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Review of face recognition algorithms

Introduction

The classification, or the system of typology or anthropological typology, intended to describe the systematic diversity within the species of *homo sapiens*, has existed for many years. As early as 1940, William Sheldon developed somatotypes, *i.e.* types of human body structure. He proposed a classification system in which all possible body types were characterised on the basis of the degree to which they matched each other and classified them into these three somatotypes[2]: ectomorphic, mesomorphic and endomorphic. Other taxonomies have been developed for body shape[3], hands[4],

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[2] Sheldon W, Atlas of Men: A Guide for Somatotyping the Adult Image of All Ages. *New York,* 1970.

[3] Alemany S *et al.*, Anthropometric survey of the Spanish female population aimed at the apparel industry <https://www.3dbodyscanning.org/cap/papers/2010/10307_11alemany.pdf>, 21 January 2021; Vinué G, Epifanio I, Alemany S, Archetypoids: A new approach to define representative archetypal data, *Computational Statistics and Data Analysis,* 2015, Vol. 87, Iss. C.

[4] Jee S.C, Yun M.H, An anthropometric survey of Korean hand and hand shape types. *International Journal of Industrial Ergonomics*, 2016, Vol. 53.

feet[5] and the head[6]. Taxonomies allow you to use common terminology to define the configuration of body parts, also providing a standardised way of their description. They are widely used in many scientific fields: humanities (archeology), engineering and technical (automation, technical informatics, biomedical engineering[7]), in all disciplines in the fields of medical sciences and health sciences, and the whole field of exact and natural sciences. In general, these types of typology systems are intended for qualitative categorisation, based on the global appearance of body parts, although in some cases, a quantitative analysis of selected characteristics has been developed to obtain classification[8].

Humans have especially developed their perceptual ability to process the face and extract information from its features[9]. The human brain has a specialised 'neural network' that processes information from images of other people's faces. It allows a person to identify other people, their gender, age and race, and even evaluate emotions. By using the human behavioural ability to perceive the face and its features, attributions such as personality are made, as well as intelligence and the credibility of the person's appearance, based on their face[10,11]. For centuries, artists, researchers and scientists have tried to develop procedures for measuring and classifying human faces. Anthropometric facial analysis is used in various fields of science, art and computer science. It is used during surgical procedures, activities related to forensics, artistic

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5 Kim N.S, Do W.H, Classification of Elderly Women's Foot Type. *Journal of* the *Korean Society* of *Clothing* and Textiles, 2014, Vol. 38.

6 Sarakon P, Charoenpong T, Charoensiriwath S, Face shape classification from 3D human data by using SVM. *Electronic source:* https://www.researchgate.net/publication/282382114_Face_shape_classification_from_3D_human_data_by_using_SVM, *accessed*: 22 January 2021.

7 Preston T.A, Singh M, Redintegrated Somatotyping. *Ergonomics*, 1972, Vol. 15; Lin Y.L, Lee K.L, Investigation of anthropometry basis grouping technique for subject classification. *Ergonomics*, 1999, Vol. 42.

8 Ritz-Timme S *et al.*, A new atlas for the evaluation of facial features: Advantages, limits, and applicability. *International Journal of Legal Medicine*, 2011, Vol. 125; Massidda M *et al.*, Somatotype of elite Italian gymnasts. *Neck Antropol*, 2013, Vol. 37; Malousaris G.G *et al.*, Somatotype, size and body composition of competitive female volleyball players. *Journal of Science and Medicine in Sport*, 2008, Vol. 11; Koleva M, Nacheva A, Boev M, Somatotype and disease prevalence in adults. *Reviews on Environmental Health*, 2002, Vol. 17, Iss. 1.

9 Damasio A.R, Prosopagnosia. *Trends Neurosciente*, 1985, Vol. 8; Bruce V, Young A, Understanding face recognition. *British Journal of Psychology*, 1986, Vol. 77, No. 3.

10 Kanwisher N, McDermott J, Chun M.M, The Fusiform Face Area: A module in human extrastriate cortex specialized for the perception of faces. *Journal of Neuroscience*, 1997, Vol. 17, Iss. 11.

11 Bruce V, Young A, Face perception. New York, 2012.

expression, facial recognition[12], emotions[13] and the assessment of human facial features[14].

In recent years, new information technologies have opened the way to the automatic assessment of facial features and facial gestures. The developed computational methods designed to make the analysis of information about the face are more and more precisely adapted to its classification based on anthropometric or emotional criteria[15].

Speaking of facial features, taxonomies are used in ergonomics, forensic anthropology, crime prevention, human-machine interaction and online activities. E-commerce, e-learning, games, dating and social networks are areas of everyday life in which various classifications of facial features are used. In such activities, it is common to use digital representations of people, known as avatars (incarnations) that symbolise our presence or act as a virtual interlocutor. Several taxonomies of facial features can be found in the literature. For example, the atlas of Peter Vanezis classifies 23 facial features, the Interpol Disaster Victim Identification[16,17] (DVI) form classifies 6 and the Data Volume Management Database[18] (DVM) classifies 45 features. Abraham Tamir divided the various shapes of the human nose into 14 groups. The division was made on the basis of 1,793 photos of the nose. A similar approach was used to classify human chins. In these works, a large collection of photographs was analysed and classified based on the similarity of the features of the chins[19,20].

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[12] Kong S.G *et al.*, Recent advances in visual and infrared face recognition – A review. *Computer Vision and Image Understanding*, 2005, Vol. 97.

[13] Tavares G, Mourão A, Magalhães J, Crowdsourcing facial expressions for affective-interaction. *Computer Vision and Image Understanding*, 2016, Vol. 147.

[14] Buckingham G *et al.*, Visual adaptation to masculine and feminine faces influences generalized preferences and perceptions of trustworthiness. *Evolution and Human Behavior*, 2006, Vol. 27, Iss. 5; Boberg M, Piippo P, Ollila E, Designing Avatars, [in:] Proceedings of the 3rd International Conference on Digital Interactive Media in Entertainment and Arts, 10–12 September 2008, Athens Greece 2008; Rojas M.M, Automatic prediction of facial trait judgments: Appearance vs. structural models. *PLoS One*, 2011, Vol. 6, Iss. 8; Laurentini A - Wikipedia, Booty A, Computer analysis of face beauty: A survey. *Computer Vision and Image Understanding*, 2014, Vol. 125.

[15] Li S.Z, Jain A.K, Handbook of Face Recognition. New York, 2005.

[16] Blameé G, Epifanio I, German S, Archetypoids…, *op.cit.*

[17] Vanezis P *et al.*, Morphological classification of facial features in adult Caucasian males based on an assessment of photographs of 50 subjects. *The Journal of Forensic Sciences*, 1996, Vol. 41.

[18] Asmann S *et al.* Anthropological atlas of male facial features. Frankfurt, 2007; Ohlrogge S *et al.*, Anthropological atlas of female facial features. Frankfurt, 2009.

[19] Tamir A, Numerical Survey of the Different Shapes of the Human Nose. *Journal of Craniofacial Surgery*, 2011, Vol. 22, Iss. 3.

[20] Tamir A, Numerical Survey of the Different Shapes of Human Chin. *Journal of Craniofacial Surgery*, 2013, Vol. 24, Iss. 5.

Automated human face recognition is a very complex computer problem of significant practical importance. It relies on numerous applications written specifically for this purpose. The applications specifically take into account automated secure access, automatic supervision, forensic analysis, quick retrieval of information from databases, *e.g.*, in police departments, automatic patient identification in hospitals, fraud and identity checking, and human-computer interactions.

In recent years, considerable research attention has been devoted to developing reliable automatic surface recognition systems that use two-dimensional images. Three-dimensional face recognition technology is emerging today due to the availability of improved 3D imaging devices and modern high-speed processing algorithms. Three-dimensional images of the face are acquired using devices for three-dimensional acquisition, similar in operation to the human visual perception system. For automatic face recognition and pattern matching, three-dimensional images have some advantages over two-dimensional images. The orientation of the face can be corrected by rotating a rigid body in 3D space. 3D images also provide information on the structure of the face, such as the surface curvature and geodetic distances. We do not get such information from a single 2D image. Recognition algorithms based on a 3D representation of a face have proven to be resistant to changes in lighting conditions during image acquisition[21,22].

Existing three-dimensional face recognition algorithms can be divided into two large classes: algorithms using “holistic” techniques and those based on local features. Face recognition techniques using holistic algorithms use information from the entire face or large regions of the face. An approach is possible in which portions of the facial surface are ranked and then compared. The second class of algorithms uses the structural properties of local physical properties[23].

Many studies have demonstrated the enormous potential of face recognition algorithms using local facial features. In the Face Recognition Vendor Test (FRVT) conducted in 2002, two face recognition algorithms based on local physical characteristics of the face were analysed. The first is based on a local analysis of its features, the second is based on[24,25] Elastic Bunch

[21] Zhao W *et al.*, Face recognition: A literature survey. *ACM Computing Surveys*, 2003, Vol. 35, Iss. 4.

[22] Kukula E.P et al., Effects of illumination changes on the performance of geometrix facevision/spl reg/ 3d frs. [in:] 38th Annual 2004 International Carnahan Conference on Security Technology, Albuquerque, 11–14 October 2004 NM. USA, 2004.

[23] Gupta S, Markey M.K, Bovik A.C, Advances and challenges in 3D and 2D+3D human face recognition, [in:] Zoeller E.A (Ed.), Pattern recognition research horizons. New York, 2007.

[24] Phillips P.J *et al.*, Face recognition vendor test 2002 evaluation report. Gaithersburg, 2003.

[25] Penev P.S, Atick J.K, Local feature analysis: a general statistical theory for object representation. *Network: Computation in Neural Systems*, 1996, Vol. 7, No. 3.

Graph Matching (hereinafter: EBGM). In EBGM, the face is represented as a "flexible graph" consisting of the wavelet coefficients of the Gabor transform at reference points of the face fragments and the Euclidean intervals between these points. Michael Hüsken *et al.* (2005) developed a 2D + 3D face recognition algorithm known as "hierarchical graph matching", which combines the results of two 2D EBGM and 3D EBGM algorithms. The algorithm was also one of the best rated during the Face Recognition Grand Challenge (FRGC) organised in 2005[26,27,28] to evaluate the performance of modern 3D face recognition algorithms, taking into account the criterion of operating time minimisation.

Face recognition algorithms in a 2D image

Facial recognition has long attracted the attention of scientists. Many of them have proposed different techniques of facial recognition in a 2D image. These techniques fall into three basic categories. The first category is a global (holistic) approach. The entire face (its surface) is used as input for the proposed recognition system. This data is then projected onto a small subspace.

The second category includes local methods. It does not include the entire face, but only some of its characteristic features or areas, then classified according to predefined statistics.

The hybrid approach (a hybrid of the two methods mentioned above) and methods based on statistical models constitute the third category. It includes hybrid approaches that use global and local features simultaneously. It accumulates the positive features of the two methods above. It also uses an approach based on statistical models. These models formalise the relationships between random variables using mathematical equations. These equations describe how one or more random variables are related to each other. A model is considered statistical when the variables are not deterministic but stochastically related.

Global Linear Face Recognition Techniques

Linear face recognition techniques use the linear projection of input data of an interesting image from a high-dimensional space to a relatively small-dimensioned space. This projection has two drawbacks. The first is the inability to preserve convex changes in the face, the second is the poor representation of the Euclidean distances between the characteristic points. Thus, the rate of face detection and recognition using linear

[26] Wiskott L *et al.*, Face recognition by elastic bunch graph matching, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 1997, Vol. 19, Iss. 7.

[27] Hüsken M *et al.*, Strategies and benefits of fusion of 2d and 3d face recognition. *Computer Vision and Pattern Recognition,* 2005, Vol. 1.

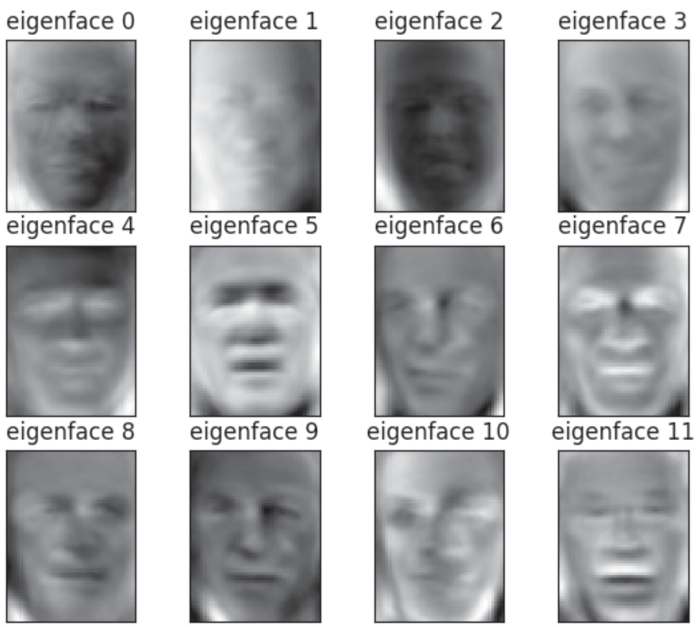
[28] Wiskott L *et al.*, Face recognition by elastic bunch graph matching. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 1997 Vol. 19, Iss. 7.

methods is low. The most popular linear recognition techniques are listed below.

— Eigenface – this technique refers to an appearance-based approach to facial recognition that aims to capture variability in a collection of facial images and use this information to encode and compare images of individual faces in a holistic way (as opposed to partial or element-based methods). In particular, eigenvectors are the major components of the surface distribution, or equivalently, of the covariance matrix of a face image set, where an image with N pixels is considered a point (or vector) in an N-dimensional space. Thus, each person's face can be reconstructed from the appropriate linear combination of "eigenes" and may be, for example, 7% of A's, 3.4% of B's, *etc.*, while someone else's face will have a different combination of its own properties. To construct a covariance matrix, each face image is converted into a vector. Each element of the vector corresponds to the intensity of the pixels. Such a transformation destroys the geometric, 2D structure of the image[29].

Picture 1

Visualisation of the author's own areas



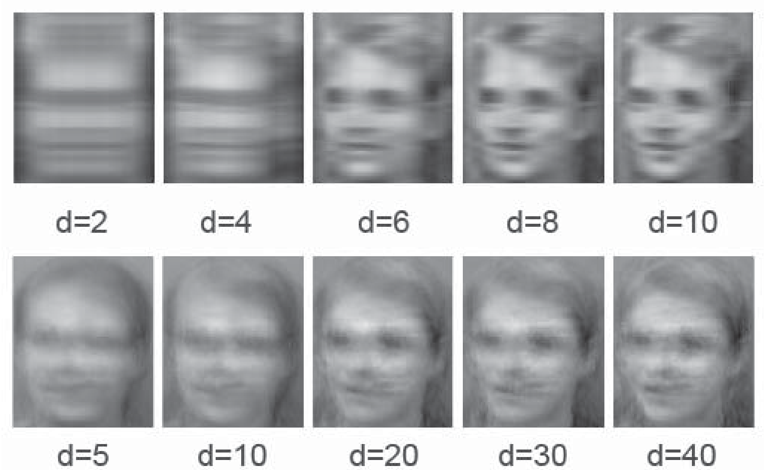
Source: Deshpande M, Face Recognition with Eigenfaces, <https://pythonmachinelearning.pro/face-recognition-with-eigenfaces/>, December 8, 2020

[29] Turk A, Pentland A.P, Face recognition using eigenfaces. *IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, 3–6 June 1991*, Maui, HI, USA, 1991.*

— 2D PCA (Principal Component Analysis; hereinafter: PCA[30] – two-dimensional PCA) – the technique is based on 2D image matrices, not 1D vectors, so the image matrix does not need to be converted into a vector before feature extraction. In this system, the covariance matrix is constructed directly using the original image matrices, and its eigenvectors are image derivatives.

Picture 2

Reconstructed images from 2D PCA (upper) and PCA (lower), where d is the number of eigenvectors



Source: Jian Y, *et al.*, Two-dimensional PCA: A new approach to appearance-based face representation and recognition, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2004, Vol. 26, Iss. 1

— ICA (Independent Component Analysis; hereinafter: ICA[31]) – to define the working principle of this algorithm, a statistical model of "hidden variables" should be used. Suppose we observe n linearly mixed elements x1, …, xn made up of n independent components. We have:

$$x_j = a_{j1}s_1 + a_{j2}s_2 + \ldots + a_{jn}s_n \text{ for every } j \qquad (1).$$

Using matrix notation, we can write the above equation as:

$$x = As \qquad (2).$$

The starting point for ICA is the assumption that the si components are statistically independent.

[30] Jian Y *et al.*, Two-dimensional PCA: A new approach to appearance-based face representation and recognition. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2004, Vol. 26, Iss. 1.

[31] Hyvärinen and Oja E, Independent Component Analysis: Algorithms and Applications. *Neural Networks*, 2000, Vol. 13, Iss. 4–5.

ICA is very closely related to a method called Blind Source Separation (BSS). “Source” here means the original signal, some independent element. ICA is one of the methods, perhaps the most widely used, to perform source separation.

- Multidimensional Scaling (hereinafter: MDSdist) – this is another well-known technique for linear reduction of the dimensions. Rather than preserving the data variance during projection, the technique tries to keep all of the distances between each pair of examples dist (x[32]$_i$, x_j), looking for a linear transformation. The minimisation problem can be solved by decomposing to eigenvalues using the Euclidean distance between the data being used. The results from this algorithm are similar to the results obtained from the PCA algorithm, but we obtain them by performing a rotation and then projecting them onto a flat area.
- Non-negative Matrix Factorization (hereinafter: NMF) – it is a method that, like the PCA algorithm, sees a person’s face as a linear vector combination without using the notion of a class. The difference between these algorithms is that, when using the NMF algorithm, it is not possible for the vectors underlying the combination of features to be negative[33].
- Linear Discriminant Analysis (hereinafter: LDA) – it functions on the principle of building a discriminant subspace that makes it possible to distinguish the faces of different people. It is also called Fisher’s Linear Discrimination. A face image, which usually consists of a large number of pixels, is reduced to a small set of linear feature combinations for that face. Then, from these linear combinations, using the Fisher classifier, its image is created, referred to as[34] Fisherface.

Regarding the linear PCA, LDA and ICA techniques of subspace analysis, work has been carried out to improve their performance. For example, work done, among others by Goncalo Tavares, improved PCA in terms of its resistance to pose / orientation changes. The probabilistic subspace has been introduced to ensure more significant similarity of the measure within the probabilistic framework. Besides this, *i.a.* Gavin Buckingham introduced the combination of D-LDA (direct LDA) and F-LDA (fractional LDA), a variant of the LDA in which weighted functions are used to avoid misclassification caused by too-close categories. Therefore, an approach based on the multi-line image set tensor distribution has been proposed

[32] Chapter 435, Multidimensional Scaling. *Electronic source*: https://www.ncss.com/wp-content/themes/ncss/pdf/Procedures/NCSS/Multidimensional_Scaling.pdf, *accessed*: January 19, 2021.

[33] Lee D.D, Seung H.S, Learning the parts of objects by non-negative matrix factorization. *Nature*, 1999, Vol. 401.

[34] Belhumeur P.N, Hespanha J.P, Kriegman D.J, Eigenfaces vs. Fisherfaces: Recognition using class specific linear projection. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 1997, Vol. 19, Iss. 7.

to reduce the influence of several factors related to the facial recognition system itself, such as lighting and orientation[35,36,37].

- Gabor wavelet – to improve facial recognition, high-intensity transformation vectors are extracted from the Gabor wavelet transform of the front face images when combined with the ICA detection model. The characteristics of the Gabor wavelet have been recognised as one of the best representations for facial recognition[38,39].

Although the linear methods described above avoid the instability of the previously developed identification methods of geometric methods, they are not precise enough. They poorly describe the subtleties of the geometric differences present in the original image. This is mainly due to limitations in the management of nonlinearity parameters during face recognition. Thus, nonlinear deformations can be smoothed out and concavities can be filled in, which can have adverse consequences in terms of facial recognition.

Other global linear techniques used for facial recognition are:

- Regularised Discriminant Analysis (hereinafter: RDA[40]) – in this technique, the class conditional covariance matrices are replaced by a regular estimate. After preliminary analysis, according to the method presented by Jerome H. Friedman, we first compute the combined (inside the class) sample covariance matrix. Then, using the regularisation parameter, we convert the class covariance matrix to a linear combination. This procedure places a two-dimensional mesh of points on two planes. It evaluates the cross-validated misclassification risk estimate at each prescribed point on a two-plane grid, and then selects the values with the lowest estimated risk as the appropriate regularisation parameter values[41].
- RLDA (*Regression*; LDA) – this technique is identical to the LDA, *i.e.* they both operate on the basis of the same basic linear models. The difference is that the RLDA optimises the conditional probability and the LDA the full probability[42].
- NLDA (*Null-space*; LDA) – is a natural extension of the conventional LDA. It is considered an NLDA when inside a class[43] singular matrix.

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35 Tavares G, Mourão A, Magellan A, Crowdsourcing…, *op.cit.*

36 Buckingham G *et al.*, Visual…, *op.cit.*

37 Boberg M, Piippo P.P, Ollila E, Designing…, *op.cit.*

38 Faculty A *et al.*, High performance human face recognition using Gabor based pseudo hidden Markov model. *International Journal of Applied Evolutionary Computation*, 2013, Vol. 4, Iss. 1.

39 Hyvärinen A, Oja E, Independent…, *op.cit.*

40 Friedman J.H, Regularized discriminant analysis. *Journal of the American Statistical Association*, 1989, Vol. 84, No. 405.

41 *Ibid.*

42 Hastie T, Lush A, Tibshirani R, Penalized discriminant analysis. *Annals of Statistics*, 1995, Vol. 23, No. 1.

43 Liu W, Null space approach of fisher discriminant analysis for face recognition, [in:] Maltoni D - Wikipedia, Jain A.K (Eds), Biometric Authentication, Proceedings of

When the matrix is single within a class, a subspace can be found in it that is an inter-class dispersion matrix. In this way, we obtain a linear criterion of Fisher's discrimination.

- Dual-space LDA – has been developed to overcome the problems associated with applying the LDA to a small number of data samples so as to take full advantage of discriminatory information in the facial space. Based on a probabilistic visual model, the spectrum of eigenvalues in the intra-class zero space of the scatter matrix is estimated, and discriminant analysis is used both in the main subspace and in the intra-class zero subspace of the dispersion matrix. Ultimately, both sets of discriminating characteristics are combined to identify a person[44].
- Boosting LDA – ensures high precision of data classification. This algorithm uses recognised groups of patterns, which are described as functions, instead of individual features. Additionally, it uses a two-stage weighted iteration that integrates weak classifiers with strong classifiers. Mutual information between classifiers is used as indicators of their weight allocation. This algorithm is faster than algorithms based on support vectors or neural networks[45].
- Block LDA – involves dividing the face image into several non-overlapping sub-images of the same size. As a result, the number of samples increases while the size of the sample decreases. In addition, all facial images used are converted to gradient images to reduce the impact of lighting variability. The resulting gradient image is divided into N[462] smaller images. Subsequently, the sub-frames are projected onto vectors using the classical LDA algorithm. Finally, a Euclidean measurement is used to determine the diagnosis score.
- Enhanced Fisher Linear Discriminant (hereinafter: FLD) – involves the simultaneous diagonalisation of two internal matrix classes and one class that arises between the matrices. First, the intra-matrix spreading matrix is cleared, and then PCA is applied between classes of the spreading matrix using the processed data. The purpose of the cleaning step is to normalise the scattering matrix within the class to

the ECCV 2004 International Workshop on Biometric Authentication, Prague, Czech Republic, 15 May 2004, Berlin-Heidelberg, 2004.

[44] Wang X, Tang X, Dual-space linear discriminant analysis for face recognition, [in:] Proceedings of the IEEE International Conference on Computer Vision and Pattern Recognition, Washington, DC, USA, 27 June–2 July 2004.

[45] Lu J.W, Plataniotis K.N, Venetsanopoulos A.N, Boosting linear discriminant analysis for face recognition. *Electronic source:* https://www.researchgate.net/profile/Konstantinos_Plataniotis/publication/224744759_Boosting_linear_discriminant_analysis_for_face_recognition/links/0f31753a4351c1151c000000/Boosting-linear-discriminant-analysis-for-face-recognition.pdf, *accessed*: January 22, 2021.

[46] Nhat V.D.M, Lee S, Block LDA for face recognition, [in:] Cabestany J, Prieto A, Sandoval F (Eds), Computational Intelligence and Bioinspired Systems. Proceedings of the 8th International Work-Conference on Artificial Neural Networks, Barcelona, Spain, 8–10 June 2005. Barcelona, 2005.

strengthen it. The second matrix operation maximises the dispersion between the classes to separate the different classes as much as possible[47].

- Discriminative Common Vectors (hereinafter: DCV) – instead of using its own scattering matrix for a given class, this algorithm uses the scattering matrix of all available classes. In this way, common vectors are obtained. A new set of vectors, called discriminating common vectors, is used for classification[48].
- Bilinear Discriminant Analysis (hereinafter: BDA or thermo-linear discriminant analysis[49]) – consists in finding a set of weights and a threshold, so that the discriminant function maximises the discrimination criterion, *e.g.* in two classes, a vector of data. Methods for determining the weights and threshold include least squares regression, logistic regression, a Fisher linear discriminator, and a single-layer perceptron. The simplicity of this algorithm makes it a good candidate for classification in situations where training data is very limited. In addition, it allows the identification of classes / elements depending on the data held.

Global nonlinear face recognition techniques

In the case when the input data structures are linear, the linear approaches described above ensure their faithful representation. However, when the data is nonlinear, a possible solution is to use kernel functions, which enables implicit comparisons of data in a high dimensional space where the nonlinear problem becomes linear. This makes it possible to use linear techniques when the internal data structure remains non-linear.

The recognition / classification process in this case is to describe the nonlinear data in terms of a linear product using the kernel function. It is in this context that several approaches for nonlinear methods have been proposed:

- Kernel Principal Component Analysis (hereinafter: KPCA) – is nothing more than the replacing of the classic linear PCA technology using the kernel function to describe non-linear data. The difference is that KPCA computes the main eigenvectors using the kernel function and not the covariance matrix directly. The PCA modified in this way can

[47] Zhou D, Yang X, Face recognition using enhanced fisher linear discriminant model with Facial combined feature, [in:] Zhang C, Guesgen H.W, Yeap W.K (Eds), PRICAI 2004: Trends in Artificial Intelligence, Proceedings of the 8th Pacific Rim International, Berlin-Heidelberg, 2004.

[48] Visani M, Garcia C, Jolion J.M, Normalized radial basis function networks and bilinear discriminant analysis for face recognition. Electronic source: https://www.researchgate.net/publication/4215341_Normalized_Radial_Basis_Function_Networks_and_Bilinear_Discriminant_Analysis_for_Face_Recognition, 22 January 2021.

[49] Cevikalp H et al., Discriminative common vectors for face recognition. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2005, Vol. 27, Iss. 1.

be viewed as an implementation of the PCA in high dimensional space by the related kernel function. Thus, the KPCA allows the construction of nonlinear mappings. Since the KPCA technique relies on kernel functions, its effectiveness largely depends on the choice of the model for this function. Typically, polynomial functions or Gaussian functions are used. The KPCA method has been used with great success to solve problems such as the recognition of people, speech or detection of new data elements of sets. Its main weakness is the size of the kernel matrix, which is the square of the number of samples of the training set, so it can quickly reach large values (large size)[50,51].

— Support Vector Machine (hereinafter: SVM*)* – is a learning technique effectively used for pattern recognition of*, e.g.*, a face, with high efficiency without the need to add more information about the object. Vladimir Naumovich Vapnik[52] had described this technique a few years earlier[53], and Guodong Guo further refined it, which used SVM, relying on the binary tree recognition strategy with particular emphasis and solving problems related to facial recognition[54].

— Kernel Independent Component Analysis (hereinafter: KICA[55]) – is an efficient algorithm for independent component analysis that estimates the source components using the generalised variance function based on a Hilbert space. The kernel of this algorithm is based on the correlation between two random variables represented in the Hilbert space related to a given map of objects. Research by Francis Bach and Michael I. Jordan, and research by, among others, Tommaso Martiriggiano showed that it is a more efficient algorithm than Fast ICA or PCA[56,57].

— Isomap[58] – is a method that has shown good results in finding low-dimensional non-uniformities from multiple samples in a multidimen-

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50 Mason T *et al.*, Performance of Geometrix Active IDTM 3D Face Recognition Engine on the FRGC Data, [in:] Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition, 21–23 September 2005, San Diego, CA, USA 2005.

51 Hoffmann H, Kernel PCA for novelty detection. *Pattern Recognit*, 2007, Vol. 40, Iss. 3.

52 Shawe-Taylor J, Cristianini N, Kernel Methods for Pattern Analysis. New York, 2004.

53 Vapnik V.N, The Nature of Statistical Learning Theory. New York, 1995.

54 Guo G, Li S.Z, Chan K, Face recognition by support vector machines, [in:] Proceedings of the Fourth IEEE International Conference on Automatic Face and Gesture Recognition, 28–30 March 2000, Grenoble, France 2000.

55 Bach F, Jordan M, Kernel independent component analysis. *Journal of Machine Learning Research*, 2002, Vol. 3.

56 *Ibid.*

57 Martiriggiano T *et al.*, Face Recognition by Kernel Independent Component Analysis. *IEEE Transactions on Neural Networks*, 2002, Vol. 13, Iss. 6.

58 Weinberger K.Q, Saul L.K, Unsupervised learning of image manifolds by semidefinite programming. *Electronic source*: https://repository.upenn.edu/cgi/

sional input space. While conventional subspace methods calculate values to represent distances between samples and use principal component analysis or something similar to induce linear varieties, the Isomap method estimates the geodetic distance between samples and then uses multivariate scaling to induce low dimensional variety.

- Maximum Variance Unfolding (hereinafter: MVU) – unlike the Isomap method, this method uses data from the determined similarities, maintaining both local distances and angles between pairs of all neighbours of each point in a given dataset. Since the method preserves the local maximum variance in dimension reduction processing, it is called maximum variance level (MVU) expansion. Similar to MDS scaling, the MVU can be applied to cases in which we have local similarities of objects in a specific data set. In such cases, the MVU tries to find the data against the input data. Technically, the MVU accepts semidefinite programming[59,60] (SDP) in order to solve the problem assumptions.
- Local Linear Embedding (hereinafter: LLE[61]) – compared to the Isomap method, has faster optimisation after implementation thanks to the use of matrix algorithms. This method begins by finding the set of the closest neighbours of each point. It then calculates a set of weights for each point that best describes that point as a linear combination of its neighbours. Finally, it uses the eigenvector optimisation technique to find low-dimensional point deposition so that each point is still described with the same linear combination of its neighbours. However, this method is poor at dealing with inhomogeneous densities of samples, as there is no fixed unit that prevents the centre of gravity of the body from shifting, and this is due to the fact that different areas differ in the density of the samples.
- Locality Preserving Projection (hereinafter: LPP[62]) – is a method that uses linear projective maps. They arise as a result of solving the

viewcontent.cgi?article=1000&context=cis_papers, *accessed*: January 22, 2021.

[59] Ming-Hsuan Yang, Face recognition using extended isomap, [in:] Proceedings of the IEEE International Conference on Image Processing, Rochester, USA, 22–25 September 2002, New York 2002.

[60] Semi-definite programming is a relatively new field of optimisation that is gaining more and more attention. Many of the practical problems of operations research and optimisation can be modelled or approximated as partially finite programming problems. In automatic control theory, SDPs are used in the context of linear matrix inequalities. All linear programs can be expressed as an SDP, and through SDP hierarchies one can approximate solutions to polynomial optimisation problems. Semi-definite programming of complex systems was used for optimisation. In recent years, some quantum query complexity problems have been framed in terms of semi-definite programs.

[61] Socolinsky D.A, Selinger A, Thermal face recognition in an operational scenario. *Conference Proceedings*, 2004, Vol. 2.

[62] He X *et al.*, Learning a locality preserving subspace for visual recognition. *Conference Proceedings*, 2003, Vol. 1.

problem of source data variation, which optimises the structure of the set of the closest neighbours. This method is an alternative to PCA, but it projects data along the directions of their maximum variance. The locality of the data is obtained by finding optimal linear approximations of the eigenfunctions of the Laplace Beltrami operator. As a result, LPP has many characteristics of nonlinear techniques such as Laplacian Eigenmaps or LLE, however it is fully linear and more importantly, it is defined in the surrounding space, not just at the training data points. LPP can be performed in the original data space or in the reconstructed Hilbert kernel space to which the data points are mapped.

— Local Tangent Space Analysis (hereinafter: LTSA) – This method models data from n-dimensional space to m-dimensional space, where m < n finds the set of k nearest neighbours of each point, then extracts local information in such a way that it computes d eigenvectors, from which the correlation matrix is then created data. The next step is to build the alignment matrix B with data beginning from B = 0. At this point, the data neighbourhood set is represented as the set of indices for the k nearest neighbours and is transposed into the new data matrix. Then the smallest eigenvectors are calculated and assigned global data coordinates corresponding to the smallest eigenvalue[63].

— Neural approaches – the development of neural networks has made it possible to change the efficiency of algorithms. The use of neural networks provides better results than the linear metric due to the fact that they create more complex decision surfaces[64]. Such networks can be trained to extract the major features of a given image in different variants. A multilayer perceptron (hereinafter: MLP) taught by the backpropagation algorithm, outputs exactly m of the first principal feature values that could be obtained from the matrix solution, provided that the neurons are linear. Otherwise, the obtained values are not exactly the values of the principal features, but better describe the vector of the input features. In this variant, we can teach the neural network in two ways. You can create one network and teach it with examples of faces from all available classes, or generate a separate network for each class and test faces in each network. Feature extraction can also be performed with the use of a network of scientists in an unsupervised manner, *e.g.* using the Hebb method. In the case where we only have one example from each class, a face typical for a given class can be obtained as an attractor using recursive neural networks, *e.g.* Hopfield networks. Based on the above, it can be said that MLP enables face detection, the Hebb network enables the extraction of its features, and the Hopfield network enables the creation of a typical face as an

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63 Wang Q, Li J, Combining local and global information for nonlinear dimensionality reduction. *Neurocomputing*, 2009, Vol. 72, Iss. 10–12.

64 Raducanu B, Dornaika F, Dynamic facial expression recognition using laplacian eigenmaps-based manifold learning. *Electronic source*: http://www.cvc.uab.es/~bogdan/Publications/raducanu_ICRA2010.pdf, *accessed*: January 22, 2021.

attractor. If we consider a face that is not exactly front-facing, we use recursive networks to find the representation of the face most independent of its rotation.

— Kohonen networks – are one of the basic types of self-organising networks. This property is adaptable to previously unknown inputs of which very little is known. Kohonen networks are synonymous with the entire group of networks in which learning is carried out using a self-organising, competitive method. It consists in feeding signals to the network inputs, and then selecting the winning neuron that best suits the input vector through competition. Given one poorly recognisable object, we can make the network try to find it in subsequent images[65].
— Convolutional neural networks[66] – a typical convolutional neural network consists of a combination of three basic layer types: convolution layer, activation layer, and size reduction layer. An additional layer is made of classic neural layers, called full connection layers. The input image is passed through the convolution layer, which performs discrete convolution operations on it. At its output, we get a map of features. In the analysis area, the convolution operation is used for filtration. The next processing step is a non-linear activation function that processes every pixel of the image. The output of the last plexus layer goes to the classical neural network. A feature that distinguishes deep neural networks from classic image classification systems is the possibility of automatic feature extraction without the participation of a researcher.

All of the above methods of image space projection based on feature space are nonlinear. Thanks to this, it is possible – to some extent – to reduce the recognition problem. However, while these techniques often improve recognition speed, they are too inflexible to be robust to new data types, unlike the linear methods described above.

Local methods of facial recognition

In local methods, we distinguish specific geometric features, such as the width of the head, the distance between the eyes or the corners of the mouth, and then these data are stored as a vector of features, making it possible to recognise / identify a given person.

We can divide these methods into two classes. One focuses on how the facial feature point detectors work, while the other deals with more elaborate representations of the information carried by these points, not just their geometric features.

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[65] Lawrence S *et al.*, Face recognition: A convolutional neural-network approach. *IEEE Transactions on Neural Networks*, 1997, Vol. 8, Iss. 1.

[66] Duffner S, Garcia C, Face recognition using non-linear image reconstruction. *Electronic source*: https://www.researchgate.net/publication/4308236_Face_recognition_using_non-linear_image_reconstruction, *accessed:* January 22, 2021; Zhang T *et al.*, Generalized discriminant analysis: A matrix exponential approach. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2010, Vol. 40, Iss. 1.

A popular method is Dynamic Link Architecture (hereinafter: DLA). Such a solution has been proposed by, among others, Daniel Saez Trigueros[67]. In this method, the face is represented by a graph of its nodes. The nodes contain Gabor wavelet coefficients highlighted for areas around a set of predefined facial landmarks. When attempting to identify faces, node plots are compared and the similarities between their graphics nodes are measured. On this basis, a decision is made to recognise the person. Another is a method that uses flexible fitting of EBGM beam graphs in which Gabor's wavelet coefficients are replaced with a histogram of directional gradients[68] - Histogram of Oriented Gradients (hereinafter: HOG)[69]. This algorithm has better performance than DLA due to the properties of the image histogram descriptors. They are more resistant to changes in lighting, rotation and small displacements. They also lead to greater distinguishability of face graphs compared to graphs obtained either by the DLA or EBGM method using Gabor wavelet coefficients.

Vector geometric functions is a method that uses a pattern image to very accurately detect the position of the source of interest in the examined image. This method determines for each point of the source the correlation coefficients between the examined image and the reference image, and then looks for the maximum values[70].

Another method is the facial statistical model – which uses multiple detectors with specific features for each part of the face, such as the eyes, nose, mouth, chin, *etc*. It is assumed that statistical models of face shapes can be built. However, despite a lot of research work, there are no sufficiently reliable and precise characteristic points that provide the possibility of identification using this[71] method.

Overall, there are many facial recognition algorithms, many more than those covered so far in this paper. The article focuses on the latest solutions – methods based on extracting features. They can be effectively used for facial recognition when, for example, only one reference photo is available. However, the performance of these algorithms depends on a number of single and effective algorithms for locating facial landmarks. In

[67] Trigueros D.S, Meng L, Hartnett M, Face recognition: From traditional to deep learning methods. *Electronic source*: https://www.researchgate.net/profile/Daniel_Saez_Trigueros2/publication/328685305_Face_Recognition_From_Traditional_to_Deep_Learning_Methods/links/5d19cd21a6fdcc2462b4a85f/Face-Recognition-From-Traditional-to-Deep-Learning-Methods.pdf, *accessed:* January 22, 2021.

[68] Lee T.S, Image representation using 2d Gabor wavelets. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 1996, Vol. 18, Iss. 10.

[69] Monzo D , *HOG-EBGM vs. Gabor-EBGM* [in:] 15th IEEE International Conference on Image Processing, ICIP 2008, Malta, 2008.

[70] Seyed M, Geometric feature descriptor and dissimilarity-based registration of remotely sensed imagery. *Electronic source*: https://doi.org/10.1371/journal.pone.0200676, *accessed*: July 19, 2018.

[71] Yang Z. *et al.,* Single Image 3D Face Reconstruction Based on Statistical Model. *Journal of Physics: Conference Series*, 2018, Vol. 1087, Iss. 5.

practice, the exact task of detecting a characteristic point is not easy and has not been completely solved, especially in cases where the characteristics of the face, *e.g.* the shape or appearance of the face image, may vary significantly.

The facial appearance algorithms above are based on defined local regions. Once defined, we choose the best way to represent the content information of each region. This is crucial for the performance of the recognition system which we want to use. Commonly used characteristics are: Gabor[72] coefficients, Haar wavelets[73], Fourier transform, descriptors such as the transformation of an invariant element, Scale-Invariant Feature Transform (hereinafter: SIFT [74]), characteristics based on the Local Binary Pattern method (hereinafter: LBP)[75], Local Phase Quantisation (LPQ) [76], Weber's Law Descriptor (WLD) [77] and Binarised Statistical Image Features (BSIF)[78].

Compared to the global approach, local methods have some advantages. First, they can provide additional information generated from local regions. Secondly, for each type of local characteristic, we can choose the most appropriate vector of features for its description. However, despite these advantages, the local approach requires the integration of local data into more general structure information. Broadly speaking, there are two ways to achieve this goal. The first way is to integrate global information about the algorithms using data structures such as a graph, where each node represents a local feature while the edge between the two nodes represents the spatial relationship between them. Facial recognition is therefore a problem of matching two graphs. The second way, however, is to use point fusion techniques: separate classifiers are used in each local characteristic to calculate similarity. Then, the obtained similarities are combined to ensure the global result of the final decision, *e.g.* Ensemble Classifiers including Random Forest or XGBoost tree classifiers and the like.

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[72] Brunelli R, Poggio T, Face recognition: Features versus templates. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 1993, Vol. 15, No. 10.

[73] Viola P, Jones M.J, Robust real-time face detection. *International Journal of Computer Vision*, 2004, Vol. 57, No. 2.

[74] Lowe D.G, Distinctive image features from scale-invariant keypoints. *International Journal of Computer Vision*, 2004, Vol. 60.

[75] Ahonen T, Hadid A, Pietikainen M, Face recognition with local binary patterns. In Computer Vision – ECCV 2004. Berlin-Heidelberg, 2004.

[76] *Ojansivu V and Heikkilä J*, Blur insensitive texture classification using local phase quantization, [in:] Elmoataz A *et al.* (Eds), Image and Signal Processing. 3rd International Conference, ICISP 2008, Cherbourg-Octeville, France, July 1–3, 2008. Proceedings, Berlin-Heidelberg 2008.

[77] Chen J *et al.*, WLD: A robust local image descriptor. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2010, Vol. 32, Iss. 9.

[78] Kannala J and Rahtu E, BSIF: Binarized statistical image features, [in:] Proceedings of the 21st International Conference on Pattern Recognition ICPR, 11–15 November 2012, Tsukuba Science City, Japan 2012.

Hybrid methods

This category includes hybrid approaches that use both global and local characteristics to make the best use of these methods. It also includes techniques based on statistical models. The latter technique formalises the relationships between variables in the form of mathematical equations that describe how one or more random variables are related to one or more random variables. This model is considered statistical when the variables are not deterministic but stochastically related. Hybrid methods include:

— Hidden Markov Models (hereinafter: HMM) – they were first used in 1975 in various fields, especially voice recognition, and have been fully used in speech recognition since 1980. They were then used for manuscript recognition, image processing, music and bioinformatics (DNA sequencing, *etc.*), as well as in cardiology (ECG signal segmentation). Hidden Markov models, also called Markov sources or probabilistic Markov functions, are powerful statistical tools for modeling stochastic signals. These models have proven successful since their invention by Leonard E. Baum and his colleagues. They can be defined by the Markov chain statistical model. This model is composed of "states" and "transitions". For face images, significant areas of the face (hair, forehead, eyebrows, eyes, nose, mouth, and chin) are placed in a natural order from top to bottom, even when the image is captured at low rotation. Each of these regions has a left-to-right state[79].

— Gabor's Wavelet Transformation based on Poisson Hidden Markov Model (hereinafter: GWT-PHMM) – is an approach that combines the multi-resolution ability of Gabor's wavelet transformation (hereinafter: GWT[80]) with the local interactions occurring in the facial structure expressed by the Poisson Hidden Markov Model (hereinafter: PHMM). Unlike the traditional zigzag scanning method for feature extraction, the continuous analysis method should run from top left to right, then top to bottom and right to left, and so on, to the bottom right of the image. Moreover, unlike traditional HMM, PHMM does not require conditional state independence for the apparent sequence of observations. This result was achieved thanks to the concept of local structures introduced by PHMM used to extract facial bands and automatically select the most informative features of the facial image. Again, using the most informative pixels instead of the entire image makes the proposed facial recognition method relatively fast.

— Recognition systems using PCA and Discrete Cosine Transform (hereinafter: DCT) in the HMM approach – when we do not use DCT, PCA is used to reduce the dimension, we arrange the facial details into blocks first, and then we apply it to these blocks DCT. The next step is to use

[79] Lal M *et al.*, Study of Face Recognition Techniques: A Survey. *International Journal of Advanced Computer Science and Applications*, 2018, Vol. 9, Iss. 6.

[80] *Ibid.*

PCA, again not using the reverse DCT transformation, which speeds up the operation of such a solution[81].

- HMM-LBP – is a hybrid approach that allows the classification of a 2D face image using the LBP tool. It consists of four stages. The first breaks down the face image into blocks. The next one extracts image features using LBP. The third one calculates the probability of a face in a given block. The last one chooses the block with the maximum probability[82].
- A hybrid approach based on Singular Value Decomposition (hereinafter: SVD) for the wavelet distribution – is an efficient facial recognition system using the eigenvalues of the wavelet transform as feature vectors and a neural network with a Radial Basis Function (RBF)[83] as a classifier. Using the 2D wavelet transform, the face image is decomposed into two levels, then the average of the wavelet coefficients is calculated in order to find the characteristic centres.
- Discriminative Gaussian Process Latent Variable Model (hereinafter: DGPLVM) – is an extension[84] Gaussian Process Latent Variable Model (GPLVM) in which the Gaussian transform process is mapped from a low-dimensional latent space to a multidimensional dataset, where the location of points in the latent space is determined by maximising the probability of the Gaussian process with respect to a matrix where the rows represent the corresponding positions of the latent X space. This model places the discriminant over the positions hidden in the matrix, using the position assignment method so that the positions of different classes are as far apart as possible, and the same ones as close as possible.
- Discriminant Analysis on Riemannian Manifold of Gaussian Distributions (hereinafter: DARG). The concept of this method is to capture the underlying distributions of the assumed data in each set of images to facilitate classification and increase its reliability. For this purpose, the image set is represented as Gaussian Mixture M-models[85] (hereinafter: GMM) containing a predetermined number of Gaussian terms with a cer-

[81] Jameel S, Face recognition system using PCA and DCT in HMM. *International Journal of Advanced Research in Computer and Communication Engineering*, 2015, Vol. 4, Iss. 1.

[82] Chihaoui M *et al.*, Face recognition using HMM-LBP, [in:] Abraham A *et al.* (Eds), Hybrid Intelligent Systems. Cham, 2015.

[83] Hashemi V.H, Gharahbagh A.A, A novel hybrid method for face recognition based on 2d wavelet and singular value decomposition. *American Journal of Networks and Communications*, 2015, Vol. 4, Iss. 4.

[84] Urtasun R, Darrell T, Discriminative Gaussian process latent variable model for classification, [in:] Ghahramani Z (Ed.), Proceedings of the 24th international conference on Machine learning, Corvallis, OR, USA, 2007. New York, 2007.

[85] Wang W *et al.*, Discriminant analysis on riemannian manifold of Gaussian distributions for face recognition with image sets. *Conference Proceedings*, 2015, Vol. 1.

tain probability. Given geometrical information, Gaussian components lie on certain Riemann manifolds. To correctly code such a Riemann collector, DARG uses several distances between Gauss components and derives a series of documented positive probabilistic cores. In the latter, a weighted differential analysis of the cores is developed to treat the GMMs as samples and their designated probabilities as their weights.

— Local affine descriptors and probabilistic similarity. This algorithm combines the affine transformation and scale-invariant feature descriptors (SIFT) according to the probabilistic similarity of their occurrence. Affine SIFT is an extension of the SIFT method. It detects local invariant descriptors, generating a number of different views using the affine transformation. In this context, it enables the differences between the face image and the pattern image to be visualised. However, the human face is not flat because it contains a 3D depth that is important for facial recognition, which the algorithm does not take into account. This approach is also ineffective for large changes in the position / orientation of a given face. In addition, it is related to the probabilistic similarity that is obtained between the face image and the pattern based on the sum of the distribution of square differences in the online learning process[86].

— Hybrid algorithm using PCA wavelets and Gabor transform – is an approach that uses a face recognition algorithm divided into two recognition steps. It is based on global and local features. In the first stage of recognising, the so-called rough algorithm uses Principal Component Analysis (PCA) to initially identify the test pattern. The face recognition phase ends at this stage if the obtained confidence level result is obtained. Otherwise, the algorithm uses the obtained result to filter images – "best candidates" with a high degree of similarity – and sends them to the next recognition stage, where Gabor filters are applied[87].

— Manual segmentation using a Gabor filter and a neural network – is another facial feature extraction technique that ensures high recognition precision. In this approach, the topographic features of the face are brought out during manual segmentation of facial areas, *i.e.* the positioning of the eyes, nose and mouth. Then the Gabor transform is performed and the maximum of these regions is defined so as to compute their local representation. In the learning phase, this approach uses the nearest neighbour method. This method is used to calculate the distance between the three feature vectors of these regions and the corresponding comparison vectors[88].

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[86] Gao Y, Lee H.J, Viewpoint unconstrained face recognition based on affine local descriptors and probabilistic similarity. *Journal of Information Processing Systems*, 2015, Vol. 11, Iss. 4.

[87] Cho H *et al.,* An efficient hybrid face recognition algorithm using PCA and GABOR wavelets. *International Journal of Advanced Robotic Systems*, 2014, Vol. 11, Iss. 1.

[88] Qasim A, Prashan P, Peter V, A hybrid feature extraction technique for face recognition. *International Proceedings of Computer Science and Information Technology*, 2014, Vol. 1.

— HMM-SVM-SVD – is a combination of two classifiers: SVM and HMM. The former is used with PCA features, while the latter is a one-dimensional seven-state model, where the features are based on SVD. Thanks to this approach, we use the rules of the above-mentioned combination to combine the results[89].
— Merging local and global features based on the Gabor and PCA transformations – is a combination of two types of features. Local features extracted by Gabor transformation and global features extracted by "contour transformation". The recognition step is finally performed by a PCA-based classifier.
— SIFT-2D-PCA – is an algorithm that combines the SIFT method with 2D-PCA. Since SIFT is used to extract characteristics that are invariant with scaling, orientation and lighting, it is a favourable combination for facial recognition even if global facial features will not be available; 2D-PCA is used to bring out global features as well as to reduce the image size[90].
— Principal Component Analysis – Local Binary Patterns PCA-LBP – is an algorithm that uses the face recognition method used when the examined individual is subject to changes such as changes in lighting, head position or facial expressions. It performs global and local extraction of features using PCA and LBP, respectively. So these global and local characteristics are introduced into a network called MLP that carries out the classification[91].
— Local Directional Pattern (hereinafter: LDP) – uses the local direction model. In this approach, the LDP feature for each pixel position is obtained by computing an image response value in eight different directions. Then the LDP image is used as input for feature extraction and representation by 2D-PCA. However, the Nearest Neighbor Classifier is used for facial recognition. While this method has good recognition accuracy under various lighting conditions, it only works with flat images acquired in a front-facing facial orientation[92].
— Wavelet and directional transformation LBP – it begins with image pretreatment with a wavelet transform to obtain a number of different subimage resolutions, and wavelet decomposition to obtain different scale terms. Then, the LBP (directionally weighted) histogram is computed for the different weighted facial image sub-regions. The chi square test is used to match the sequence of the histogram. This method reduces

[89] Nebti S, Fadila B, Combining classifiers for enhanced face recognition. *Advances in Information Science and Computer Engineering*, 2015, Vol. 82.

[90] Singha M, Deb D, Roy S, Hybrid feature extraction method for partial face recognition. *International Journal of Emerging Technology and Advanced Engineering,* 2014, Vol. 4.

[91] Sompura M, Gupta V, An efficient face recognition with ANN using hybrid feature extraction methods. *International Journal of Computer Applications*, 2015, Vol. 117, No. 17.

[92] Kim D.J, Lee S.H, Shon M.Q, Face recognition via local directional pattern. *International Journal of Security and its Applications*, 2013, Vol. 7, Iss. 2.

computational complexity and improves recognition rate, but is sensitive to face orientation changes[93].

Face recognition algorithms in a 3D image

Over the years of modification and development of numerical methods, 2D face recognition has reached a high "level of maturity" and a high rate of precision. After years of research, the state of the art of facial recognition continues to improve and provides ever more accurate results. It is one of the most active areas of research in the field of computer image processing. However, in the last few years, as a result of technological development, new, very promising research directions have emerged. Despite the high rate of effectiveness achieved in face recognition in flat images, problems with obtaining insensitivity to changes in lighting, face orientation in relation to the camera axis, and acquisition parameters remain noticeable. The panacea turned out to be facial recognition in a 3D representation that contains information about the shape of the face surface. Several of the latest techniques using 3D data have been described in the literature[94].

Analysing the issue, recent studies indicate that the combination of multimodal 2D and 3D face recognition is much more accurate and reliable than unimodal recognition. Research in this area can be found in the literature. It is devoted to highlighting the advantages of combining 2D and 3D face representations. In subsequent work, deep learning techniques have been added to the 3D techniques. Deep Neural Networks (DNN) seem to be the best techniques for carrying out broadly understood tasks in the field of image classification, speech recognition

[93] Wu F, Face recognition based on wavelet transform and regional directional weighted local binary pattern. *Journal of Multimedia*, 2014, Vol. 9, No. 8.

[94] Huang D *et al.*, 3D face analysis: Advances and perspectives, [in:] Sun Z *et al.* (Eds), Biometric Recognition: 9th Chinese Conference on Biometric Recognition, CCBR 2014, Shenyang, China, November 7–9, 2014, Proceedings. New York, 2014; Drira H *et al.*, 3D face recognition under expressions, occlusions, and pose variations. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2013, Vol. 35; Huang D *et al.*, 3-D face recognition using eLBP-based facial description and local feature hybrid matching. *IEEE Transactions on Information Forensics and Security*, Vol. 7; Said S *et al.*, Fast and efficient 3D face recognition using wavelet networks, [in:] 16th IEEE International Conference on Image Processing (ICIP), 7–10 November 2009, Cairo. Egypt, 2009; Borgi M.A, M. El'Arbi Amar C.B, Wavelet network and geometric features fusion using belief functions for 3D face recognition, [in:] Wilson R *et al.* (Eds), Computer Analysis of Images and Patterns. Proceedings of the 15th International Conference, CAIP 2013, York, UK, 27–29 August 2013. Berlin-Heidelberg, 2013; Soltana W.B *et al.*, Multi library wavelet neural networks for 3D face recognition using 3D facial shape representation. *Electronic source*: https://zenodo.org/record/41296#.YA6mM3ZKh9M, *accessed*: 25 January 2021.

and face recognition. In particular, Convolutional Neural Networks (CNN) have shown promising results in facial recognition. A new approach to facial recognition, namely infrared imaging, has also been developed. It aims to overcome the limitations of facial recognition, such as changes in lighting, as well as disguising faces, which can significantly reduce recognition accuracy. Infrared images are a modality that has attracted special attention due to the invariability of the classification result against changes in lighting. Indeed, the data obtained with infrared cameras have many advantages over ordinary cameras that operate in the visible spectrum. For example, infrared face images can be obtained in any lighting condition, even in completely dark environments. This infrared technique allows you to achieve a higher degree of resistance to changes in facial expression[95,96,97,98,99,100].

There are works that combine several of the techniques listed above. For example, the authors of the work Multimodal biometrics based on near-infrared face combined multimodal face recognition, and Jiquan Ngiam, used both multimodal facial recognition and deep learning[101,102].

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[95] Bowyer K.W, Chang K, Flynn P, A survey of approaches and challenges in 3D and multimodal 3D+2D face recognition. *Computer Vision and Image Understanding*, 2006, Vol. 101, Iss. 3.

[96] Lakshmiprabha N.S, Bhattacharya J, Majumder S, Face recognition using multimodal biometric features, [in:] Proceedings of the IEEE International Conference on Image Information Processing (ICIIP), 3–5 November 2011 Shimla. India, 2011; Radhey S, Narain S.Y, Identifying individuals using multimodal face recognition techniques. *Procedia Computer Science*, 2015, Vol. 48.

[97] Balaban S, Deep learning and face recognition: The state of the art. *Proceeding SPIE* 2015, Vol. 9457.

[98] Li S.Z *et al.*, Illumination invariant face recognition using near-infrared images. *IEEE Transactions on Pattern Analysis and Machine Intelligence, 2007* Vol. 29; Huang D, Wang Y, Wang Y, A robust method for near infrared face recognition based on extended local binary pattern. *Electronic source*: http://rendicahya.lecture.ub.ac.id/files/2018/03/A-Robust-Method-for-Near-Infrared-Face-Recognition-Based-on-Extended-Local-Binary-Pattern.pdf, *accessed*: 22 January 2021.

[99] Friedrich G, Yeshurun Y, Seeing people in the dark: Face recognition in infrared images, [in:] Bülthoff H *et al.* (Eds), Biologically Motivated Computer Vision. Berlin, 2003.

[100] Jeni L.A, Hashimoto H, Kubota T, Robust facial expression recognition using near infrared cameras. *Journal of Advanced Computational Intelligence and Intelligent Informatics,* 2012, No. 16.

[101] Wang R *et al.*, Multimodal biometrics based on near-infrared face, [in:] Boulgouris N.V, Plataniotis K.N, Micheli-Tzanakou E (Eds), Biometrics: Theory, Methods, and Applications. New Jersey, 2009.

[102] Ngiam J *et al.*, Multimodal deep learning, [in:] Proceedings of the 28th International Conference on Machine Learning, ICML 2011, 28 June–2 July 2011, Bellevue, WA, USA, 2011.

Overview of 3D models of the face

From the point of view of appearance, an important feature of a 3D surface is its shape. A common way to represent it is through a 3D surface mesh of lines and their 3D coordinates. Given a set of vertices, we can associate with each vertex not only its geometric information (3D coordinates) but also its RGB values. When working with faces, it is necessary to register them in a common vertex mesh where each vertex has a specific identity. This can be achieved through the registration process, which maps the canonical grid template to a 3D face image[103,104].

The human face is a deformable object. Its shape changes dynamically with gender, age and ethnicity. Basically, the basics of a shape model can capture these shape changes. However, constructing such a model would require a huge training set of 3D face images containing all of the interesting shapes. Suppose we have a large number of training samples for a specific group of people identified by their age and gender. A more efficient solution is to transfer the internal variations of one particular face group to another. For this, the most sensible model seems to be the Morphable 3D face model with Gaussian mixture (GM-3DMM)[105].

Local methods

One local method is to use Gaussian curvature to find 5 landmarks in a 3D model. Its originators assumed that the correct landmark may have a maximum error of ± 4 mm. On the other hand, the use of Gaussian curvature and pure curvature in combination with depth maps to isolate the eye and nose areas was proposed by Gaile G. Gordon[106,107]. By matching the eye and nose areas to each other, a recognition precision of 97% for a dataset of 24 images has been achieved. The use of both the median and Gaussian curvature to select 35 facial features describing the area of the nose and eyes is described in Face Recognition using 3D Surface-Extracted Descriptors. The best recognition rate with this method has been achieved on neutral (non-expressive) faces with a recognition rate of 78%[108].

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[103] One of the models of the colour space, described by RGB coordinates. Its name is based on the first letters of the English colour names: Red, Green, and Blue.

[104] Rodriguez J.T, 3D Face Modelling for 2D+3D Face Recognition, Surrey University, Guildford, UK, 2007, PhD thesis.

[105] Headings P *et al.*, Gaussian mixture 3D morphable face model. *Pattern Recognition*, 2018, Vol. 74.

[106] Suikerbuik C.A.M *et al.*, Automatic feature detection in 3D human body scans. *Electronic source*: https://www.researchgate.net/publication/46659540_Automatic_Feature_Detection_in_3D_Human_Body_Scans, *accessed*: January 25, 2021.

[107] Gordon G.G, Face Recognition Based on Depth Maps and Surface Curvature. *Geometric Methods in Computer Vision*, 1991, Vol. 1570.

[108] Moreno A.B *et al.*, Face Recognition using 3D Surface-Extracted Descriptors. *Irish Machine Vision and Image Processing Conference Proceedings*, 2003, Vol. 2.

The use of Gauss-Hermit moments as local descriptors in conjunction with the global grid has been proposed by the authors of the article: Automatic 3D Face recognition combining global geometric features with local shape variation information. In their attempts, the authors of the method achieved a recognition rate of 96.1% for 30 tested objects. With 120 properties, the index dropped to 72.4%. This difference is due to the type of database used. For 30 objects, it was a manual database (Manual DataBase; MDB), for 120 objects, it was an automatically created database (Automatic DataBase; ADM). Data in a MDB has better quality than that in an ADB[109].

Another local method is to use specific features of points – their signatures – in order to describe points in detail in 3D. Here, the forehead, nose and eyes are described using point signatures. This method achieved a recognition rate of 100% for six objects. Tests carried out on 50 objects were compared with the results of Gabor's wavelet approach. The level of recognition of the proposed method was determined at 85%, and Gabor's wavelet approach at 87%. After combining 2D and 3D landmarks, both methods achieved a recognition rate of 89%[110].

The use of Gaussian curvature to define square features of the patches was proposed in the publication Three-Dimensional Surface Curvature Estimation using Quadric Surface Patches. Its authors claim that their method can be used to recognise all kinds of 3D models, not just faces. Another local shape descriptor that proved to work well on human bodies was the Paquet shape descriptor[111,112].

Global methods

One global curvature method is presented in the article: 3D head model classification by evolutionary optimisation of the extended Gaussian image representation[113]. The surface of the facial model is presented in the

[109] Coins C *et al.*, Automatic 3D Face recognition combining global geometric features with local shape variation information, [in:] Proceedings of the IEEE International Conference on Automatic Face and Gesture Recognition, 19 May 2004, Seoul. South Korea, 2004.

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publication using the extended Gaussian image (EGI), which enables the reduction of facial recognition problems in the 3D model to a problem similar to comparing histograms for a 2D face model. The Multiple Conditional Probability Mass Function Classifier (MCPMFC) was used as a classifier. This classifier, tested on a dataset of five objects, achieved a recognition rate of 80.08%. The Minimum Distance Classifier (MDC) achieved a recognition rate of 67.40%. However, further tests showed that for both methods, the diagnosis rate drops by 10% when the data is increased set to 21 people.

A proposal to use a combination of a 3D model and facial texture has been suggested by Theodoros Papatheodorou and Daniel Rueckert, including measures of similarity for two 3D models without texture and in combination with a texture. The results have shown an increase in the number of correctly classified front images when texture was included[114].

Charles Beumier and Marc Acheroy have proposed the use of 3D model profiles for facial recognition. The first face recognition attempt was based on three profiles of one face and exhibited an error rate of 9%. In the second attempt, including gray value information in the matching process reduced the error rate to 2.5%. Yijun Wu, Gang Pan and Zhaohui Wu have proposed a 3D face recognition technique by extracting multiple horizontal profiles (layers) of the 3D model, and matching the profiles to each other. They achieved an error rate of 1% to 5.5% for data with 30 objects[115,116,117].

Summary

The article presents the latest methods of identifying people, developed in various research centres. The analysis has been carried out in terms of a reliable presentation of the above-mentioned methods in order to be able to refer to them later and compare them with the algorithm developed as part of our own work and research. The literature analysis shows that there is no scientific study that would indicate a change in the biometric parameters of the human face due to uncoordinated head movements in the three Euler axes. The next stage of the work will be the analysis of

[114] Papatheodorou T, Rueckert D, Evaluation of automatic 4D Face recognition using surface and texture registration. Proceedings of the IEEE International Conference in Automatic Face and Gesture Recognition, 19 May 2004, Seoul. South Korea, 2004.

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such changes and an attempt to determine the parameters enabling the transformation of the changed features to the base features. The next stage is a thorough comparison of the speed and accuracy of the functioning of the algorithms available on the market with the algorithm developed as part of our own work.

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Keywords: face recognition, biometrics, image processing, pattern recognition, neural network

Summary: Information technology of the 20th and 21st centuries "opened the way" to the automatic assessment of anthropometric facial features, facial gestures and other characteristic behaviours. Recognition is a very complex technical problem with a significant practical effect. There are dedicated applications for this purpose. The article presents face recognition algorithms for 2D images, for three-dimensional spaces, and methods using neural networks. Linear and nonlinear, local and global, and hybrid methods of facial recognition are presented. The study understands the strengths and weaknesses of the laws governing the use of face recognition technology and, if possible, analyses their efficiency. The methodological review has been created in connection with the idea of the author's own fast algorithms and facial recognition.