ then no domain of the set G_n contains a point within C. Let Q_m denote the point set

[1]) This proof has much in common with the proof of Theorem 2 in my above mentioned paper.

obtained by adding together all circular domains q such that, for some i less than or equal to m, q belongs to the set G_i and lies in R. The point set Q'_m is the sum of a finite number of circles plus their interiors. Hence Q'_m is connected im kleinen at the point P. Since Q'_m is a subset of R' and contains every point of R' that lies within C therefore R' is connected im kleinen at P. Thus R' is connected im kleinen at every point of $R'-R'.T$. If there exist any points of R' at which it is not connected im kleinen there must exist a continuum of such points and this continuum must be a subset of T, contrary to the hypothesis that T is totally disconnected. It follows that R' is a continuous curve. Let E denote the unbounded complementary domain of R' and let β denote the boundary of E. The point set β is [1]) a continuous curve. Hence the outer boundary of E with respect [2]) to R is a simple closed curve J. The curve J is a subset of the boundary of R and it encloses R, and therefore L. Since it is a subset of the boundary of R it contains no point of $H+K-T$. But it is a subset of $D+H$. Therefore it contains points of D. Hence there are points of D without J. Therefore, since D is bounded, there are points of H without J. But there are points of K within J and $K+H$ is connected. Hence J contains at least one point of $K+H$. Thus it contains at least one point of T.

[1]) Loc. cit., page 475.

[2]) Loc. cit.