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Convergence of the identification algorithm applied to the mutual inductance of the induction motor

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Abstract: A new observer of induction motor state variables is proposed in the paper. A nonlinearity of the main magnetic path is expressed as a function of a properly chosen parameter versus the position vector length. The value of the mutual inductance received in the identification algorithm is calculated exploiting the estimated values of the state variables. The coefficients appearing in the differential equations of the observer system are modified in each step of the algorithm on the basis of the calculated mutual inductance. The analysis of convergence of the identification algorithm is shown in this paper.

Key words: induction motor, mutual inductance, identification, convergence

1. Introduction

Simple construction, low production costs and easy operation are the reasons behind the fact that induction motors are commonly used electrical machines. The drives with controlled shaft speed contain the induction motor, power electronic converter and a control system. An ongoing progress in the field of production of fast solid state switching devices makes it possible to construct the converters of an ever-increasing power, which are the sources of alternating voltage or current with low harmonic content.

The mathematical model with constant parameters used commonly in drive control systems with induction motors does not satisfy the requirements concerning the accuracy of estimating a rotor flux vector put forward by precise drive systems.

A synthesis of a control system with structure resulting from the implementation of a mathematical model of the induction motor based on space vector components as state variables [1] requires the application of variable estimating system. Quick estimation of the rotor flux plays a significant role in control systems based on non-linear feedback. A simplified model of induction motor taking the saturation of the main magnetic path into account has been proposed in [2, 4] using the position vector method presented in [5, 6].

An analysis of convergence of the identification algorithm applied to estimation of the mutual inductance using the nonlinear observer of state variables is presented in the paper. Results of simulation and experimental investigations are presented and discussed.

2. The observer of induction motor state variables with nonlinearity of main magnetic path taken into account

On the basis of mathematical model of the induction motor taking into account the saturation of the main magnetic path referred to in papers [2-4], as well as on the basis of an observer of state variables of induction motor proposed by Krzemiński [6-8], the following equations of a state variable observer are written in stationary coordinate system:

$$\frac{d\hat{i}_{sx}}{dt}=\frac{1}{L_{p2}}\left(-R_s\hat{i}_{sx}-L_{p1}\left(R_rL_{p1}\hat{i}_{sx}-R_rL_p\hat{\psi}_{rx}-\omega_r\hat{\psi}_{ry}\right)+u_{sx}\right)+k_i\left(i_{sx}-\hat{i}_{sx}\right), \tag{1}$$

$$\frac{d\hat{i}_{sy}}{dt}=\frac{1}{L_{p2}}\left(-R_s\hat{i}_{sy}-L_{p1}\left(R_rL_{p1}\hat{i}_{sy}-R_rL_p\hat{\psi}_{ry}-\omega_r\hat{\psi}_{rx}\right)+u_{sy}\right)+k_i\left(i_{sy}-\hat{i}_{sy}\right), \tag{2}$$

$$\frac{d\hat{\psi}_{rx}}{dt}=R_rL_{p1}\hat{i}_{sx}-R_rL_p\hat{\psi}_{rx}-\omega_r\hat{\psi}_{ry}+k_{f1}\left(i_{sx}-\hat{i}_{sx}\right)-k_{f2}\omega_r\left(i_{sy}-\hat{i}_{sy}\right), \tag{3}$$

$$\frac{d\hat{\psi}_{ry}}{dt}=R_rL_{p1}\hat{i}_{sy}-R_rL_p\hat{\psi}_{ry}-\omega_r\hat{\psi}_{rx}+k_{f1}\left(i_{sy}-\hat{i}_{sy}\right)-k_{f2}\omega_r\left(i_{sx}-\hat{i}_{sx}\right), \tag{4}$$

where:

$$L_p=\frac{1}{L_r}, \tag{5}$$

$$L_{p1}=1-L_{\sigma r}L_p, \tag{6}$$

$$L_{p2}=L_{\sigma s}+L_{\sigma r}L_{p1}, \tag{7}$$

$$L_r=L_{\sigma r}+M. \tag{8}$$

M is the mutual inductance of the windings, i_{sx}, i_{sy}, u_{sx}, u_{sy} are the components of space vectors of stator voltage and current, $\hat{i}_{sx}, \hat{i}_{sy}, \hat{\psi}_{rx}, \hat{\psi}_{ry}$ are the components of estimated space vectors of current, stator and rotor flux linkage, R_r, R_s are the rotor and stator resistances, k_i, k_{f1}, k_{f2} are the coefficients of weight matrix.

The saturation of the main magnetic path is represented by the following equation:

$$L_p = g\left(\left|\mathbf{v}_p\right|\right), \tag{9}$$

where:

$$\mathbf{v}_p = L_{\sigma r}\hat{\mathbf{i}}_s + \hat{\boldsymbol{\psi}}_r \tag{10}$$

is the position vector introduced in [5], $\hat{\mathbf{i}}_s$, $\hat{\boldsymbol{\psi}}_r$ are the stator current and rotor flux vectors.

3. Algorithm for identification of the mutual inductance

For the purpose of forming the equation for computing the value of mutual inductance M, relationships were used between the variables of multiscalar induction motor described in [6]. In that model the variables were proposed which were the results of the scalar and vector multiplications of the vectors.

The algorithm for estimation of the mutual inductance created on the basis of the multiscalar model of the induction motor was described in [2-4].

The formula to calculate the value of mutual inductance $\hat{M}_{(k)}$ estimated in a k sampling step of the observer takes the following form:

$$\hat{M}_{(k)} = \frac{\hat{x}_{21(k)}}{\hat{x}_{22(k)}}, \tag{11}$$

where $\hat{x}_{21(k)}, \hat{x}_{22(k)}$ are the values of multiscalar model variables estimated in the k sampling step of the observer defined as follows:

$$x_{21} = \Psi_r^2, \tag{12}$$

$$x_{22} = \Psi_{rx} i_{sx} + \Psi_{ry} i_{sy}. \tag{13}$$

At every sampling step of the observer the calculation of estimated parameters of the motor takes place. The coefficients in the observer equations are modified using the calculated values of estimated parameter. The value of mutual inductance M is therefore constantly updated on the basis of the current values of the motor state variables.

4. Convergence analysis of identification algorithm

During the convergence analysis of identification algorithm of magnetisation curve in the observer of induction motor state variables, the procedure presented in [2] was employed. In a coordinate system rotating with an angular speed equal to the rotation speed of one of the space vectors, the component derivatives of this vector is equal to zero in the steady-state. After the transformation of observer Equations (1)-(4) the following algebraic equations for the steady-state were formed, written in coordinate system rotating with angular velocity equal to the pulsation of the first harmonic of the stator voltage $\overline{u}_s$:

$$0=\frac{1}{\hat{L}_{p2}}\left(-R_s\hat{i}_{sx}-\hat{L}_{p1}\left(R_r\hat{L}_{p1}\hat{i}_{sx}-R_r\hat{L}_p\hat{\Psi}_{rx}-\omega_r\hat{\Psi}_{ry}\right)+u_{sx}\right)+\omega_o i_{sy}+k_i\left(i_{sx}-\hat{i}_{sx}\right), \tag{14}$$

$$0=\frac{1}{\hat{L}_{p2}}\left(-R_s\hat{i}_{sy}-\hat{L}_{p1}\left(R_r\hat{L}_{p1}\hat{i}_{sy}-R_r\hat{L}_p\hat{\Psi}_{ry}-\omega_r\hat{\Psi}_{rx}\right)+u_{sy}\right)+\omega_o i_{sx}+k_i\left(i_{sy}-\hat{i}_{sy}\right), \tag{15}$$

$$0=R_r\hat{L}_{p1}\hat{i}_{sx}-R_r\hat{L}_p\hat{\Psi}_{rx}+\left(\omega_o-\omega_r\right)\hat{\Psi}_{ry}-k_{f1}\left(i_{sx}-\hat{i}_{sx}\right)-k_{f2}\omega_r\left(i_{sy}-\hat{i}_{sy}\right), \tag{16}$$

$$0=R_r\hat{L}_{p1}\hat{i}_{sy}-R_r\hat{L}_p\hat{\Psi}_{ry}-\left(\omega_o-\omega_r\right)\hat{\Psi}_{rx}-k_{f1}\left(i_{sy}-\hat{i}_{sy}\right)+k_{f2}\omega_r\left(i_{sx}-\hat{i}_{sx}\right), \tag{17}$$

where ω_o is the pulsation of the first harmonic of the stator current.

After multiplication of (14) and (15) by the components of the rotor flux, and subsequently adding both sides of the equation and assuming that

$$\hat{\Psi}_{rx}i_{sx}+\hat{\Psi}_{ry}i_{sy}\approx x_{22}\ \text{ and }\ \hat{\Psi}_{rx}i_{sy}-\hat{\Psi}_{ry}i_{sx}\approx x_{12}, \tag{18}$$

the following equation for the steady-state is received:

$$-\hat{x}_{21}+\hat{M}\,\hat{x}_{22}-\frac{L_r}{R_r}k_{f1}\left(x_{22}-\hat{x}_{22}\right)-\frac{L_r}{R_r}k_{f2}\omega_r\left(x_{12}-\hat{x}_{12}\right)=0. \tag{19}$$

After the transformations described in [2], omission of the components with values close to zero and assumption that

$$\hat{M}_{n+1}=\frac{x_{21}+\Delta x_{21n}}{x_{22}+\Delta x_{22n}}, \tag{20}$$

the following relationship occurs:

$$\begin{aligned}&\left(\frac{k_{f2}\omega_r}{\omega_o-\omega_r}-M\right)\Delta x_{22n}+\left(M-\frac{k_{f2}\omega_r}{\omega_o-\omega_r}+\frac{L_r}{R_r}k_{f1}\right)\Delta x_{22n+1}+\\&+\frac{R_r\hat{M}+L_rk_{f1}}{L_r\left(\omega_o-\omega_r\right)}\left(\Delta x_{12n}-\Delta x_{12n+1}\right)+\frac{L_r}{R_r}\omega_rk_{f2}\Delta x_{12n+1}=0,\end{aligned} \tag{21}$$

where $\Delta x_{21n}, \Delta x_{22n,}\ \Delta x_{12n}, \Delta x_{21n+1}, \Delta x_{22n+1}, \Delta x_{12n+1}$ are errors of multiscalar variable estimations in subsequent steps of identification algorithm and M is the actual value of the mutual inductance of the windings.

For the purpose of determining a relationship between estimation errors it is advisable to employ equations of the mathematical model of the induction motor described in [2]. The equations for the derivatives of stator current vector components written in the steady-state for coordinate system rotating with an angular speed ω_o take the following form:

$$0=\frac{1}{L_{p2}}\Big(-R_s\,i_{sx}-L_{p1}\left(R_rL_{p1}i_{sx}-R_rL_p\Psi_{rx}-\omega_r\Psi_{ry}\right)+u_{sx}\Big)+\omega_o i_{sy}, \tag{22}$$

$$0=\frac{1}{L_{p2}}\Big(-R_s\,i_{sy}-L_{p1}\left(R_rL_{p1}i_{sy}-R_rL_p\Psi_{ry}+\omega_r\Psi_{rx}\right)+u_{sy}\Big)+\omega_o i_{sx}. \tag{23}$$

After simple transformation the following equation is received:

$$-\frac{R_s+L_{p1}^2R_r}{L_{p2}}i_s^2+\frac{R_rL_p}{L_{p2}}x_{22}-\frac{\omega_r}{L_{p2}}x_{12}+\frac{1}{L_{p2}}\left(u_{sx}\,i_{sx}+u_{sy}\,i_{sy}\right)=0. \tag{24}$$

The equation of the steady-state for the variables estimated in the observer take the following form:

$$-\frac{R_s+L_{p1}^2R_r}{L_{p2}}\hat{i}_s^2+\frac{R_rL_p}{L_{p2}}\hat{x}_{22}-\frac{\omega_r}{L_{p2}}\hat{x}_{12}+\frac{1}{L_{p2}}\left(u_{sx}\,\hat{i}_{sx}+u_{sy}\,\hat{i}_{sy}\right)-k_i\Delta i_s^2=0. \tag{25}$$

where $\hat{i}_s$ is the estimated module of the stator current vector, Δi_s is the estimation error of the module of stator current vector.

Assumption of large weight matrix coefficients of the observer makes the estimation error of stator current vector $\overline{i}_s$ after the completion of transient process negligible. One may therefore make the following assumptions:

$$\Delta i_s\approx 0,\ \hat{i}_{sx}\approx i_{sx},\ \hat{i}_{sy}\approx i_{sy},\hat{i}_s\approx i_s, \tag{26}$$

where i_s is the module of stator current space vector.

After the transformations an approximate relationship between errors of estimation occurs:

$$\Delta x_{12}=\frac{R_r}{L_r\omega_r}\Delta x_{12}. \tag{27}$$

The convergence of the iterative algorithm is conditioned on the following inequality being true:

$$\left|\Delta x_{22n+1}\right|<\left|\Delta x_{22n}\right|, \tag{28}$$

where: $n\in N,\ n\to\infty.$

Substituting (27) into (21) results in establishing relationships between errors of estimation in the subsequent sampling steps of the observer. After taking into account (28) the following convergence condition is received:

$$\left(k_{f1}L_r+k_{f2}R_r\right)\left(\omega_o-\omega_r\right)\omega_r>0. \tag{29}$$

Fulfillment of the condition (29) results in the choice of the values of coefficients k_{f1} and k_{f2} depending on the motor working point. Iterative algorithm is convergent for the following conditions:

1) Motor working area: $\omega_r<\omega_o\wedge\omega_r<0,$ inequality (29) is satisfied for

$$k_{f1} + k_{f2}\frac{R_r}{L_r} > 0, \tag{30}$$

2) Generator working area: $\omega_r < \omega_o \wedge \omega_r < 0$ inequality (29) is satisfied for

$$k_{f1} + k_{f2}\frac{R_r}{L_r} < 0, \tag{31}$$

3) Brake working area: $\omega_r < \omega_o \wedge \omega_r < 0$ inequality (29) is satisfied for

$$k_{f1} + k_{f2}\frac{R_r}{L_r} < 0. \tag{32}$$

If the parameters representing the saturation of the main magnetic path in the observer equations are estimated with an error, then in the steady-state there shall be a constant error of estimating the stator and rotor flux current vector components. If – calculated using the relationships (5)-(7), on the basis of the state variables values – the parameters $\hat{L}_p$, $\hat{L}_{p1}$, $\hat{L}_{p2}$ are introduced to differential equations of the observer, then the error of state variables estimation should be reduced. The error of calculating the parameters $\hat{L}_p$, $\hat{L}_{p1}$, $\hat{L}_{p2}$ should also be reduced.

For the purpose of checking the correctness of the above-mentioned reasoning, a simulation researches were carried out. Differential equations of the motor model and the observer have been solved according to Runge-Kutta method as well as algebraic equations of the steady-state in order to calculate the state variables and identification error of $\hat{L}_p$, $\hat{L}_{p1}$, $\hat{L}_{p2}$ parameters. The initial value of the mutual inductance was assumed to be equal to 10% of the rated value. The results of the simulations are presented in Figure 1.

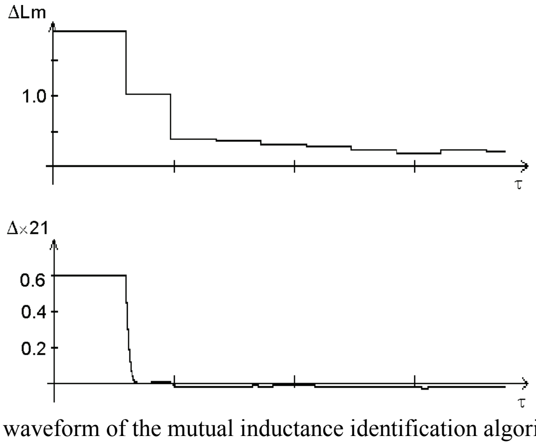

Fig. 1. Convergence waveform of the mutual inductance identification algorithm, τ = 100 ms/div

Simulation of the identification algorithm proved that after substitution of the parameter appearing in the equations of the observer by another one – calculated on the basis of the estimated state variables – representing the saturation of the main magnetic path, a reduction of variable state estimation error $\Delta \times 21$ as well as the mutual inductance identification error ΔLm is received at each subsequent sampling step.

5. Results of experimental research

Experimental research of state variable observer of induction motor with windings' mutual inductance identification algorithm was conducted at a research station with an induction motor loaded on a shaft with a direct current machine. The experiments were carried out feeding the induction motor from PWM voltage source inverter. Output voltage frequency was approximately 10 Hz at an amplitude reduced to 65 V. Observer algorithm was realised through the use of a 32-bit signal procesor, while the control of measuring transducers, the control algorithm of PMW converter and the control of emergency state signalisation – through the use of ALTERA programmable logic system. The observer algorithm as well as the control of measuring transducers were called every 100 µs. In Figure 2 is shown schematic diagram of the experimental stand.

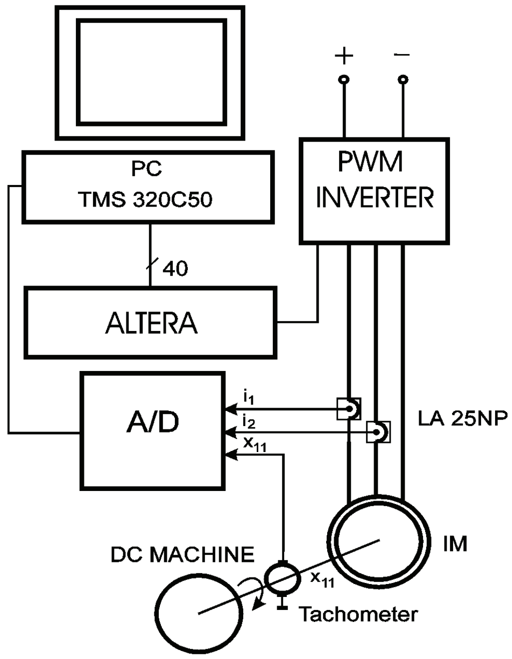

Fig. 2. Schematic diagram of the experimental stand

Waveforms of the estimated variables of the observer system by load torque equal to zero are presented in Figure 3. Waveforms of the estimated variables for the step-change of the value of motor load torque by 15% are presented in Figure 4. The following notations of variables were used: *x*12*o* – estimated electromagnetic torque of a motor, *isxo*-*x*-axis of the estimated stator current, *Lmo* – estimated mutual inductance of the windings.

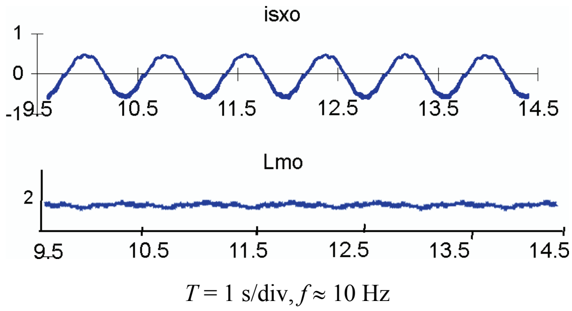

Fig. 3. Waveforms of the estimated variables of the observer system by load torque equal to zero

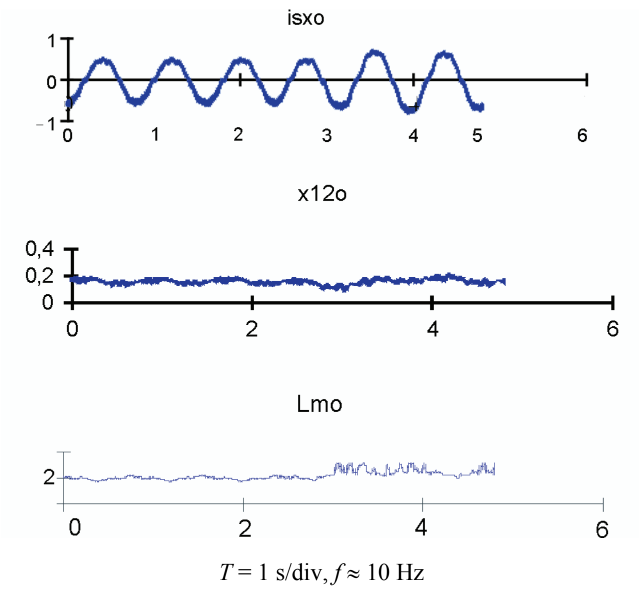

Fig. 4. Waveforms of the estimated variables for the step-change of the value of motor load torque by 15%

6. Conclusions from the experimental research

Experimental research of the induction motor fed by PWM voltage source inverter showed correct functioning of the system of state variable observer with windings' mutual inductance estimating algorithm.

The presented identification method makes it possible the functioning of a non-linear state variable observer in the drive control system without the need of experimental determining of magnetisation curve of the main magnetic path. Simulation and experimental research have proved the correctness of mathematical proof of the convergence of the identification algorithm for the mutual inductance.

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