Tomasz TRAWIŃSKI

USING THE BLOCK MATRICES IN THE MODELING OF DRIVING AND CONTROL SYSTEM OF HARD DISK DRIVES

ABSTRACT *In the article block matrix theory is employed to model of the branched kinematics chains. It allows for convenient construction of total dynamic matrix of complex branched kinematic system of head positioning system used in hard disk drives, with respect to enlargement of numbers of branches in kinematics chain. It allows giving the general expressions for individual matrix elements (in terms of basic kinematic parameters) before and after it inversion. In chapter 3 the block matrix inversion is discussed and finally in cheapter 4 the exemplary simulation results of time optimal control is presented.*

Keywords: *block matrices, partitioned matrices, hard disk drives, branched kinematic systems, time optimal control.*

1. HEAD POSITIONING SYSTEM, SCHEMATIC REPRESENTATION

The head positioning system of modern magnetic mass storage devices – hard disk drives – belongs to very complicated system resulting from construction. The construction features of head positioning system results from mutual cooperation with spindle system (which usually drives a few data disk). Typical head

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positioning system may be regarded as specific mechatronics system which consists of such components as: driving system, control system and measurement system. This article focus on driving system and some parts of control system. The block schema of head positioning system regarded as mechatronics system is in Figure 1 presented [1].

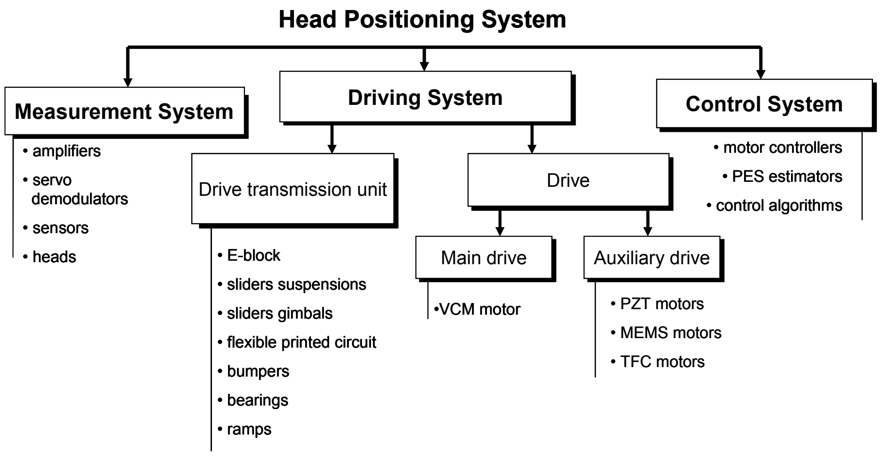


Fig. 1. Block schema of head positioning system regarded as mechatronics system

The driving system consists of such components as: E-block, slider suspensions, sliders gimbals, flexible printed circuit etc. The shapes of above mentioned components depending on data areal densities of hard disk drive (which determine the length and the width of data track on rotating disk) and numbers of disk in spindle system. In Figure 2 the exemplary drive transmission unit taken from HDD (Western Digital WD400 40GB) in which the head positioning system cooperate with one side of data disk is presented.

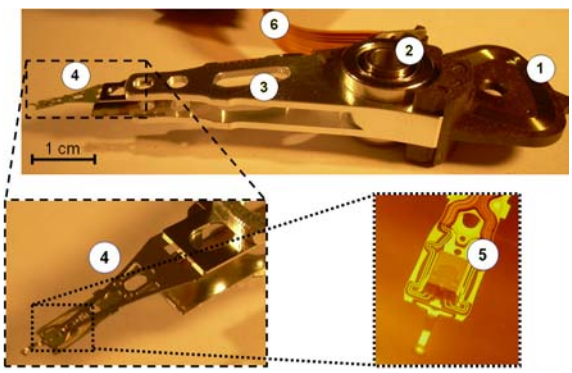
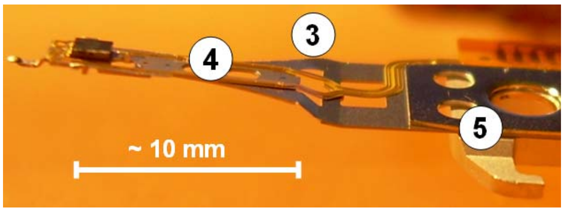


Fig. 2. Basic components of typical drive transmission unit working with one side of data disk:
(1) VCM motor winding,
(2) pivot,
(3) E-block,
(4) slider suspension,
(5) slider,
(6) flexible printed circuit

The brand new construction of driving system of head positioning system uses additional micromotors (PZT motors) [10, 11], beside the main VCM motor, as an auxiliary drive. The additional micromotors are helping in proper seeking and following the data tracks [5, 7-9]. In Figure 3 two different construction of slider suspensions are shown.

a)



b)

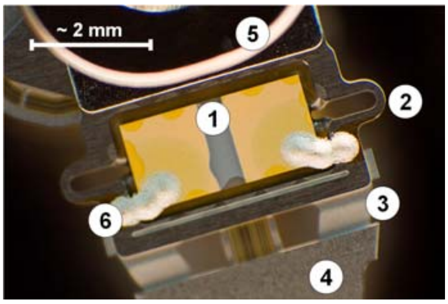


Fig. 3. Slider suspensions: a) without PZT micromotor, b) with PZT micromotor (1)

Analyzing the fixation system (Fig. 3a) between slider suspension (4) and end tip of E-block (5), which consists of two elastic stripes (3), it is easy to spot that it may be regarded as rotating joint. In this joint acts torque generated by spring (which is formed by two elastic bars). In Figure 3b the connection between end tip of E-block (5) and suspension (4) may be regarded as two successive rotating joints with rotating axes perpendicular to each other. In the first joint acting torque generated by PZT micromotor (1) and spring (2) formed by so-called Φ-shape hinge. The second rotating joint is formed, like in previous case, by two elastic stripes (3). Seeing some similarities of the robot manipulators to the structure of drive transmission unit, one can be represented by kinematic chain consisting with kinematic pairs (perfectly stiff) connected by rotating (or prismatic) joints (with one degree of freedom). The real kinematic structure of the drive transmission unit presented in Figure 3 may be represented in simplified form as is shown in Figure 4. For simplicity the spring elements producing torque in joints are not represented in this figure.

Rotating joint in Figure 4a here and after will be denoted by small letter "r", the first rotating joint in Figure 4b will be denoted by capitol letter "R", the second one, as in previous case will be denoted by small letter "r".

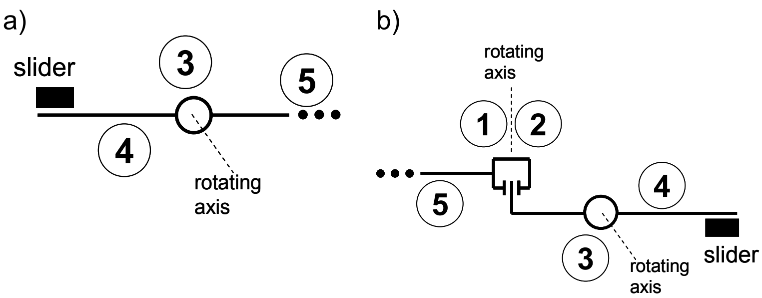


Fig. 4. Schematic representation of real kinematic chain of slider suspensions

Whole kinematic chain of the drive transmission unit may be expressed by connection of bough and branches. Bough consists with E-block and rotating joint which is directly driven by VCM motor. Branches consist with slider suspension, slider gimbals, slider, heads etc. The number of branches joints and links depends on interpretations and what way the real kinematic chain of branches was replaced by simplified kinematic chain, it may have at least one degree of freedom up to few (for example 3) degrees of freedom. Here and after kinematic chain of drive transmission unit represented by bough and branches will be called as the branched kinematic chain. Exemplary branched kinematic chain of drive transmission unit of head positioning system consists with bough and four branches with two degrees of freedom. Total numbers of degrees of freedom is equal 9 DoF in this case. This branched kinematic chain will be denoted by symbolic form "4G9rp" what means: four branches (G) nine degrees of freedom – branches type – "rp" (single branch consist of two joints rotating "r" and prismatic "p"). Small letter "p" denotes that translation axis of prismatic joint is perpendicular to rotating axis of bough.

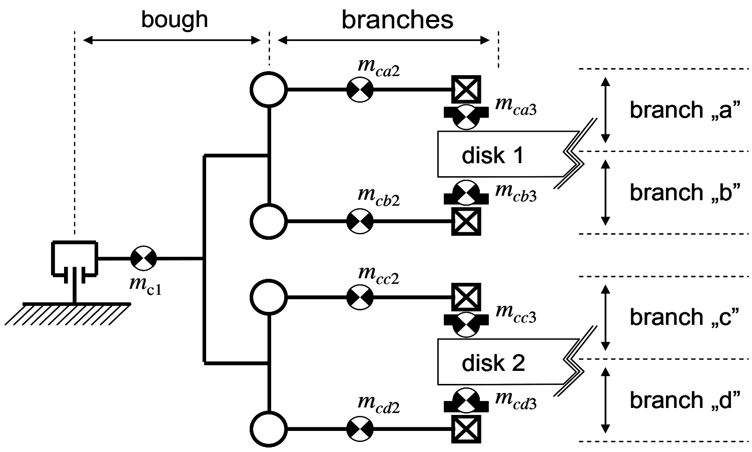


Fig. 5. 4G9rp branched kinematic chain

2. FORMULATION OF DYNAMIC EQUATIONS – BLOCK MATRIX REPRESENTATION

Mathematical model of branched kinematic chain may be expressed in terms of Lagrange equation, in matrix form as follows:

$$\boldsymbol{D}_{\mathrm{r}}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{G} = \boldsymbol{Q} \tag{1}$$

where $\boldsymbol{D}_{\mathrm{r}}, \boldsymbol{C}, \boldsymbol{G}, \boldsymbol{Q}$ – matrices respectively: inertial, centrifugal and Coriolis force, gravitational force or torque, generalized forces; $\ddot{\boldsymbol{q}}$, $\dot{\boldsymbol{q}}$ – vectors of generalized: acceleration and speed.

The crucial problem lies in haw formulate the inertial matrix $\boldsymbol{D}_{\mathrm{r}}$ for branched kinematic chains. The procedure of $\boldsymbol{D}_{\mathrm{r}}$ matrix formulation basing on following steps depicted in Figure 6. The procedure of formulation of drive system of head positioning system consists with eleven steps, but we focus on only the most important steps crucial to proper and convenient formulation of dynamic equations of branched kinematics chain.

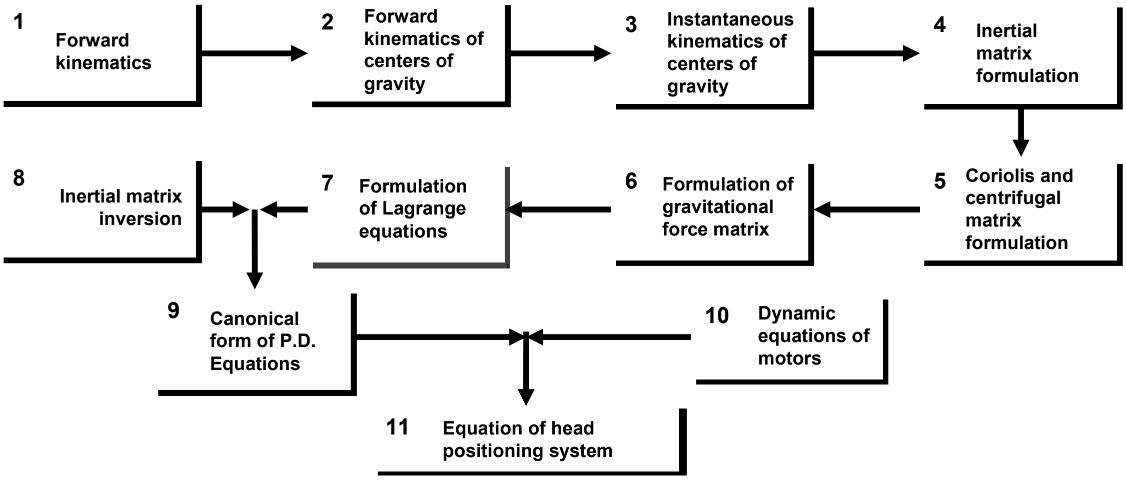
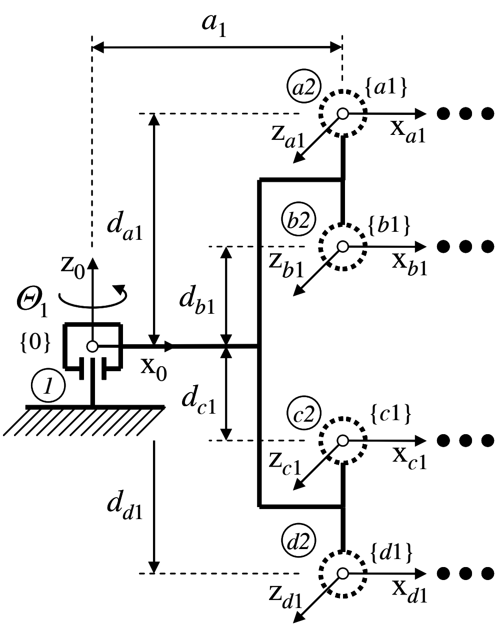


Fig. 6. Block diagram describing the way of dynamic model of head positioning system formulation

Step 1. Forward kinematics – in this step it is necessary to describe the kinematic chain following the Denavit-Hartenberg rules [2]. Finishing this step the homogenous transformation matrix will be given for all coordinate system fixed with branched kinematic chain [3, 4, 6]. For chosen exemplary branched kinematic chain the described chain according to Denavit-Hartenberg rule is presented in Figure 7, separately for bough and single branch "a".

a)



b)

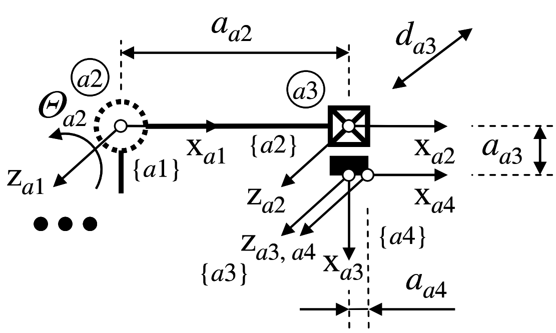


Fig. 7. Description of kinematic chains:
a) bough, b) exemplary branch. The common joint are denoted by bold dotted line

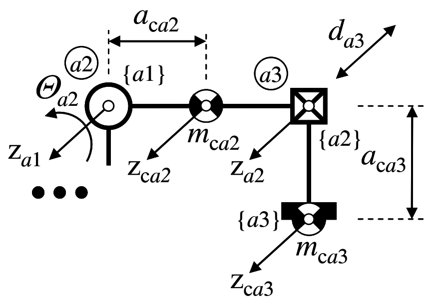


Fig. 8. Positions of mass centers of rp branch links

Step 2. Forward kinematics of centers of gravity – in this step it is necessary to formulate homogenous matrices which describes the position and orientation for centers of gravity of every link in branched kinematic chain, in terms of base coordinate system (fixed with bough). In Figure 8 positions of mass centers for branches "*a*" is shown.

General homogenous transformations matrices, determined with help of Denavit-Hartenberg rules, of mass centers of bough and first, second links of branches are given by:

$$\boldsymbol{T}_0^{\mathrm{c}1} = \left[\begin{array}{c:c} \boldsymbol{R}_0^{\mathrm{c}1} & \boldsymbol{d}_0^{\mathrm{c}1} \\ \hdashline \boldsymbol{0} & \boldsymbol{1} \end{array}\right] = \left[\begin{array}{ccc:c} c_1 & -s_1 & 0 & a_{\mathrm{c}1}c_1 \\ s_1 & c_1 & 0 & a_{\mathrm{c}1}s_1 \\ 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right] \tag{2}$$

$$\boldsymbol{T}_0^{\mathrm{c}g2} = \left[\begin{array}{c:c} \boldsymbol{R}_0^{\mathrm{c}g2} & \boldsymbol{d}_0^{\mathrm{c}g2} \\ \hdashline \boldsymbol{0} & \boldsymbol{1} \end{array}\right] = \left[\begin{array}{ccc:c} c_1 c_{g2} & -c_1 s_{g2} & s_1 & (a_{\mathrm{c}g2}c_{g2} + a_1)c_1 \\ s_1 c_{g2} & -s_1 s_{g2} & -c_1 & (a_{\mathrm{c}g2}c_{g2} + a_1)s_1 \\ s_{g2} & c_{g2} & 0 & \pm d_{g1} + a_{\mathrm{c}g2}s_{g2} \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right] \tag{3}$$

$$\boldsymbol{T}_0^{\mathrm{c}g3} = \left[\begin{array}{c:c} \boldsymbol{R}_0^{\mathrm{c}g3} & \boldsymbol{d}_0^{\mathrm{c}g3} \\ \hdashline \boldsymbol{0} & \boldsymbol{1} \end{array}\right] = \left[\begin{array}{ccc:c} c_1 s_{g2} & c_1 c_{g2} & s_1 & a_1 c_1 + a_{g2}c_1 c_{g2} + d_{g3}s_1 \pm a_{\mathrm{c}g3}c_1 s_{g2} \\ s_1 s_{g2} & s_1 c_{g2} & -c_1 & a_1 s_1 + a_{g2}s_1 c_{g2} - d_{g3}c_1 + a_{\mathrm{c}g3}s_1 s_{g2} \\ -c_{g2} & s_{g2} & 0 & \pm d_{g1} \pm (-a_{\mathrm{c}g3})c_{g2} + a_{g2}s_{g2} \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right] \tag{4}$$

where $a_{\mathrm{c}1}$, $a_{\mathrm{c}g2}$, $a_{\mathrm{c}g3}$, d_{g1} – positions and offsets of mass centers of branches $g \in \{a, b, c, \ldots\}$, d_{g3} – translations of prismatic joints, s_1, s_{g2}, c_1, c_{g2} – abbreviations of sine and cosine functions of angle Θ_1, Θ_{g2}. Sign "±" changes to "+" for branches situated above base coordinate system.

Step 3. Instantaneous kinematics of centers of gravity – in this step it is necessary to formulate jacobian matrices of every center of masses. Jacobian matrices expressing relation between vector of generalized joint speeds of branched kinematic chain and linear and angular speed vector in base coordinate system (fixed with bough). The general form of jacobian matrices is given by following formulas:

$$\boldsymbol{J}_{\mathrm{c}1}=\left[\begin{array}{c:c:c}\boldsymbol{S}(\boldsymbol{z}_0)(\boldsymbol{o}_{\mathrm{c}1}-\boldsymbol{o}_0) & \boldsymbol{0} & \boldsymbol{0}\\ \hdashline \boldsymbol{z}_0 & \boldsymbol{0} & \boldsymbol{0}\end{array}\right] \tag{5}$$

$$\boldsymbol{J}_{\mathrm{c}g2}=\left[\begin{array}{c:c:c}\boldsymbol{S}(\boldsymbol{z}_0)(\boldsymbol{o}_{\mathrm{c}g2}-\boldsymbol{o}_0) & \boldsymbol{S}(\boldsymbol{z}_{g1})(\boldsymbol{o}_{\mathrm{c}g2}-\boldsymbol{o}_{g1}) & \boldsymbol{0}\\ \hdashline \boldsymbol{z}_0 & \boldsymbol{z}_{g1} & \boldsymbol{0}\end{array}\right] \tag{6}$$

$$\boldsymbol{J}_{\mathrm{c}g3}=\left[\begin{array}{c:c:c}\boldsymbol{S}(\boldsymbol{z}_0)(\boldsymbol{o}_{\mathrm{c}g3}-\boldsymbol{o}_0) & \boldsymbol{S}(\boldsymbol{z}_{g1})(\boldsymbol{o}_{\mathrm{c}g3}-\boldsymbol{o}_{g1}) & \boldsymbol{z}_{g2}\\ \hdashline \boldsymbol{z}_0 & \boldsymbol{z}_{g1} & \boldsymbol{0}\end{array}\right] \tag{7}$$

where $\boldsymbol{z}_0 = [0,0,1]^{\mathrm{T}}$; $\boldsymbol{z}_{g1}$, $\boldsymbol{z}_{g2}$ – unit vectors of coordinate systems which are given in homogenous matrices by Eqn. (2) and Eqn. (3) (every first three elements of third column), $\boldsymbol{o}_0$, $\boldsymbol{o}_{g1}$, $\boldsymbol{o}_{\mathrm{c}1}$, $\boldsymbol{o}_{\mathrm{c}g1}$, $\boldsymbol{o}_{\mathrm{c}g2}$, $\boldsymbol{o}_{\mathrm{c}g3}$ – vectors representing origins of coordinates systems fixed with base, “r” joints, all center of masses (see Figure 5), $\boldsymbol{S}(\boldsymbol{z}_i)$ – skew symmetric matrix for vector $\boldsymbol{z}_i$.

Step 4. Inertial matrix formulation – this is crucial point of the algorithm. In this step it is necessary to formulate the kinetic energy of branched kinematic chain. The kinetic energy of branched kinematic chain may be expressed by sum of kinetic energy of bough and consecutive branches:

$$\begin{aligned}E_{\mathrm{k}} &= \frac{1}{2}\dot{\boldsymbol{q}}_0^T\,(m_{\mathrm{c}1}\boldsymbol{J}_{\mathrm{vc}1}^T\boldsymbol{J}_{\mathrm{vc}1}+\boldsymbol{J}_{\omega\mathrm{c}1}^T\boldsymbol{R}_{\mathrm{c}1}\boldsymbol{I}_{\mathrm{c}1}\boldsymbol{R}_{\mathrm{c}1}^T\boldsymbol{J}_{\omega\mathrm{c}1})\dot{\boldsymbol{q}}_0+\\ &+\frac{1}{2}\sum_g\dot{\boldsymbol{q}}_g^T\sum_{s=2}^{n}(m_{cgs}\boldsymbol{J}_{\mathrm{v}cgs}^T\boldsymbol{J}_{\mathrm{v}cgs}+\boldsymbol{J}_{\omega cgs}^T\boldsymbol{R}_{cgs}\boldsymbol{I}_{cgs}\boldsymbol{R}_{cgs}^T\boldsymbol{J}_{\omega cgs})\dot{\boldsymbol{q}}_g\end{aligned} \tag{8}$$

where $\dot{\boldsymbol{q}}_g$ – vector of generalized joint speeds, $\boldsymbol{J}_{\mathrm{vca}j}$, $\boldsymbol{J}_{\omega caj}$ – jacobians of linear [first rows of Eqns. (5-7)] and angular speed [second rows of Eqns. (5-7)], $\boldsymbol{R}_{\mathrm{c}gs}$ – rotation matrices in homogenous transformation matrices given by Eqns. (2-4), $\boldsymbol{I}_{\mathrm{c}gs}$ – matrix of mass moment of inertia, $m_{\mathrm{c}gs}$ – mass of the link concentrated in the center of masses.

After that it is possible to rearrange the expression of kinetic energy according to generalized vector of joint speed to the quadratic form [1]:

$$E_k=\frac{1}{2}\dot{\boldsymbol{q}}^T\boldsymbol{D}_{\mathrm{r}}\dot{\boldsymbol{q}} \tag{9}$$

Matrix $\boldsymbol{D}_{\mathrm{r}}$ present in Eqn. (8) has internal structure correlated with real structure of branched kinematic chain. The block structure of inertial matrix is as follow:

$$\boldsymbol{D}_{\mathrm{r}}=\left[\begin{array}{c:c:c}\boldsymbol{k} & \boldsymbol{g_k} & \cdots\\ \hdashline \boldsymbol{g_k^T} & \boldsymbol{g} & \cdots\\ \vdots & \vdots & \ddots\end{array}\right] \tag{10}$$

where $\boldsymbol{g} \in \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \ldots\}$ – branch name, $\boldsymbol{k}$ – bough inertial matrix – 1×1, $\boldsymbol{g}_k$ – bough-branch mutual inertial coupling matrix – $1\times n$, $\boldsymbol{g}$ – branch inertial matrix – $(n\text{-}1)\times(n\text{-}1)$, n – sum of bough and single branch degrees of freedom.

Matrix $\boldsymbol{k}$ of block matrix $\boldsymbol{D}_{\mathrm{r}}$ consists on only one element, which is given by general form as follows:

$$k_{11} = m_{\mathrm{c}1}\sum_{i=1}^{3} J_{\mathrm{vc}1\_i1}^{2} + I_{\mathrm{zc}1}\Big(\sum_{i=1}^{3} J_{\omega\mathrm{c}1\_i1} r_{\mathrm{c}1\_i3}\Big)^2 + \sum_{g\in\{z_{\mathrm{n}}\cup z_{\mathrm{p}}\}}\sum_{s=2}^{n} m_{\mathrm{c}gs}\sum_{i=1}^{3} J_{\mathrm{vc}gs\_i1}^{2} + \\ + \sum_{g\in\{z_{\mathrm{n}}\cup z_{\mathrm{p}}\}}\sum_{s=2}^{n} I_{zgs}\Big(\sum_{i=1}^{3} J_{\omega\mathrm{c}gs\_i1} r_{\mathrm{c}gs\_i3}\Big)^2 \tag{11}$$

where $J_{\mathrm{vc1\_ij}}$, $J_{\mathrm{vcgs\_ij}}$, $J_{\omega\mathrm{c1\_ij}}$, $J_{\omega\mathrm{cgs\_ij}}$ – elements of jacobian matrices linear speed of bough and branches mass centers and angular speed of bough and branches mass centers respectively, $r_{\mathrm{c1\_ij}}$, $r_{\mathrm{vcgs\_ij}}$, $r_{\omega\mathrm{cgs\_ij}}$ – elements of rotation matrices of center of masses for bough and branches.

For freely chosen branch "g", general elements of inertial branch matrix are as follow, for diagonal elements (with column index $k \geq 2$):

$$g_{k-1,k-1} = \sum_{s=2}^{n}\left(m_{\mathrm{c}gs}\sum_{i=1}^{3} J_{\mathrm{vc}gs\_ik}^{2} + I_{zgs}\Big(\sum_{i=1}^{3} J_{\omega\mathrm{c}gs\_ik} r_{\mathrm{c}gs\_i3}\Big)^2 \right), \tag{12}$$

for elements lying outside the diagonal for rows $w \geq 2$ and columns $k > 2$ with different subscripts:

$$g_{w,k} = \sum_{s=2}^{n}\left(m_{\mathrm{c}gs}\sum_{i=1}^{3}\prod_{j\in\{w,k\}} J_{\mathrm{vc}gs\_ij} + I_{zgs}\prod_{j\in\{w,k\}}\sum_{i=1}^{3} J_{\omega\mathrm{c}gs\_ij} r_{\mathrm{c}gs\_i3} \right). \tag{13}$$

The general forms for elements of inertia bough-branch mutual inertial coupling matrix are as follows:

$$g_{\mathrm{k}(w,k-1)} = \sum_{s=2}^{n}\left(m_{\mathrm{c}gs}\sum_{i=1}^{3}\prod_{j\in\{w,k\}} J_{\mathrm{vc}gs\_ij} + I_{zgs}\prod_{j\in\{w,k\}}\sum_{i=1}^{3} J_{\omega\mathrm{c}gs\_ij} r_{\mathrm{c}gs\_i3} \right) \tag{14}$$

for row w = 1 and column $k \geq 2$.

For specific "4G9rp" branched kinematic chain which is presented in Figure 5 the following block matrix elements are as follow:

- k_{11} for $\boldsymbol{k}$ matrix:

$$k_{11} = m_{c1}a_{c1}^2 + I_{zc1} + \sum_{g\in\{a,b,c,d\}} m_{cg2}(a_1 + a_{cg2}c_{g2})^2 + \sum_{g\in\{a,c\}} m_{cg3}((a_1 + a_{g2}c_{g2} - a_{cg3}s_{g2})^2 + d_{g3}^2) +$$
$$+ \sum_{g\in\{b,d\}} m_{cg3}((a_1 + a_{g2}c_{g2} + a_{cg3}s_{g2})^2 + d_{g3}^2) \tag{15}$$

- elements g_{ij} inertial branches matrices $\boldsymbol{g}$ are as follows:

$$\begin{cases} g_{11} = m_{cg2}a_{cg2}^2 + I_{zg2} + m_{cg3}(a_{g2}^2 + a_{cg3}^2) + I_{zg2} \\ g_{22} = m_{cg3} \\ g_{12} = 0 \end{cases} \tag{16}$$

- elements of inertia bough-branch mutual inertial coupling matrix $\boldsymbol{g}_k$:
  - for branches $g \in \{a, c\}$:

$$\begin{cases} g_{k11} = -d_{g3}m_{cg3}(a_{cg3}c_{g2} + a_{g2}s_{g2}) \\ g_{k12} = -m_{cg3}(a_1 + a_{g2}c_{g2} - a_{cg3}s_{g2}) \end{cases} \tag{17}$$

  - for branches $g \in \{b, d\}$:

$$\begin{cases} g_{k11} = -d_{g3}m_{cg3}(-a_{cg3}c_{g2} + a_{g2}s_{g2}) \\ g_{k12} = -m_{cg3}(a_1 + a_{g2}c_{g2} + a_{cg3}s_{g2}) \end{cases} \tag{18}$$

The general structure of inertial matrix for “x_G_y_rp” kind of branched kinematic chain is in Figure 9 presented. When number of branches increases the inertial matrix extends for another column and row as is shown in Figure 9.

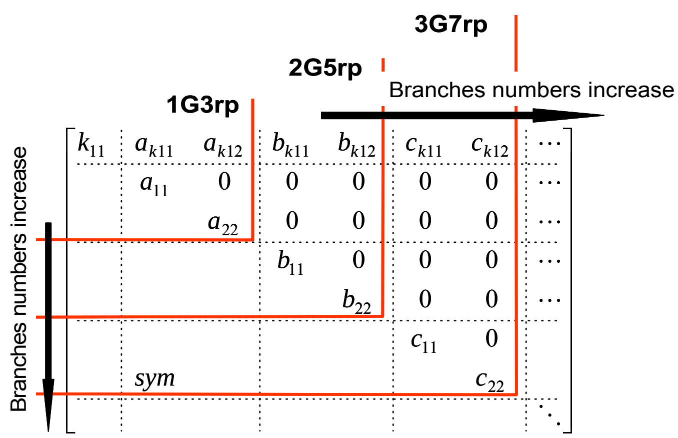


Fig. 9. Inertial matrix expansion

Step 5. Coriolis and centrifugal force matrix formulation. These matrices are formulated basing on formulas given in [1-3].

Step 6. Calculation of gravitational force matrix – it depends on position of whole drive with respect to gravitation field [2].

Step 7. Formulation of Lagrange equations. Having all necessary matrices: inertial matrix $\boldsymbol{D}_{\mathrm{r}}$, centrifugal and Coriolis force matrix $\boldsymbol{C}$, gravitational force or torque matrix $\boldsymbol{G}$ it is possible to rewrite Lagrange equations (1) to canonical form as follows:

$$\ddot{\boldsymbol{q}} = \boldsymbol{D}_{\mathrm{r}}^{-1}(\boldsymbol{Q} - \boldsymbol{C}\dot{\boldsymbol{q}} - \boldsymbol{G}) \tag{19}$$

In Eqn.(19) is necessary to invert inertial matrix $\boldsymbol{D}_{\mathrm{r}}$, it may be accomplish using its block structure.

3. BLOCK MATRIX INVERSION

For exemplary “4G9rp” kinematic chain total dimension of the block matrix is 9×9, the internal structure of inertial matrix consists of 1×1 dimensional inertial matrix of bough, eight 1×2 dimensional bough-branch inertial coupling matrices and four 2×2 branch inertial matrices:

$$\boldsymbol{D}_{\mathrm{r}} = \left[\begin{array}{c|cccc} \boldsymbol{k} & \boldsymbol{a}_{\mathrm{k}} & \boldsymbol{b}_{\mathrm{k}} & \boldsymbol{c}_{\mathrm{k}} & \boldsymbol{d}_{\mathrm{k}} \\ \hline \boldsymbol{a}_{\mathrm{k}}^{T} & \boldsymbol{a} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{b}_{\mathrm{k}}^{T} & \boldsymbol{0} & \boldsymbol{b} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{c}_{\mathrm{k}}^{T} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{c} & \boldsymbol{0} \\ \boldsymbol{d}_{\mathrm{k}}^{T} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{d} \end{array}\right] \tag{20}$$

For convenient matrix inversion we use definition of invert matrix:

$$\boldsymbol{D}_{\mathrm{r}}^{-1}\boldsymbol{D}_{\mathrm{r}} = \boldsymbol{1} \tag{21}$$

and after some rearrangement we have:

$$adj\boldsymbol{D}_{\mathrm{r}}\,\boldsymbol{D}_{\mathrm{r}} = \boldsymbol{1}\det\boldsymbol{D}_{\mathrm{r}} \tag{22}$$

where $adj\boldsymbol{D}_{\mathrm{r}}$ – adjunction matrix.

Eqn. (22) may be written in block matrix form, as follow [1, 3, 4]:

$$\begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} & \boldsymbol{A}_{13} & \boldsymbol{A}_{14} & \boldsymbol{A}_{15} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} & \boldsymbol{A}_{23} & \boldsymbol{A}_{24} & \boldsymbol{A}_{25} \\ \boldsymbol{A}_{31} & \boldsymbol{A}_{32} & \boldsymbol{A}_{33} & \boldsymbol{A}_{34} & \boldsymbol{A}_{35} \\ \boldsymbol{A}_{41} & \boldsymbol{A}_{42} & \boldsymbol{A}_{43} & \boldsymbol{A}_{44} & \boldsymbol{A}_{45} \\ \boldsymbol{A}_{51} & \boldsymbol{A}_{52} & \boldsymbol{A}_{53} & \boldsymbol{A}_{54} & \boldsymbol{A}_{55} \end{bmatrix} \begin{bmatrix} \boldsymbol{k} & \boldsymbol{a}_{\mathrm{k}} & \boldsymbol{b}_{\mathrm{k}} & \boldsymbol{c}_{\mathrm{k}} & \boldsymbol{d}_{\mathrm{k}} \\ \boldsymbol{a}_{\mathrm{k}}^{T} & \boldsymbol{a} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{b}_{\mathrm{k}}^{T} & \boldsymbol{0} & \boldsymbol{b} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{c}_{\mathrm{k}}^{T} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{c} & \boldsymbol{0} \\ \boldsymbol{d}_{\mathrm{k}}^{T} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{d} \end{bmatrix} = \boldsymbol{1}\det\boldsymbol{D}_{\mathrm{r}} \tag{23}$$

In Eqn. (23) it is necessary to calculate unknown block elements $\boldsymbol{A}_{ij}$ of block adjunction matrix. It results in 25 algebraic matrix equations to be solved. After inversion we get final inverted inertial block matrix [1]:

$$
\boldsymbol{D}_{\mathrm{r}}^{-1} = \left[\begin{array}{c:c:c:c:c}
\boldsymbol{k}_1 & -\boldsymbol{k}_1\boldsymbol{a}_{\mathrm{k}}\boldsymbol{a}^{-1} & -\boldsymbol{k}_1\boldsymbol{b}_{\mathrm{k}}\boldsymbol{b}^{-1} & -\boldsymbol{k}_1\boldsymbol{c}_{\mathrm{k}}\boldsymbol{c}^{-1} & -\boldsymbol{k}_1\boldsymbol{d}_{\mathrm{k}}\boldsymbol{d}^{-1} \\ \hdashline
 & \boldsymbol{a}_1 & \boldsymbol{a}^{-1}\boldsymbol{a}_{\mathrm{k}}^T\boldsymbol{k}_1\boldsymbol{b}_{\mathrm{k}}\boldsymbol{b}^{-1} & \boldsymbol{a}^{-1}\boldsymbol{a}_{\mathrm{k}}^T\boldsymbol{k}_1\boldsymbol{c}_{\mathrm{k}}\boldsymbol{c}^{-1} & \boldsymbol{a}^{-1}\boldsymbol{a}_{\mathrm{k}}^T\boldsymbol{k}_1\boldsymbol{d}_{\mathrm{k}}\boldsymbol{d}^{-1} \\ \hdashline
 & & \boldsymbol{b}_1 & \boldsymbol{b}^{-1}\boldsymbol{b}_{\mathrm{k}}^T\boldsymbol{k}_1\boldsymbol{c}_{\mathrm{k}}\boldsymbol{c}^{-1} & \boldsymbol{b}^{-1}\boldsymbol{b}_{\mathrm{k}}^T\boldsymbol{k}_1\boldsymbol{d}_{\mathrm{k}}\boldsymbol{d}^{-1} \\ \hdashline
 & & & \boldsymbol{c}_1 & \boldsymbol{c}^{-1}\boldsymbol{c}_{\mathrm{k}}^T\boldsymbol{k}_1\boldsymbol{d}_{\mathrm{k}}\boldsymbol{d}^{-1} \\ \hdashline
sym & & & & \boldsymbol{d}_1
\end{array}\right] \tag{24}
$$

The $\boldsymbol{k}_1$ elementary bock matrix will be called as the leading elementary matrix of inverted bock matrix, and it is given by expression:

$$
\boldsymbol{k}_1 = (\boldsymbol{k} - \boldsymbol{a}_{\mathrm{k}}\boldsymbol{a}^{-1}\boldsymbol{a}_{\mathrm{k}}^T - \boldsymbol{b}_{\mathrm{k}}\boldsymbol{b}^{-1}\boldsymbol{b}_{\mathrm{k}}^T - \boldsymbol{c}_{\mathrm{k}}\boldsymbol{c}^{-1}\boldsymbol{c}_{\mathrm{k}}^T - \boldsymbol{d}_{\mathrm{k}}\boldsymbol{d}^{-1}\boldsymbol{d}_{\mathrm{k}}^T)^{-1} \tag{25}
$$

Expression of the leading elementary matrix extends when numbers of branches increases in very simple way which is shown in Figure 10.

$$
\boldsymbol{k}_1 = (\underbrace{\underbrace{\overbrace{\overbrace{\overbrace{\boxed{\boldsymbol{k}} - \boxed{\boldsymbol{a}_{\mathrm{k}}\boldsymbol{a}^{-1}\boldsymbol{a}_{\mathrm{k}}^T}}^{\text{one branch}} - \boxed{\boldsymbol{b}_{\mathrm{k}}\boldsymbol{b}^{-1}\boldsymbol{b}_{\mathrm{k}}^T}}^{\text{two branches}} - \boxed{\boldsymbol{c}_{\mathrm{k}}\boldsymbol{c}^{-1}\boldsymbol{c}_{\mathrm{k}}^T}}^{\text{three branches}} - \boxed{\boldsymbol{d}_{\mathrm{k}}\boldsymbol{d}^{-1}\boldsymbol{d}_{\mathrm{k}}^T}}_{\text{four branches}} - \boxed{\boldsymbol{e}_{\mathrm{k}}\boldsymbol{e}^{-1}\boldsymbol{e}_{\mathrm{k}}^T}}_{\text{five branches}} - \ldots)^{-1}
$$

Fig. 10. Expansion of the leading elementary matrix expression vs. number of branches increase

The rest of diagonal elements are as follow:

$$
\begin{aligned}
\boldsymbol{a}_1 &= (\boldsymbol{a} - \boldsymbol{a}_{\mathrm{k}}^T(\boldsymbol{k}_1^{-1} + \boldsymbol{a}_{\mathrm{k}}\boldsymbol{a}^{-1}\boldsymbol{a}_{\mathrm{k}}^T)^{-1}\boldsymbol{a}_{\mathrm{k}})^{-1} \\
\boldsymbol{b}_1 &= (\boldsymbol{b} - \boldsymbol{b}_{\mathrm{k}}^T(\boldsymbol{k}_1^{-1} + \boldsymbol{b}_{\mathrm{k}}\boldsymbol{b}^{-1}\boldsymbol{b}_{\mathrm{k}}^T)^{-1}\boldsymbol{b}_{\mathrm{k}})^{-1} \\
\boldsymbol{c}_1 &= (\boldsymbol{c} - \boldsymbol{c}_{\mathrm{k}}^T(\boldsymbol{k}_1^{-1} + \boldsymbol{c}_{\mathrm{k}}\boldsymbol{c}^{-1}\boldsymbol{c}_{\mathrm{k}}^T)^{-1}\boldsymbol{c}_{\mathrm{k}})^{-1} \\
\boldsymbol{d}_1 &= (\boldsymbol{d} - \boldsymbol{d}_{\mathrm{k}}^T(\boldsymbol{k}_1^{-1} + \boldsymbol{d}_{\mathrm{k}}\boldsymbol{d}^{-1}\boldsymbol{d}_{\mathrm{k}}^T)^{-1}\boldsymbol{d}_{\mathrm{k}})^{-1}
\end{aligned} \tag{26}
$$

When we add new fifth branch, then appears diagonal elements $\boldsymbol{e}_1$ described by formula:

$$
\boldsymbol{e}_1 = (\boldsymbol{e} - \boldsymbol{e}_{\mathrm{k}}^T(\boldsymbol{k}_1^{-1} + \boldsymbol{e}_{\mathrm{k}}\boldsymbol{e}^{-1}\boldsymbol{e}_{\mathrm{k}}^T)^{-1}\boldsymbol{e}_{\mathrm{k}})^{-1}. \tag{27}
$$

4. IMPLEMENTATION OF "4G9RP" HEAD POSITIONING SYSTEM

Mathematical model of head positioning system with branched kinematic chain "4G9rp" with time optimal control system for position control is in Figure 11 shown. Mathematical model of VCM motor is implemented in block subscribed by "VCM motor" and it is described in detail in [1]. In position time control algorithm the exactly knowledge about actual position and angular speed of bough is assumed. Time optimal control is implemented in block denoted by "TOC". Mathematical model of branched kinematic chain was in "4G9rp" block implemented using "Matlab Function" sub block. The parameters of slider suspension driven by electrostatic motor, like stiffness and masses, were taken from [18] and uniformly used for all prismatic joints present in branched kinematic chain.

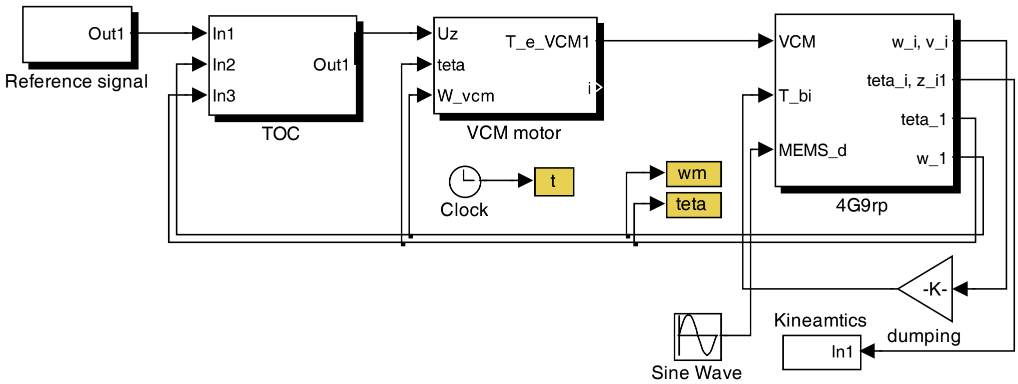


Fig. 11. Block schema of time optimal control of head positioning system with branched kinematic chain

The angular displacement was used as reference signal in bough joint: 45 degrees (at time t_0 = 0 s), 10 degrees (at time t_1 = 50 ms) and 45 degrees (at time t_2 = 100 ms). Results of simulation are in Figures 12-17 presented. Angular displacement of bough is presented in Figures 12 and 13 presents the displacement of the head in perpendicular direction to the data track centre.

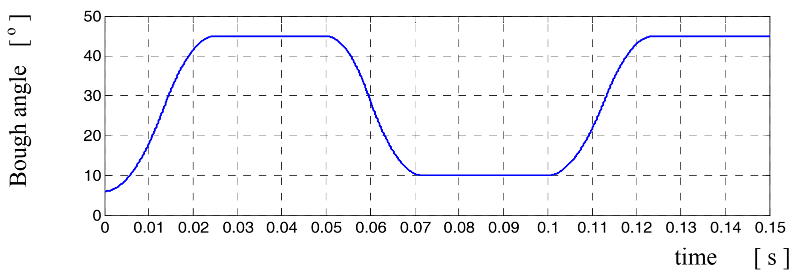


Fig. 12. Angular displacement of bough joint

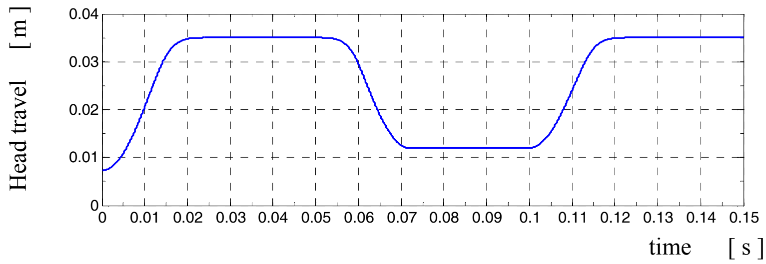


Fig. 13. Displacement of head in perpendicular direction to the center of data track

In Figure 14, the characteristic triangle shape of angular speed during time optimal control of bough joint is presented. The speed values reached more than 55 rad/s.

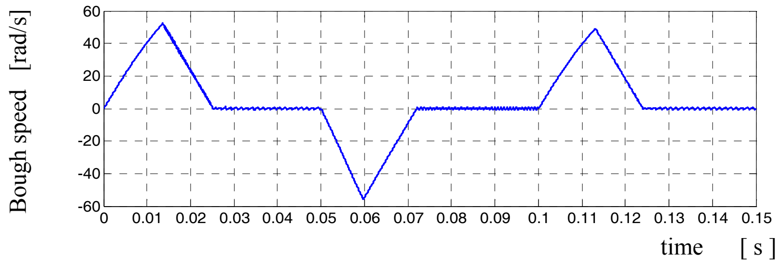


Fig. 14. Angular speed of bough joint

Linear displacement of slider/head in branch “a” relative to suspension is in Figure 15 presented. Maximum value of slider/head displacement in perpendicular direction to the center of data track reaching 6 nm during acceleration process (because of extremely high dumping ratio was assumed). When dumping ratio decreases 100 times (Fig. 16) the displacement of head during acceleration reached values close to 10 μm (this values is not reached in real case because of limited range of electrostatic motor motion to about 1 μm).

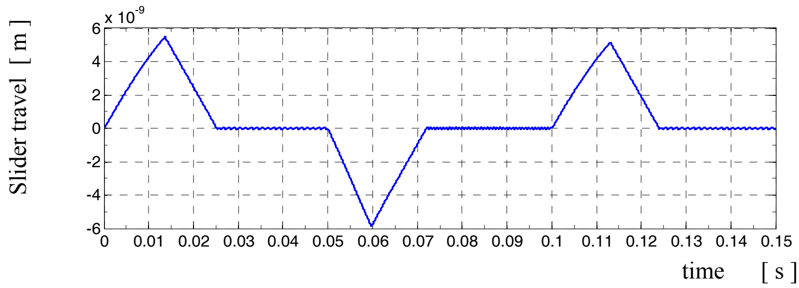


Fig. 15. Linear displacement of prismatic joint – slider displacement in branch “a”

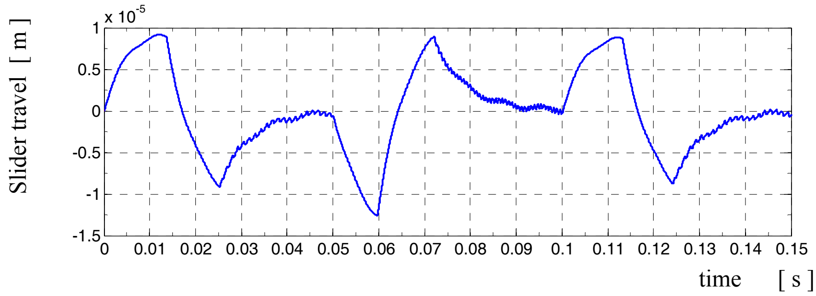


Fig. 16. Linear displacement of prismatic joint – slider displacement in branch “a” – dumping ratio decreases 100 times

In next figures the electromagnetic variable of VCM motor during time optimal control of head positioning system are presented. In Figure 17 the torque generated by VCM motor is shown and corresponding to the torque armature current is presented in Figure 18.

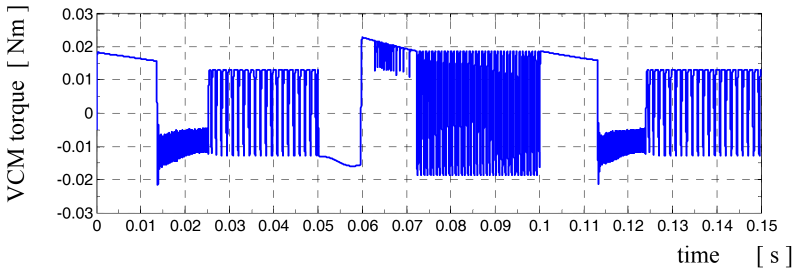


Fig. 17. Electromagnetic torque generated by VCM motor

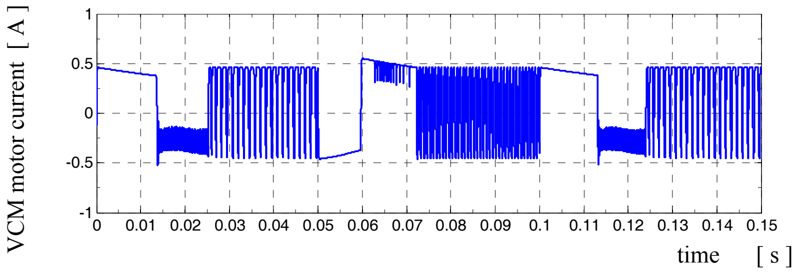


Fig. 18. Current flowing by VCM motor winding

5. CONCLUSIONS

Mathematical model of branched kinematic chain of head positioning system can be derived with the help of block matrix theory as well as in field of kinematics and dynamics. Formulated block matrices in chapter 2 have internal structure which corresponds to the structure of the branched kinematic chain; they consist with elementary inertial matrices related to bough, branches and expressions described mutual dynamic interactions between bough and branches links. The general expressions for all block matrices internal elements were given, as well as the method of inertial block matrix inversion.

Presented method allows for fast implementation of different structure of branched kinematic chains for head positioning system, what is very valuable when the new methods supporting data areal densities increase are searching by manufacturers of hard disk drives.

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WYKORZYSTANIE MACIERZY BLOKOWYCH DO MODELOWANIA UKŁADU NAPĘDOWEGO ORAZ STEROWANIA DYSKÓW TWARDYCH

Tomasz TRAWIŃSKI

STRESZCZENIE *W artykule przedstawiono zastosowanie teorii macierzy blokowych do formułowania modeli matematycznych rozgałęzionych systemów pozycjonowania głowic pamięci masowych, z uwzględnieniem zwiększania liczby gałęzi łańcucha kinematycznego.*

Metoda pozwala na zapisanie ogólnej postaci na każdy element macierzy bezwładnościowych przed i po jej odwróceniu. W rozdziale 3 przedstawiono proces odwracania macierzy blokowej, a w rozdziale 4 przedstawiono przykładowe wyniki symulacyjne, potwierdzające prawidłowe sformułowanie modeli.