

Jiles–Atherton Magnetic Hysteresis Parameters Identification

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This paper presents a method of fitting the magnetic hysteresis curves using a calculating algorithm within a computer programme. The calculating programme uses the equations of the Jiles–Atherton model to simulate the $M(H)$ magnetization curve. The aim of this program is to find the values of the parameters which are part of the Jiles–Atherton model once the regression curve for a major magnetization curve obtained experimentally is found. The values of these parameters can be increased or decreased gradually. The parameters modification takes place in such a way that the root mean square deviation between the points of the two graphs, also experimentally simulated, should decrease gradually and, in the end, it should be found below the preestablished percentage value in comparison to the previous value. The procedure for minimizing the root mean square deviation has as a result the fact that the simulated curve represents the regression curve. The calculating programme considers a second method of calculating the values of the parameters by using the already found regression curve. This paper also presents the values of the Jiles–Atherton model parameters obtained by fitting the magnetization curves of several cobalt ferrites samples, sintered at different temperatures.

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1. Introduction

This paper presents a calculating algorithm with two complementary methods of determining the regression curve in the case of major experimental magnetization curves. For this purpose the equations of the Jiles–Atherton (J–A) model [1] are used, obtaining the following simulated magnetization curves: the $M_{an}(H)$ anhysteretic curve, the $M(H)$ first magnetization curve with H increasing from 0 to $+H_{max}$ and the major magnetization curve with H in the $[-H_{max}, +H_{max}]$ interval. The major curve is given by two equations, as it will be seen in Sect. 2, according to the way H increases or decreases in the mentioned range. H_{max} is the maximum value of the applied magnetic field which magnetizes a ferro or ferrimagnetic sample.

In the J–A model equations there are four parameters: a , α , k and c . To obtain the regression curve it is necessary to find the values of these parameters. Of course, attempts have been made to find the regression curve for a given experimental magnetization curve and to calculate the coefficients of the J–A model using different programming methods or different mathematical adjustments of the used equations to simplify the achievement of this objective [2–12]. Taking into consideration that one of the model’s equations is a differential equation which does not have an analytical solution, the attempts of fitting experimental magnetization curves were based on numerical methods. Unlike the already taken actions [5–8], what we suggest is a calculating algorithm which determines the values of the J–A model parameters in

two ways. The algorithm made to obtain the regression curve is based on repeated calculating structures which gradually draw the simulated major magnetization curve nearer to the experimental one. This approach is based on a simple principle: the a , α , k , c coefficients are gradually increased or decreased so that in the end the difference between the graphs should be minimum. Within the calculating programme the initial values of these parameters are random. The root mean square deviation could be considered as measure of the distance between the simulated curve and the experimental one and is given by the equation

$$S = \sqrt{\frac{\sum_{i=1}^N (M_i - M_{exp,i})^2}{N}}, \quad (1)$$

where N is the number of points on the experimental major magnetization curve, M_i represents the value of magnetization calculated by the computer programme for a field identical or very close to the experimental $H_{exp,i}$ and $M_{exp,i}$ is the value of magnetization experimentally determined for $H_{exp,i}$. The calculating programme achieves a leveled variation of the magnetic field. The calculated values of H do not exactly overlap the $H_{exp,i}$ experimental values but some of them could be “as close as possible”. In other words, the terms of a subsequence of the H values are very close to the sequence of the $H_{exp,i}$, $i = 1 \dots N$ experimental values. Minimizing S leads to a hysteresis loop having approximately the same shape and area as the experimental one, as it will be seen in Sect. 4. The simulated curve obtained through minimizing the root mean square deviation is called regression curve and has the property that the sum of the squares of the distances, measured along the ordinate, from its points to the experimental ones is minimum. Thus the distance be-

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tween two points on the two curves, corresponding to the same magnetic field, is the difference of the magnetizations which appears in Eq. (1): $M_i - M_{\text{exp},i}$. The values of the J–A model coefficients for three cobalt ferrite samples obtained through sintering at different temperatures will also be shown in Sect. 4.

2. The equations of the J–A model

The magnetization calculated according to the J–A model has two components, an irreversible and a reversible one corresponding to the irreversible or reversible phenomena which take place within the magnetic material during the magnetization. The domain walls moving over the pinning sites represent irreversible displacing and their movement between two successive pinning sites or the rotations of the magnetic moments which lead to the decrease of the magnetization at field inverting after achieving the saturation represent reversible phenomena. Thus the total magnetization is $M = M_{\text{rev}} + M_{\text{irr}}$.

The irreversible component of the magnetization is given by the following differential equation:

$$dM_{\text{irr}} = \frac{M_{\text{an}} - M_{\text{irr}}}{k\delta - \alpha(M_{\text{an}} - M_{\text{irr}})} dH, \quad (2)$$

where M_{an} is the anhysteretic magnetization and k is a coefficient which appears as a factor of proportionality between the wasted energy when a domain wall crosses a pinning site and the variation of the sample's magnetization as an effect of this movement. Within this equation also, $\delta = 1$ if $dH/dt > 0$ and $\delta = -1$ if $dH/dt < 0$.

The anhysteretic magnetization is the magnetization of an ideal ferromagnet which does not have pinning sites and thus, its magnetization curve does not present hysteresis, the magnetization being made only through the rotating mechanism of the magnetic moments. Langevin's function specific to magnetizing paramagnetic substances is used within the J–A model for anhysteretic magnetization, in which case an effective field, H_e , within the sample replaces the magnetic field, H , external to the sample. The equation of the anhysteretic magnetization is

$$M_{\text{an}} = M_s \left(\coth \frac{H_e}{a} - \frac{a}{H_e} \right), \quad (3)$$

where $a = k_B T / (\mu_0 m)$ is another parameter within the model which is obtained as a result of the fitting process, m is the magnetic moment of a typical magnetic domain and $H_e = H + \alpha M$ (the polycrystalline ferromagnet is treated [1] as a canonical ensemble of interacting magnetic domains each carrying a magnetic moment \mathbf{m}).

Sablick nad Jiles [13] consider that $M_s = mn$, where M_s is the saturation magnetization and n is the density of magnetic domains of ferromagnetic sample. Therefore $n = \mu_0 M_s a / (k_B T)$ and $n \sim a M_s$. a is a coefficient which can be interpreted as a measure of the coupling between the adjoining magnetic domains. The reversible component of the magnetization is a fraction c of the difference

of the magnetization anhysteretic and the irreversible one

$$M_{\text{rev}} = c(M_{\text{an}} - M_{\text{irr}}). \quad (4)$$

Finally, the value of the total magnetization is given by the following equation:

$$M = cM_{\text{an}} + (1 - c)M_{\text{irr}}. \quad (5)$$

From (5) results that

$$\frac{dM}{dH} = c \frac{dM_{\text{an}}}{dH_e} \frac{dH_e}{dH} + (1 - c) \frac{dM_{\text{irr}}}{dH} \quad (6)$$

and

$$M_{\text{irr}} = \frac{m - cM_{\text{an}}}{1 - c}. \quad (7)$$

Substituting (7) into (2) we obtain

$$dM_{\text{irr}} = \frac{M_{\text{an}} - M}{k\delta(1 - c) - \alpha(M_{\text{an}} - M)} dH. \quad (8)$$

Differentiating (3) we get

$$\frac{dM_{\text{an}}}{dH_e} = \frac{M_s}{a} \left[1 - \left(\coth \frac{H_e}{a} \right)^2 + \left(\frac{a}{H_e} \right)^2 \right]. \quad (9)$$

But

$$\frac{dH_e}{dH} = 1 + \alpha \frac{dM}{dH}. \quad (10)$$

Substituting (8), (9) and (10) into (6) we obtain the equation

$$\chi' = \frac{dM}{dH} = \left\{ c \frac{M_s}{a} \left[1 - \left(\coth \frac{H_e}{a} \right)^2 + \left(\frac{a}{H_e} \right)^2 \right] + (1 - c) \frac{M_{\text{an}} - M}{k\delta(1 - c) - \alpha(M_{\text{an}} - M)} \right\} / (1 - \alpha c). \quad (11)$$

This is the final differential equation that can be reached using J–A model, equation which gives the dependence between the magnetization of a magnetic sample and the applied magnetic field.

Jiles et al. [2, 3] obtained the a , α , k , c coefficients which appear in Eqs. (2)–(5) or (11) by calculating some differential magnetic susceptibilities at different points of the anhysteretic magnetization curves, major or first magnetization. These differential susceptibilities, χ' (dM/dH or dM_{an}/dH) are the derivatives of the magnetization in relation to the H field at different points of the magnetization curve or at different points of the anhysteretic curve. The initial, remanent, coercive and saturation anhysteretic differential susceptibilities will be marked by χ'_{an} , χ'_{anmax} , χ'_{anc} , χ'_{anm} . The differential susceptibilities on the major magnetization curve corresponding to the mentioned areas will also appear in these equations: χ'_{in} , χ'_{max} , χ'_c , χ'_m .

The Taylor series expansion of the anhysteretic magnetization in the vicinity of $H = 0$, $M = 0$, gives

$$M_{\text{an}} \cong M_s \frac{H_e}{3a} = \frac{M_s}{3a} (H + \alpha M). \quad (12)$$

Substituting (5) into (12) we obtain

$$M_{\text{an}} = \frac{M_s}{3a} [H + \alpha c M_{\text{an}} + \alpha(1 - c)M_{\text{irr}}]. \quad (13)$$

For $H = 0$ we will have $M_{\text{an}} = 0$. We consider that in this zone the irreversible component of the magnetization is null (for very small fields the domain walls perform reversible motions). $M_{\text{irr}} = 0$. What results from Eq. (2) is

$$\left. \frac{dM_{\text{irr}}}{dH} \right|_{H=0} \cong 0. \quad (14)$$

Taking into consideration Eqs. (5) and (14) we can write

$$\chi'_{\text{in}} = \left. \frac{dM}{dH} \right|_{H=0} \cong c \left. \frac{dM_{\text{an}}}{dH} \right|_{H=0}. \quad (15)$$

As a matter of fact, Eq. (15) presents c as the relation between the differential magnetic susceptibilities calculated in the beginning zone of the first magnetization curve

$$c \cong \frac{\chi'_{\text{in}}}{\chi'_{\text{an}}}. \quad (16)$$

The following two equations are presented as they were calculated by Jiles et al. [3]:

$$k = \frac{M_{\text{an}}(H_c)}{1 - c} \left(\alpha + \frac{1}{\chi'_{\text{max}} - \frac{c}{1-c} \frac{dM_{\text{an}}(H_c)}{dH}} \right), \quad (17)$$

$$M_{\text{R}} = M_{\text{an}}(M_{\text{R}}) + \frac{k}{\frac{\alpha}{1-c} + \frac{1}{\chi'_{\text{rem}} - c \frac{dM_{\text{an}}(M_{\text{R}})}{dH}}}. \quad (18)$$

M_{R} is the remanent magnetization, $M_{\text{an}}(M_{\text{R}}) = M_{\text{an}}(0)$ and $dH/dt < 0$, $dM_{\text{an}}(M_{\text{R}})/dH = dM_{\text{an}}(0)/dH$, $dH/dt < 0$ [2, 3].

The c coefficient being known from (16), k and α , respectively, can be found from (17) and (18).

Differentiating (13) in $H = 0$ and taking (14) into account, we have

$$\left. \frac{dM_{\text{an}}}{dH} \right|_{H=0} \cong \frac{M_s}{3a} \left(1 + \alpha c \left. \frac{dM_{\text{an}}}{dH} \right|_{H=0} \right). \quad (19)$$

Equation (19) leads to

$$a \cong \frac{M_s}{3\chi'_{\text{an}}} + \frac{\alpha c M_s}{3}. \quad (20)$$

In conclusion, if the graph of a major magnetization curve is known, including the anhysteretic and the first magnetization curves, the a , α , k , c coefficients can be calculated using Eqs. (16)–(18) and (20) first obtaining the susceptibilities involved in these equations.

3. The method of fitting the experimental hysteresis curves

The structure of the calculating programme which fits the experimental magnetization curve has the following calculating steps:

1) Initialization of the physical units which are the object of the calculation;

2) Calculation of the coordinates of the simulated magnetization curve;

3) Calculation of the average square deviation S using Eq. (1);

4) Gradual increasing or decreasing the a , α , k , c parameters in order to minimize S ;

5) Displaying the calculated physical units including the values of the a , α , k , c parameters, of the experimental and regression magnetization curves;

6) Recalculation of the values of the a , α , k , c parameters using Eqs. (16)–(18), (20) and displaying of these values.

Detailing stage 1:

The experimental data are “read” first so that a set of variables corresponding to the magnetic field gets the corresponding numerical values from a text file. Another set of variables receive the values corresponding to the magnetization. Using the second set of variables the value of the saturation magnetization M_s is determined through interpolation. The magnetic field is gradually increased or decreased with dH . If dH_{exp} is the average distance between two successive values of the measured magnetic field intensity, in order to obtain calculated values of the magnetic field intensity as close as possible to the experimental ones, it is necessary that dH should be much smaller than dH_{exp} . Thus the preestablished vicinity of a random experimental value of the $H_{\text{exp},i}$ magnetic field, in which there is only one calculated value H of the magnetic field, will be $V = (H_{\text{exp},i} - dH/2, H_{\text{exp},i} + dH/2)$. If the distance between any two consecutive experimental values of the magnetic field is constant then $dH = dH_{\text{exp}}$. In the same stage the a , α , c , k parameters will receive random values. The maximum value of the calculated H_{max} is initialized with the maximum experimental value of the magnetic field. Also, the initial values of the calculated magnetization M and of the calculated magnetic field H are null.

Detailing stages 2 and 3:

The calculation of the coordinates (H, M) on the simulated magnetization curve takes place as follows:

— for a given H the effective H_e field is calculated using the equation $H_e = H + \alpha M$ (initially H and M are null);

— the anhysteretic magnetization is calculated using Eq. (3);

— the variation of the irreversible component of dM_{irr} magnetization is calculated using Eq. (2), where dH is the variation step of the calculated magnetic field;

— the value of the irreversible component of the magnetization is found: $M_{\text{irr}} = M'_{\text{irr}} + dM_{\text{irr}}$, where M'_{irr} is the previous value of this magnetization corresponding to the previous point on the simulated magnetization curve;

— finally, the total magnetization M , which includes the irreversible component is calculated. Equation (5) is used;

— after calculating M for a given H field, the value of the field becomes $H + dH$ on the $0 \rightarrow H_{\text{max}}$, $-H_{\text{max}} \rightarrow +H_{\text{max}}$ intervals and $H - dH$ on

the $+H_{\max} \rightarrow -H_{\max}$ interval; after this modification of the field the sequence of operations described above starts again. The repetition of the calculation sequence stops after the values were included once in the intervals and reached their extremes each time. The first interval corresponds to the calculation of the coordinates of the simulated first magnetization curve and the other two together correspond to the simulated major magnetization curve.

The calculation of the average square deviation S takes place simultaneously with the previously mentioned calculations.

The calculation of S takes place as follows:

— at each repetition of the calculation sequence previously described, for each H value of the magnetic field calculated in the variation intervals $+H_{\max} \rightarrow -H_{\max}$ and $-H_{\max} \rightarrow +H_{\max}$ (the major magnetization curve), it is tested if this value is in the preestablished vicinity V of an experimental value $H_{\text{exp},i}$, $i = 1 \dots N$, N being the number of experimental data on the major experimental magnetization curve;

— if the previous condition is fulfilled, then the value of the calculated magnetization $M(H)$ is memorized by a variable of the programme, M_i , and then the next modification of the square of S takes place: $S^2 \rightarrow S^2 + [M_i - M_{\text{exp},i}]^2$. If the condition is not fulfilled, S^2 keeps its value. $M_{\text{exp},i}$ is the measured magnetization corresponding to the measured magnetic field $H_{\text{exp},i}$. At first $S = 0$.

When ending these calculations, after all the coordinates of the simulated magnetization curve have been calculated, the value of S becomes $(S^2/N)^{1/2}$; thus the calculation of the average square deviation is finalized according to Eq. (1).

Stage 4 follows, which means the modification of one of the four parameters. This modification aims at obtaining a simulated curve closer to the experimental one, this being possible only if, as a result of the modification, the value of S is smaller than the previous one.

Detailing stage 4:

After modifying one of the mentioned parameters with a fraction z of its previous value, for example increasing, the calculation programme recalculates the coordinates of the simulated curve $M(H)$ and recalculates the average square deviation S . In this way, a repetition of stages 2–3 takes place. If S 's new value is smaller than the previous one, the modification of that parameter continues increasing it again with the z fraction of its new value (for example, at the beginning $z = 50\%$). The repetition of this operation takes place until the value of S becomes bigger than the previous one. Then, that parameter, and also S , returns to the value considered before the last modification. After this the modification of the next parameter starts, etc. If from the very first increasing modification of one of the parameters, the decrease of S is not possible, then there is an alternative option to have a decreasing modification. If this does not have as a result the decrease of S , then the next parameter is taken into consideration. The decreasing modification takes place

according to the same procedure as the increasing modification. Therefore, the gradual modifications of these parameters lead to the step-by-step decrease of S . After the successive modifications of all the parameters the program evaluates if the difference between the last two values of S is smaller than a certain percentage of S 's last value. If this condition is not fulfilled, then the series of modifications of these parameters starts again. If this condition is fulfilled, then the z fraction decreases twice and the succession of the parameters' modifications resumes. The decrease of z implies a finer modification of each of the parameters thus a more precise calculation of them. The decrease of the z fraction takes place until $z < 0.001\%$.

Then the computer screen displays the experimental and the calculated magnetization curves, the a , α , k , c coefficients, the remanent magnetization, the coercive field and the susceptibilities mentioned in Sect. 2.

The susceptibilities mentioned in Sect. 2 are the tangents to the magnetization curve at different points, and the programme calculates them simultaneously with the coordinates of the simulated magnetization curve.

Stage 6:

Immediately after finalizing the fitting, there is the possibility for the user to calculate the a , α , k , c parameters using Eqs. (16)–(18), (20). By pressing a button on the interface the values of the parameters calculated in this way are displayed. These new values can be compared to those obtained through minimizing the root mean square deviation. Therefore, there are two complementary methods of calculating the parameters of the J–A model. The first method corresponds to the values directly obtained through fitting and the second method corresponds to the use of the above mentioned equations immediately after finding the regression curve. The second method would not be possible without finding the regression curve because all the susceptibilities which are part of these equations are calculated using this curve.

4. Results

The experimental data used by the application are determined by a vibrating samples magnetometer (VSM). We will further present the fitting results for three samples of CoFe_2O_4 obtained by coprecipitation method and sintered for 5 h in air atmosphere at temperatures of 1050 °C, 1100 °C and 1150 °C. The experimental and calculated values are presented in Table I. The experimental and calculated hysteresis loop (regression curve) for sample 1 (sintered at 1050 °C) are presented in Fig. 1. In Table II there are presented the values of the J–A model's parameters for sample 1 obtained by the two calculation modes. We can observe that in the case of these materials the values of the a , k , α parameters determined with the two methods are relatively close. The parameters used to determine the quality of the fittings were the quality factor, ε [12], and the Pearson coefficient, r^2 . The expression of the quality factor is given by equation

$$\varepsilon = \frac{S}{M_s}. \quad (21)$$

It represents a percentage value of the root mean square deviation.

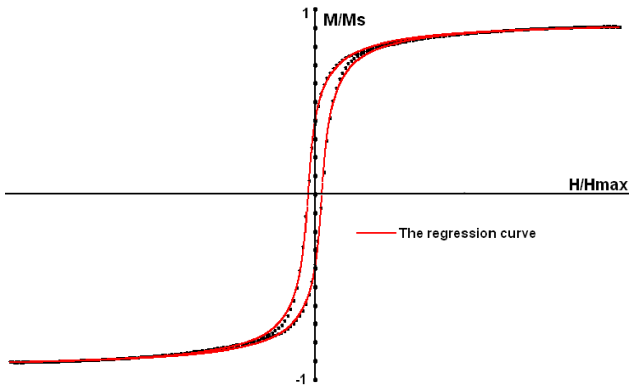


Fig. 1. Regression curve for sample 1.

TABLE I

The values of the Jiles coefficients for the three samples of CoFe_2O_4 obtained through mode 2 of calculation.

	Sample 1	Sample 2	Sample 3
T [°C]	1050	1100	1150
ρ [g/cm ³]	4.89	4.92	5.01
M_s [A/m]	414800	424000	439600
M_{rem} [A/m]	174100	87600	77000
H_c [A/m]	18480	12610	7920
α	0.176	0.126	0.198
k [A/m]	18964	12755	7978
c	0.00009	0.00014	0.00023
a [A/m]	33661	36610	42860
r^2 [%]	99.98	99.96	99.94
ε [%]	1	0.78	0.83

The r^2 coefficient is given by the equation

$$r^2 = 1 - \frac{\sum_{i=1}^N (M_i - M_{\text{exp},i})^2}{\sum_{i=1}^N (\overline{M}_{\text{exp}} - M_{\text{exp},i})^2}, \quad (22)$$

where M_i is the calculated value of the magnetization for the $H_{\text{exp},i}$ field and $\overline{M}_{\text{exp}}$ is the arithmetic average of the magnetization experimental values. As the value of r^2 is closer to 1, the regression curve is more accurate.

The values of these two quantities, presented in Table I, show a relatively high precision of the obtained regression curves. Regarding the values obtained for the four coefficients we can state that:

— the decrease of k with the increase of the sintering temperature indicates a decrease of the blocking areas density in the studied samples (in the considered temperature range) and thus a reduction of the energetic losses;

— the increase of c with the sintering temperature indicates an augmentation of the reversible component of the magnetization which can be caused by the decrease of the blocking areas density.

TABLE II

A comparison between the values of the Jiles coefficients for sample 1 obtained through the two modes of calculation.

	a [A/m]	k [A/m]	α	c
mode 1	33679	19083	0.178	1×10^{-6}
mode 2	33661	18964	0.176	9×10^{-5}

Considering the values for M_s and a from Table I, we can say that in the studied temperature range, the density of the magnetic domains ($n \sim aM_s$) of cobalt ferrite samples increases with the sintering temperature.

5. Conclusion

The designed software based on J–A model can be used in fitting the experimental hysteresis loops of magnetic materials with high accuracy. The purpose of the fitting programme was the well-precise determination of the values of the model's coefficients for a certain material. This purpose was achieved. Moreover, this calculating programme can also be used for fitting the magnetization curves of samples under mechanical stress or under other interaction conditions and, in this way, the dynamic of the model's coefficients in the given situations can be observed. These coefficients, through their physical meanings, can provide useful information regarding the microstructure of magnetic materials and of the interactions at this level.

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