ARCHIVES OF ELECTRICAL ENGINEERING VOL. 64(3), pp. 379-389 (2015)

DOI 10.2478/aee-2015-0030

Estimation of coefficients of multivariable power series approximating magnetic nonlinearity of AC machines*

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(Received: 31.09.2014, revised: 15.01.2015)

Abstract: Energy based approach was used in the study to formulate a set of functions approximating the magnetic flux linkages versus independent currents. The simplest power series that approximates field co-energy and linked fluxes for a two winding core of an induction machine are described by a set of common unknown coefficients. The authors tested three algorithms for the coefficient estimation using Weighted Least-Squared Method for two different positions of the coils. The comparison of the approximation accuracy was accomplished in the specified area of the currents. All proposed algorithms of the coefficient estimation have been found to be effective. The algorithm based solely on the magnetic field co-energy values is significantly simpler than the method based on the magnetic flux linkages estimation concept. The algorithm based on the field co-energy and linked fluxes seems to be the most suitable for determining simultaneously the coefficients of power series approximating linked fluxes and field co-energy.

Key words: magnetism, magnetic field co-energy, mathematical modelling, parameters estimation

1. Introduction

Determining winding inductances of electrical machines with nonlinear cores is very important for their circuital models [1, 2]. Usually magnetic field computation techniques are employed to obtain the physical quantities which are necessary in calculations procedures [7]. Accurate field method was described in [5]. The simplified method based on the distribution of magnetic field density in an air gap of an electric machine was mentioned in [10]. The indirect but more simple method based on the magnetic co-energy function used the set of functions approximating linked fluxes of windings was proposed in [4]. This method has also been described in [9]. An inverse modelling method using the functions approximating independent currents versus linked fluxes of windings was presented in [12]. The aim of this paper

* This is extended version of a paper which was presented at the 23th Symposium on Electromagnetic Phenomena in Nonlinear Circuits, Pilsen, Czech Republic 02-04.07, 2014.

is to test the accuracy of different versions of the method based on the magnetic co-energy and to indicate their usefulness.

To create machine equations using the Lagrange's formalism it is sufficient to know the co-energy E_{mo} as a function of all independent currents, constituting the vector $\mathbf{i} = [i_1 \cdots i_{\mathrm{N}}]$, and the rotary angle φ of a rotor. Then a set of the electric equations for N – windings takes the form:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial E_{\mathrm{mo}}(\mathbf{i},\varphi)}{\partial i_n}\right) + \mathrm{R}_n \cdot i_n = u_n \quad n \in \{1, 2, ..., \mathrm{N}\}, \tag{1a}$$

where: R_n is a resistance, u_n is a total voltage of the n-winding. The mechanical equation can be written as:

$$\mathrm{J}\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} + D\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{\partial E_{mo}(\mathbf{i},\varphi)}{\partial \varphi} + T_{\mathrm{m}}, \tag{1b}$$

where: J is a moment of inertia, D is a friction coefficient, T_{m} is a load torque.

The flux linkage $\psi_n(\mathbf{i}, \varphi)$ of any individual machine winding and the magnetic field co-energy $E_{\mathrm{mo}}(\mathbf{i}, \varphi)$ are related as follows:

$$\psi_n(\mathbf{i},\varphi) = \frac{\partial\, E_{mo}(\mathbf{i}, \varphi)}{\partial i_n}, \; n \in \{1, 2, ..., \mathrm{N}\}. \tag{2}$$

The dynamic inductances are uniquely determined by the linkage fluxes:

$$L^d_{n,k}(\mathbf{i}, \varphi) = \frac{\partial \psi_n(\mathbf{i}, \varphi)}{\partial i_k}, \quad k, n \in \{1, 2, ..., \mathrm{N}\}. \tag{3}$$

The co-energy and flux linkages are multivariable functions of independent currents of machine windings and all of them can be approximated by mutually related Taylor series. In [4] it has been proposed to rewrite the Taylor series for the co-energy function in the form bellow:

$$E_{\mathrm{mo}}(\mathbf{i},\varphi) = \frac{1}{2}\mathbf{i}^T \cdot \mathbf{A}_2(\varphi)\cdot\mathbf{i} \;+\; \frac{1}{4}\mathbf{i}^T \cdot \mathbf{A}_4(\mathbf{i},\varphi)\cdot\mathbf{i} \;+\; \frac{1}{6}\mathbf{i}^T \cdot \mathbf{A}_6(\mathbf{i},\varphi)\cdot\mathbf{i} \;+\; \cdots \tag{4}$$

It only contains even terms because the co-energy is an even function of all currents $E_{\mathrm{mo}}(\mathbf{i}, \varphi) = E_{\mathrm{mo}}(-\mathbf{i}, \varphi)$. The first term of this series describes the co-energy when linearity of machine's magnetic core is assumed and it has a well known quadratic form of the currents. All others terms in that series appear when magnetic nonlinearity is considered. They are written also as the quadratic forms of currents but using the current dependant matrices. Such form of the co-energy function allows one to write the vector of flux linkages $\Psi = [\psi_1(\mathbf{i},\varphi) \cdots \psi_{\mathrm{N}}(\mathbf{i},\varphi)]$ – satisfying the relations (2) – in the form:

$$\mathbf{\Psi}(\mathbf{i}, \varphi) = (\mathbf{A_2}(\varphi) + \mathbf{A_4}(\mathbf{i}, \varphi) + \mathbf{A_6}(\mathbf{i}, \varphi) + \cdots)\cdot\mathbf{i} \tag{5}$$

using the same matrices as in the co-energy function (4). The matrices $\mathbf{A_2}(\varphi), \mathbf{A_4}(\mathbf{i},\varphi)$, $\mathbf{A_6}(\mathbf{i},\varphi),\cdots$ can be written in rather well-ordered forms. It should be noticed that the sum of matrices in parenthesis creates the matrix of nonlinear inductances of machine windings. The $\mathrm{N} \times \mathrm{N}$ matrix $\mathbf{A}_2(\varphi)$ contains constant elements, which can be angle-dependent. For the machine with N windings it has the following form:

$$\mathbf{A}_2(\varphi) = \begin{bmatrix} \mathrm{A}_{1,1}(\varphi) & \cdots & \mathrm{A}_{1,\mathrm{N}}(\varphi) \\ \vdots & & \vdots \\ \mathrm{A}_{\mathrm{N},1}(\varphi) & \cdots & \mathrm{A}_{\mathrm{N},\mathrm{N}}(\varphi) \end{bmatrix}. \tag{6}$$

In a linear case the elements of this matrix are self- and mutual inductances of machine windings respectively.

The matrix $\mathbf{A}_4(\mathbf{i},\varphi)$, in the first term representing magnetic nonlinearity in the co-energy function, is also of the dimension $\mathrm{N} \times \mathrm{N}$. It can be written as a product of a matrix with constant elements and a specially arranged vector of the currents:

$$\mathbf{A}_4(\mathbf{i},\varphi) = \begin{bmatrix} [i_1 \cdots i_\mathrm{N}] & & \\ & \ddots & \\ & & [i_1 \cdots i_\mathrm{N}] \end{bmatrix} \cdot$$

$$\cdot \begin{bmatrix} \begin{bmatrix} \mathrm{A}_{1,1,1,1}(\varphi) & \cdots & \mathrm{A}_{1,1,1,\mathrm{N}}(\varphi) \\ \vdots & & \vdots \\ \mathrm{A}_{1,1,\mathrm{N},1}(\varphi) & \cdots & \mathrm{A}_{1,1,\mathrm{N},\mathrm{N}}(\varphi) \end{bmatrix} & \cdots & \begin{bmatrix} \mathrm{A}_{1,\mathrm{N},1,1}(\varphi) & \cdots & \mathrm{A}_{1,\mathrm{N},1,\mathrm{N}}(\varphi) \\ \vdots & & \vdots \\ \mathrm{A}_{1,\mathrm{N},\mathrm{N},1}(\varphi) & \cdots & \mathrm{A}_{1,\mathrm{N},\mathrm{N},\mathrm{N}}(\varphi) \end{bmatrix} \\ \vdots & & \vdots \\ \begin{bmatrix} \mathrm{A}_{\mathrm{N},1,1,1}(\varphi) & \cdots & \mathrm{A}_{\mathrm{N},1,1,\mathrm{N}}(\varphi) \\ \vdots & & \vdots \\ \mathrm{A}_{\mathrm{N},1,\mathrm{N},1}(\varphi) & \cdots & \mathrm{A}_{\mathrm{N},1,\mathrm{N},\mathrm{N}}(\varphi) \end{bmatrix} & \cdots & \begin{bmatrix} \mathrm{A}_{\mathrm{N},\mathrm{N},1,1}(\varphi) & \cdots & \mathrm{A}_{\mathrm{N},\mathrm{N},1,\mathrm{N}}(\varphi) \\ \vdots & & \vdots \\ \mathrm{A}_{\mathrm{N},\mathrm{N},\mathrm{N},1}(\varphi) & \cdots & \mathrm{A}_{\mathrm{N},\mathrm{N},\mathrm{N},\mathrm{N}}(\varphi) \end{bmatrix} \end{bmatrix} \cdot \begin{bmatrix} \begin{bmatrix} i_1 \\ \vdots \\ i_\mathrm{N} \end{bmatrix} & & \\ & \ddots & \\ & & \begin{bmatrix} i_1 \\ \vdots \\ i_\mathrm{N} \end{bmatrix} \end{bmatrix} \tag{7}$$

The matrix with the angle dependent constant elements is of the dimensions ($\mathrm{N}^2 \times \mathrm{N}^2$) and it is preceded by the diagonal matrix with transposed current vectors repeated N-times in the main diagonal, so its total dimensions ($\mathrm{N}^2 \times \mathrm{N}$). The last matrix is diagonal one with the vector of currents repeated N-times in the main diagonal, having the total dimension ($\mathrm{N} \times \mathrm{N}^2$). The subscripts k, l, m, n of the elements $\mathrm{A}_{k,\,l,\,m,\,n}(\varphi)$ in [H/A^2] for $k, l, m, n \in \{1,\dots,N\}$ indicate the numbers of currents multiplying a specific element. The elements differing only in the sequence of the indexes are equal. The matrix $\mathbf{A}_6(\mathbf{i},\varphi)$, and all the subsequent matrices in (4) and (5) are arranged in the same way [11]. However, the number of the unknown elements increases very rapidly in successive matrices. The estimation of the constant elements in the matrices $\mathbf{A}_2(\varphi), \mathbf{A}_4(\mathbf{i},\varphi), \mathbf{A}_6(\mathbf{i},\varphi)$, can be realized based on the values of co-energy and flux linkages [9]. All necessary data can be obtained only by computing a magnetic field distribution in the magnetic circuit of a machine. The post-processing procedures of commercial FEM packages provide the values of the co-energy as well as linked fluxes for any modelled individual machine winding. This paper describes an algorithm for the estimation of those constant elements, which combines the values of co-energy and/or flux linkages, respectively. The proposed algorithm is described based on an example of a machine with two windings, when the co-energy and flux linkages are approximated taking into account the first nonlinear term only. The dependence of coefficients on the rotary angle φ is neglected.

2. Description of algorithm based on co-energy and flux linkages for a case of two windings

Under mentioned above assumptions only the matrices $\mathbf{A_2}$ and $\mathbf{A_4}(\mathbf{i})$ appear in the formulas (4), (5) and they have the following forms:

$$\mathbf{A_2} = \begin{bmatrix} \mathrm{A}_{2,0} & \mathrm{A}_{1,1} \\ \mathrm{A}_{1,1} & \mathrm{A}_{0,2} \end{bmatrix} \tag{8}$$

$$\mathbf{A_4}(i_1, i_2) = \begin{bmatrix} i_1 & i_2 & 0 & 0 \\ 0 & 0 & i_1 & i_2 \end{bmatrix} \cdot \begin{bmatrix} \mathrm{A}_{4,0} & \mathrm{A}_{3,1} & \mathrm{A}_{3,1} & \mathrm{A}_{2,2} \\ \mathrm{A}_{3,1} & \mathrm{A}_{2,2} & \mathrm{A}_{2,2} & \mathrm{A}_{1,3} \\ \mathrm{A}_{3,1} & \mathrm{A}_{2,2} & \mathrm{A}_{2,2} & \mathrm{A}_{1,3} \\ \mathrm{A}_{2,2} & \mathrm{A}_{1,3} & \mathrm{A}_{1,3} & \mathrm{A}_{0,4} \end{bmatrix} \cdot \begin{bmatrix} i_1 & 0 \\ i_2 & 0 \\ 0 & i_1 \\ 0 & i_2 \end{bmatrix} \tag{9}$$

They contain eight unknown coefficients: three coefficients in the matrices $\mathrm{A}_{2,0}$, $\mathrm{A}_{1,1}$, $\mathrm{A}_{0,2}$ for the linear term and the remaining five in $\mathrm{A}_{4,0}$, $\mathrm{A}_{3,1}$, $\mathrm{A}_{2,2}$, $\mathrm{A}_{1,3}$, $\mathrm{A}_{0,4}$, for the nonlinear term. As such, for any pair of the currents $\mathrm{I}_{1,n}, \mathrm{I}_{2,n}$ those coefficients have to fulfil the equations:

$$\begin{bmatrix} \mathrm{E}_{\mathrm{mo}}(n) \\ \psi_1(n) \\ \psi_2(n) \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\mathrm{I}_{1,n}{}^2 & \mathrm{I}_{1,n}\mathrm{I}_{2,n} & \frac{1}{2}\mathrm{I}_{2,n}{}^2 \\ \mathrm{I}_{1,n} & \mathrm{I}_{2,n} & 0 \\ 0 & \mathrm{I}_{1,n} & \mathrm{I}_{2,n} \end{bmatrix} \cdot \begin{bmatrix} \mathrm{A}_{2,0} \\ \mathrm{A}_{1,1} \\ \mathrm{A}_{0,2} \end{bmatrix} +$$

$$+ \begin{bmatrix} \frac{1}{4}\mathrm{I}_{1,n}{}^4 & \mathrm{I}_{1,n}{}^3 \cdot \mathrm{I}_{2,n} & \frac{3}{2}\mathrm{I}_{1,n}{}^2 \cdot \mathrm{I}_{2,n}^2 & \mathrm{I}_{1,n} \cdot \mathrm{I}_{2,n}{}^3 & \frac{1}{4}\mathrm{I}_{2,n}{}^4 \\ \mathrm{I}_{1,n}{}^3 & 3\mathrm{I}_{1,n}{}^2 \cdot \mathrm{I}_{2,n} & 3\mathrm{I}_{1,n} \cdot \mathrm{I}_{2,n}{}^2 & \mathrm{I}_{2,n}{}^3 & 0 \\ 0 & \mathrm{I}_{1,n}{}^3 & 3\mathrm{I}_{1,n}{}^2 \cdot \mathrm{I}_{2,n} & 3\mathrm{I}_{1,n} \cdot \mathrm{I}_{2,n}{}^2 & \mathrm{I}_{2,n}{}^3 \end{bmatrix} \cdot \begin{bmatrix} \mathrm{A}_{4,0} \\ \mathrm{A}_{3,1} \\ \mathrm{A}_{2,2} \\ \mathrm{A}_{1,3} \\ \mathrm{A}_{0,4} \end{bmatrix} \tag{10}$$

or in the abbreviated form $\mathbf{Y}(n) = \mathbf{I}(n) \cdot \mathbf{A}$. The equations (10) relate the values of the co-energy and the winding flux linkages with values of coefficients for a given pair of currents. Those equations can be very easily modified when only the co-energy or only the flux linkages are used. It is essential that those relationships are linear with respect to coefficients and they can be recorded based on only one case of field computations. However, the matrix $\mathbf{I}(n)$ is rectangular and the vector $\mathbf{A}$ cannot be found directly. The linear regression method [3, 8] should be used for that purpose. For this purpose field computation should be provided for the numerous pairs $\{I_1, I_2\}$ in the two dimensional current space and for each of the pair the Equation (10) should be written and arranged into the equation set:

$$\begin{bmatrix} \mathbf{Y}(1) \\ \mathbf{Y}(2) \\ \vdots \\ \mathbf{Y}(\mathrm{N}) \end{bmatrix} = \begin{bmatrix} \mathbf{I}(1) \\ \mathbf{I}(2) \\ \vdots \\ \mathbf{I}(\mathrm{N}) \end{bmatrix} \cdot \mathbf{A} \tag{11}$$

or in a more general form $\mathbf{Y} = \mathbf{I} \cdot \mathbf{A}$. The application of the linear regression method leads to the solution:

$$\mathbf{A} = \left(\left(\mathbf{I}^T \cdot \mathbf{I} \right)^{-1} \cdot \mathbf{I}^T \right) \cdot \mathbf{Y}, \tag{12}$$

where the matrix $(\mathbf{I}^T \cdot \mathbf{I})^{-1} \cdot \mathbf{I}^T$ is known as the pseudo-inverse Moore-Penrose matrix [8]. It is important that all coefficients can be found in one step without any iterations. However, for AC machines with multiple windings the number of coefficients grows very rapidly with the number of nonlinear terms in the formulas (4) and (5) and the number N in the equations (11) should be sufficiently high. It means that field computations have to be performed N times.

3. Estimation method

The estimation of the sought parameters of the Equation (12) was carried out by the Weighted Least Squared Method in three ways based on: 1^0 – the field co-energy values, 2^0 – the linked flux values, 3^0 – both the linked flux values and the field co-energy values. It was assumed, that relative uncertainties of the data are equal and the weight matrix takes the form below:

$$W = diag\left[1/y_1^2, \ldots, 1/y_N^2 \right]. \tag{13}$$

That provides normalization for the data with different units. The Equation (12) for each method was formulated with the matrix of weights as follows:

$$\mathbf{A} = \mathbf{C} \cdot \mathbf{I}^T \cdot \mathbf{W} \cdot \mathbf{Y}, \tag{14}$$

where $\mathbf{C} = (\mathbf{I}^\mathbf{T} \cdot \mathbf{W} \cdot \mathbf{I})^{-1}$.

In the first, second and third method the elements of equation (14) assume successively the following forms:

1^0 $\mathbf{I} = [I_E]$ $\mathbf{W} = [W_E]$ $\mathbf{Y} = [E_{co}]$,

2^0 $\mathbf{I} = [I_{\psi 1}, I_{\psi 2}]^T$ $\mathbf{W} = [W_{\psi 1}, W_{\psi 2}]^T$ $\mathbf{Y} = [\psi_1, \psi_2]^T$,

3^0 $\mathbf{I} = [I_E, I_{\psi 1}, I_{\psi 2}]^T$ $\mathbf{W} = [W_E, W_{\psi 1}, W_{\psi 2}]^T$ $\mathbf{Y} = [E_{co}, \psi_1, \psi_2]^T$.

Calculated values of the coefficients A should comply with the conditions set resulting from the physical nature of magnetic core saturation. The nonlinear static and dynamic self inductances of the coils and a determinant of inductances matrix should be positive decreasing functions. The simplest series approximating dynamic inductances arise from (10) and take a form:

$$\begin{bmatrix} L^d_{1,1}(n) \\ L^d_{1,2}(n) \\ L^d_{2,2}(n) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} A_{2,0} \\ A_{1,1} \\ A_{0,2} \end{bmatrix} + \begin{bmatrix} 3I_{1,n}^2 & 6I_{1,n} \cdot I_{2,n} & 3I_{2,n}^2 & 0 & 0 \\ 0 & 3I_{1,n}^2 & 6I_{1,n} \cdot I_{2,n} & 3I_{2,n}^2 & 0 \\ 0 & 0 & 3I_{1,n}^2 & 6I_{1,n} \cdot I_{2,n} & 3I_{2,n}^2 \end{bmatrix} \cdot \begin{bmatrix} A_{4,0} \\ A_{3,1} \\ A_{2,2} \\ A_{1,3} \\ A_{0,4} \end{bmatrix} \tag{15}$$

From this follow that the coefficients must fulfill the conditions: $A_{2,0} > 0$, $A_{0,2} > 0$, $A_{4,0} < 0$, $A_{0,4} < 0$, $A_{2,2} < 0$ and $A_{1,3}$, $A_{3,1}$ of the opposite sign to $A_{1,1}$.

4. Testing calculations

Testing calculations have been made for two coil assemblies distributed in a stator of a two-phase asynchronous 5 kW motor. For this purpose a field model of the magnetic core with an un-slotted rotor was used. The magnetization characteristic of the core corresponded to the characteristics of the typical dynamo steel sheets. An unambiguous magnetisation curve was determined on the basis of a collection of hysteresis loops presented in [14] for non-oriented steel assumed as isotropic.

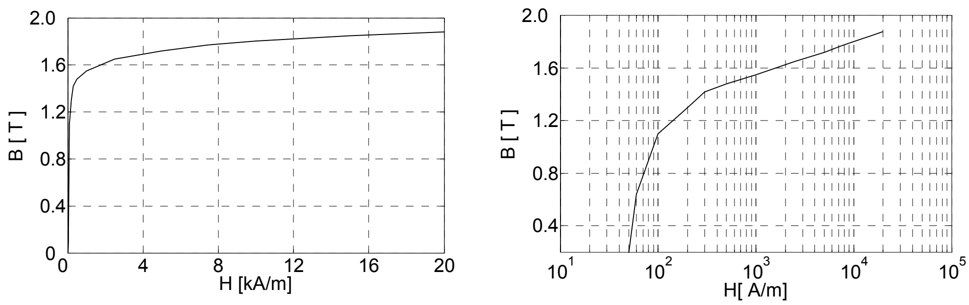

Fig. 1. Non-linear unambiguous magnetisation curve in linear scale and semi-log scale of modelled dynamo sheets marked as M530-50 A

Magnetostatic field distributions were determined by supplying the direct currents in the two pairs of the coils. The first consists of two identical diametrical coils arranged perpendicularly to one another. The second consists of two different coils arranged in parallel. The ranges of both current values were limited by the local field density less than 1.85 T.

The magnetic vector potential over the core cross section was computed by solving non-linear Poisson equation in Cartesian coordinates using the commercial package MagNet 2D [15]. The FEM solving parameters were as follows: Newton-Raphson method, Newton tolerance = 1%, CG tolerance = 0.01%, typical number of the triangle elements was 77000, for approximating polynomials of degree 1, or 34000 for approximating polynomials of degree 2.

The flux linkage was computed using vector potential circulation based on the calculated vector potential values over cross section of the coils regions [9, 13, 15]. The co-energy was calculated using the formula applicable in nonlinear magnetic materials. The differences between co-energy and linked fluxes values obtained using these two methods were lower than 1%. To provide a similar level of result uncertainties all calculations were performed with the same solution parameters. A data set of (30-60) nodes of the currents space containing the linked fluxes values and field co-energy values was created for each case.

a) b)

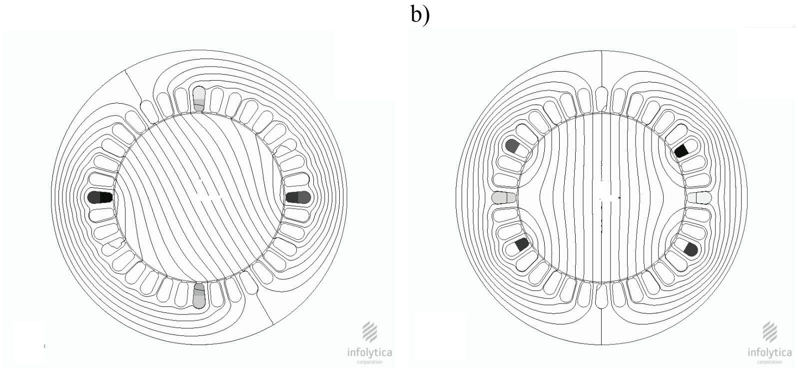

Fig. 2. Modelled magnetic core of an induction motor with unslotted rotor and chosen pair of the tested stator coils: a) with perpendicular axes, b) with parallel axes

The estimation process was executed using the Matlab software [16]. The estimation results are listed in Table 1a for the perpendicular coils and in Table 2a for the parallel coils. The Main Square Errors (MSE) for each method are specified in Table 1b and Table 2b respectively.

Table 1a. Estimated coefficients for the perpendicular coils

No	$A_{2.0}$ [H]	$A_{1.1}$ 10^{-6} [H]	$A_{0.2}$ [H]	$A_{4.0}$ 10^{-6} H/A^2]	$A_{3.1}$ 10^{-6} [H/A^2]	$A_{2.2}$ 10^{-6} [H/A^2]	$A_{1.3}$ 10^{-6} [H/A^2]	$A_{0.4}$ 10^{-6} [H/A^2]
1^0	0.0210	–0.00	0.0210	–4.14	0.00	–2.02	0.00	–4.05
2^0	0.0210	–0.17	0.0211	–5.33	0.00	–1.78	0.00	–5.53
3^0	0.0211	–0.17	0.0211	–4.84	0.00	–1.92	0.00	–4.92

Table 1b. Approximation MSE errors for the perpendicular coils

No	δE_{co} 10^{-3} [J^2]	$\delta\Psi_1$ 10^{-3} [Wb^2]	$\delta\Psi_2$ 10^{-3} [Wb^2]
1^0	0.90	0.077	0.080
2^0	2.64	0.055	0.048
3^0	1.00	0.056	0.053

Table 2a. Estimated coefficients for the parallel coils

No	$A_{2,0}$ [H]	$A_{1,1}$ [H]	$A_{0,2}$ [H]	$A_{4,0}$ 10^{-6} [H/A^2]	$A_{3,1}$ 10^{-6} [H/A^2]	$A_{2,2}$ 10^{-6} [H/A^2]	$A_{1,3}$ 10^{-6} [H/A^2]	$A_{0,4}$ 10^{-6} [H/A^2]
1^0	0.0202	0.0229	0.0410	−1.22	−2.81	−4.29	−5.64	−7.20
2^0	0.0206	0.0233	0.0417	−3.52	−4.96	−7.36	−10.37	−14.14
3^0	0.0204	0.0234	0.0415	−3.22	−4.65	−6.83	−9.63	−13.10

Table 2b. Approximation MSE errors for the parallel coils

No	δE_{co} 10^{-3} [J^2]	$\delta\Psi_1$ 10^{-3} [Wb^2]	$\delta\Psi_2$ 10^{-3} [Wb^2]
1^0	0.87	0.25	0.48
2^0	2.26	0.06	0.12
3^0	2.02	0.07	0.14

The results from all tested method are consistent and fulfill the conditions mentioned in the previous subsection. The estimation process based on co-energy values used the smaller data set and yields the smallest MSE error of the co-energy approximation. But with this method the MSE error of the linked fluxes approximation is (2-4) times greater than when using the estimation method based on the linked flux values. However, the smaller error of the linked fluxes approximations is accompanied by the three times greater error of the co-energy approximation. Although the both kinds of functions should be approximated equally well the combined third method seems to be most appropriate.

The estimated coefficients are sufficient to generate the two-variable functions approximating the magnetic field co-energy, the flux linkages and the inductances of the modelled coils in a limited space of the two currents. One example of an obtained function approximating the flux linkage vs one chosen current (in limited scope) at maintaing the second one constant is shows in Figure 3.

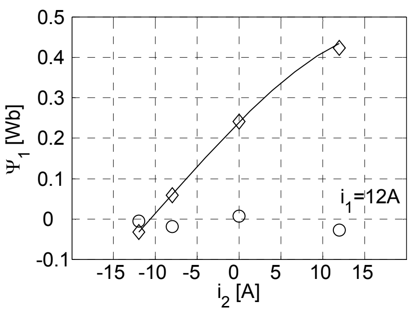

Fig. 3. Example: flux linkage vs current curve. $\Psi_1(i_1 = 12$ A, $i_2 =$ var) for parallel coils. The diamond markers – results extracted from FE analysis, solid line – approximating curve obtained with the first method, round marker – relative error of approximation

When both currents are variable the graphs of the functions become surfaces. The exemplary surfaces of the approximated functions and the residual distributions are presented in Figures 4-6. The maximum residual values for the linked fluxes approximation do not exceed 5 per cent.

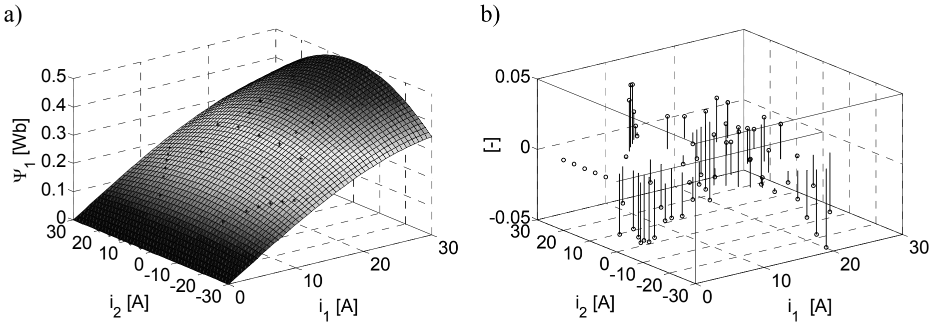

Fig. 4. a) Surface of the function approximating flux linked with one coil, b) distribution of relative residuals obtained with the third method for the perpendicular identical coils. The effect of cross saturation can be noticed

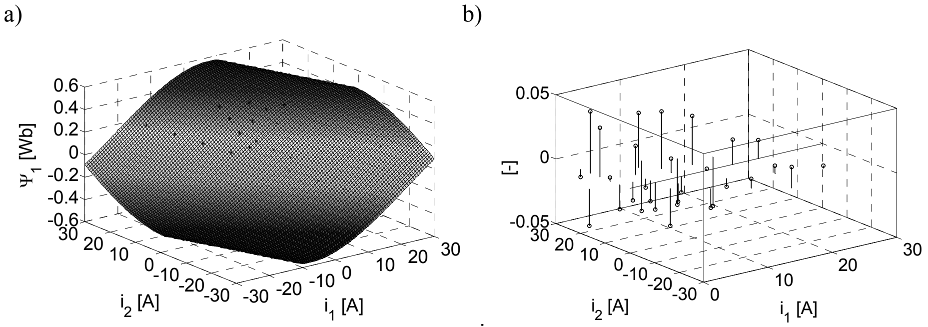

Fig. 5. a) Surface of the function approximating flux linked with coil no 1, b) distribution of relative residuals obtained with the third method for the parallel coils.

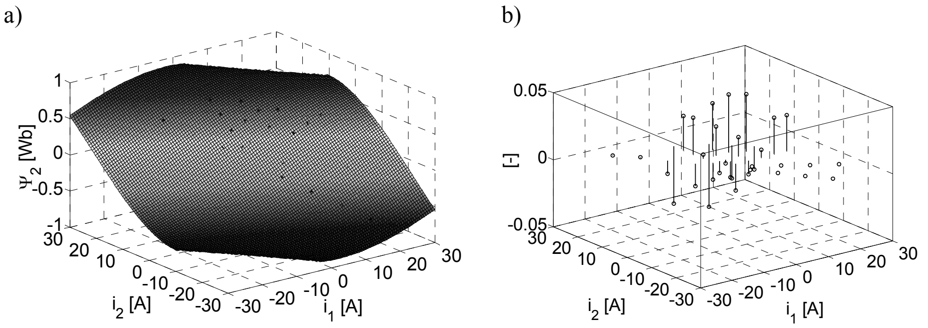

Fig. 6. a) Surface of the function approximating flux linked with coil no 2, b) distribution of relative residuals obtained with the third method for the parallel coils

4. Conclusions

All three estimation algorithms are equally effective for the processed data. Singularities in the model matrix appeared only in the first algorithm for a small number of points. Residual distributions indicate the periodic nature of the deviations for large currents. This indicates that the area of qualitatively correct approximations by third-degree polynomials has been exceeded outside. The trends of accuracy changes are similar for weakly coupled windings (e.g. with perpendicular axes) and for highly coupled windings (with parallel axes). The co-energy approximation error is the smallest when the coefficients are estimated solely on the basis of co-energy. Coefficients estimation on the basis of linked flux of both coils reduces its function approximation error and increases the co-energy approximation error. The estimation of the coefficients on the basis of the linked fluxes and the field co-energy modifies these results slightly. The obtained results show that the parameters which are estimated only on the basis of the co-energy values provide the linked fluxes approximation quality on a similar level in spite of the smaller data set.

Acknowledgements
This paper was supported by research grant No DEC-2011/01/B/ST7/04479: "Modelling of nonlinearity, hysteresis, and anisotropy of magnetic cores in electromechanical converters with rotating magnetic field" financed by the National Science Centre (Poland).

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