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Simulation of thermomechanical processes in disc brakes of wheeled vehicles

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ABSTRACT

Purpose: Ensuring the required operational reliability of disc brakes by forecasting their technical condition taking into account thermomechanical processes.

Design/methodology/approach: Differential equations of rotation of a rigid body around a fixed axis are solved, it is established that the equations of motion and the equations of thermal conductivity are indirectly related. The use of these analytical dependences provides a better understanding of thermomechanical transients.

Findings: The solution is obtained on the basis of the differential equation of thermal conductivity of the hyperbolic type, which does not allow an infinite velocity of propagation of temperature perturbations in contrast to the differential equation of thermal conductivity of the parabolic Fourier type. The obtained analytical dependences provide a better understanding of thermomechanical transients and develop a theoretical basis for determining stresses and heat fluxes in solving problems of reliability and durability of disc brakes.

Research limitations/implications: The work uses generally accepted assumptions and limitations for thermomechanical calculations.

Practical implications: It is shown, that transients in a mechanical system - a brake disk at impulse loadings cause emergence of thermal effects which arise under the influence of external loadings.

Originality/value: The application of these analytical dependences provides a better understanding of thermomechanical transients and develops a theoretical basis for solving problems of reliability and durability of disc brakes.

Keywords: Disc brake, Wheeled vehicle, Angular velocity, Thermodynamically reversible process, Centrifugal force, Time of relaxation, Isotrope elastic medium, Torsional moment

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ANALYSIS AND MODELLING

1. Introduction

The mechanical system, the brake disc of a wheeled vehicle, in the mode of impulse loads are examined. It is shown that transients processes cause heating effects that occur under external loads. These solutions are achieved based on the hyperbolic type differential equation of thermal conductivity, which eliminates an infinite velocity of temperature perturbations propagation in contrast to the thermal conductivity differential equation of the parabolic Fourier type. Differential equations of a rigid body rotation around a fixed axis are solved. It is established that the motion equations and the thermal conductivity equations are tangentially related. The application of these analytical dependences provides a better understanding of thermomechanical transients and develops a theoretical basis for solving problems of reliability and life time of disc brakes.

In conditions of high competition in the vehicle market, reliability is one of the determining factors of competitiveness. An effective method of guaranteeing the required operational reliability of equipment is prediction of the technical condition [1,2,3] to prevent possible failures. Forecasting the possible moment of failure provides adequate technical service, taking into account operating conditions, climatic conditions in a particular region, season, type and version of vehicle. Analysis [1, 4-6] of the causes of vehicle failures indicates that for each model in certain operating conditions at a fixed mileage there is a number of parts that often, relative to others, fail, according to this, their reliability is critical. For wheeled vehicles, in particular truck tractors, most often these parts are brake discs, clutch discs, etc.

Studies of statistics on failures of car tractors, which were carried out taking into account the failure rate (per car), proved the importance of the influence of climatic conditions [1]. The analysis of data on failures of the examined equipment established a causal relationship. It is an intense spread of the specific failures number through countries. As a result, a grouping of countries by climate type according to the Köppen-Geiger climate classification was performed [7]. According to the distribution of the specific failures number and the function of the failures distribution, for example, the specific number of failures for Angola 0.75, for Sudan 0.6 at a correlation coefficient $R^2 = 0.7866$, for Algeria 1.7 at $R^2 = 1$ [1]. The correlation coefficient of distribution law for each of the climatic groups of countries is significantly bigger than in their general consideration. It points to the necessity of having regard to the climatic conditions factor while forecasting failures of critical parts of wheeled vehicles.

According to studies of vehicles disc brakes [8], a thermostructural model was built based on 3D modelling constructions. Therefore, the transition temperature field of a car brake in certain operating conditions (hard braking, etc.) was analysed. The temperature curves of the brake disc in the radial and circumferential directions were constructed, stresses and strains were calculated with the help of 3D modelling. In this case, the stress-strain state of the disk hub has not been sufficiently studied [9-10], taking into account the stress level of the material (Figs. 1, 2).

Since thermal stresses are additional and significantly affect the overall level of stress-strain state (mechanical stresses and strains) [9-12]. It can lead to the formation and propagation of cracks and as a consequence spontaneous destruction of metal structures [2-6, 14-20]. Therefore, it is efficient to polish up the theoretical foundations in order to improve the understanding of thermomechanical processes in this metal structure. Overheating and thermomechanical or mechanical fatigue are the causes of degradation of metal structures materials (brake discs, tubes, vehicle transmissions, undercarriage) [8,9, 15-20]. During the study of mechanical transients, changes in the time of the angular velocity are determined after the perturbation occurred. It should be stated that the mechanical systems

a) b)

c)

Fig. 1. VOLVO FH 500 truck tractor: a) general view; b) a fragment of the running gear; c) brake disc

are usually studied in steady-state processes. Such solutions about steady-state processes are only a partial case of solving the problem, because after any change in the system over time, the angular velocities do not change or stop instantly. The mechanical system is affected by various variable and impulse processes. Therefore, it is important to check the behaviuor of these systems under pulse loads, which are significantly different from the steady-state processes. In addition, transients make it possible to ensure the appearance of thermal effects caused by external loads. External loads lead to the occurrence of deformations and changes in temperature in the researched body. Any deformation, including elastic, is accompanied by thermal effects, and therefore the attempt to describe the behaviuor of a continuous medium only by a mechanical scheme, ignoring the thermomechanical interactions within the medium, is difficult [8, 9, 21-25].

2. Material and methods

At researches the shaft, or the cylinder of final length continuous or with the central opening or a disk of constant thickness are studied. The solution of these problems can be combined, as the exact solution of the equations of the theory of elasticity is given based on the method which was developed in [10]. It is known that in the various papers [26-30] there is no exact solution of the problem of the theory of elasticity even for a disk of constant thickness rotating at a constant speed. This problem is solved based on hypotheses of the plane-stress state [31].

Let's assume that the volumetric forces and heat flux that result from the action of external forces on the body change slowly over time. Then we can neglect the inertial terms in the equations of motion and consider motion as a sequence of equilibrium. This approach to solving problems

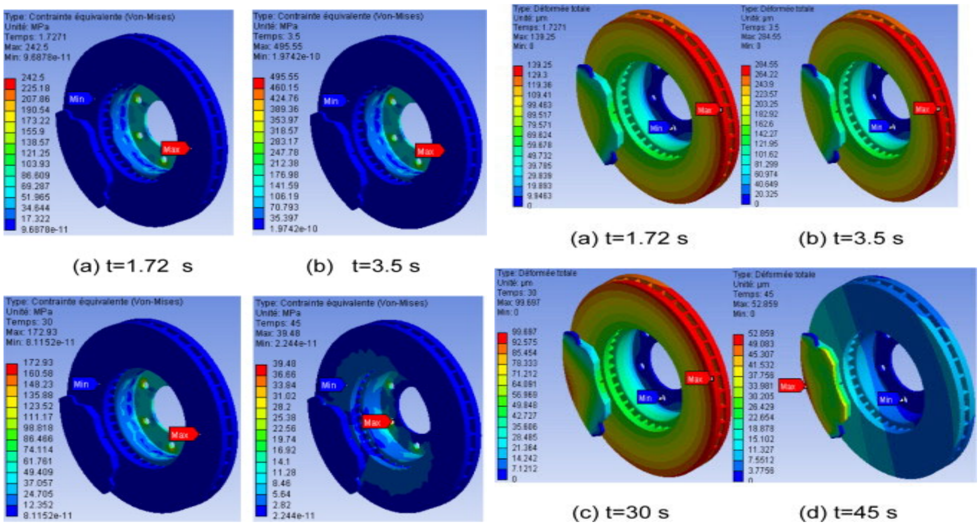

Fig. 2. Von Mises stress distribution and total distortion distribution [9]

of dynamic elasticity theory is called quasi-static. In the quasi-static consideration of unidentified stresses, the time it is a parameter [27,30,31] and therefore it is possible to use solutions for the corresponding stationary problems.

As mentioned above, the deformation of a continuous elastic medium is not always a purely mechanical phenomenon: the change in body temperature can occur as a result of the deformation process itself, and due to external causes [1,2,14,15,18,24,25,29]. Next, the theory of elasticity considers the change in temperature, which will change under external forces. In particular, for a cylinder or a rotating disk, the external volumetric forces are represented by a radial centrifugal force:

$$F_r = \rho \omega^2 \tau \tag{1}$$

where ρ is the density of the material, r is the radial coordinate, ω = ω (t) is the change in angular velocity over time.

Suppose that in the undeformed and unstressed state the cylinder has a temperature T_0=0. Due to external loads and centrifugal force of inertia (1), the cylinder will be deformed and its temperature will change. The change in temperature is $\theta = T - T_0$, where T is the absolute temperature of the body point. In the first approximation, the change in temperature $\theta\,(r, z, t)$ is insignificant and does not cause existing changes in the physical and mechanical characteristics of the body material [23-36].

Therefore, during deformation, the temperature of the point of the body changes and as a result, absorption or release of heat by an elastic uninsulated body during its interaction with the environment can occur [34]. If the deformation of the body is slight, then the volumetric forces (i.e. $F_r = \rho\omega^2\tau$) that cause the deformation are ceased, and the body (cylinder or disk) returns to the initial undeformed state. Such deformations are called elastic. The deformation process is very slow, i.e. it will be thermo-dynamically reversible [34].

In [37], a solution for a rotating circular hollow cylinder of finite length or a disk of constant thickness is given, without any special hypotheses, except for the general hypotheses of the linear theory of elasticity. It is shown that as centrifugal force occurs, a temperature field $\theta\,(z)$, that depends on the speed of rotation, appears. The solution was obtained with a steady process of the system, i.e. without changing the time (stationary problem). In formula (1), the angular velocity ω is constant and independent of time. The obtained formulas for stresses that coincide with the stresses found based on hypotheses of the plane-stress state have the form:

$$\begin{aligned}
\sigma_{11} &= \frac{3+\nu}{8}\rho\omega^2\left(\alpha_1^2 + \alpha_2^2 - \frac{\alpha_1^2\alpha_2^2}{r^2} - r^2\right)\\
\sigma_{22} &= \frac{3+\nu}{8}\rho\omega^2\left(\alpha_1^2 + \alpha_2^2 - \frac{\alpha_1^2\alpha_2^2}{r^2} - \frac{1+3\nu}{3+\nu}r^2\right)\\
\sigma_{13} &= \sigma_{33} = 0
\end{aligned} \tag{2}$$

Temperature change $\theta\,(z)$:

$$\theta(z) = -\frac{\nu}{4G\alpha} z^2 \rho\omega^2 \tag{3}$$

The heat flux power is also determined on the basis of the Fourier thermal conductivity equation, regardless of the time of temperature change $\theta(z)$:

$$W=\frac{\nu\lambda}{2G\alpha}\rho\omega^2 \tag{4}$$

Now, taking into consideration mentioned above, we will analyse the problem of transients in cylinders or rotating disks. The temperature at an arbitrary point of the body in cylindrical coordinates at time t is described by the Fourier thermal conductivity equation in partial derivatives [34]:

$$\Delta T-\frac{1}{\alpha}T_{,t}+\frac{1}{\lambda}\omega=0 \tag{5}$$

where $\alpha=\lambda/c\rho$ — thermal conductivity, λ – thermal conductivity coefficient, c — specific thermal conductivity, ρ – body density, ω — the amount of heat generated or absorbed per unit volume of the body per unit time. In the case if the change in temperature field $\theta(r,z,t)$ has the form $\theta=T-T_0$, the thermal equation (5) will look like this:

$$\Delta\theta-\frac{1}{\alpha}\theta_{,t}+\frac{1}{\lambda}W=0 \tag{6}$$

3. Research results and discussion

The main disadvantage of the classical theory of Fourier thermal conductivity is that heat distribution is described by a differential equation of parabolic type (6), which assumes an infinite rate of heat distribution [39]. This is incorrect according to physical essence of the considered processes in the hubs of the brake discs and the discs. Another direction of development of the theory of thermal conductivity is various generalizations of the classical theory. Thus, in [38], by introducing the characteristic of the thermal energy velocity, a more general equation of thermal conductivity of the hyperbolic type was obtained, which does not allow an infinite velocity of propagation of temperature perturbations. Since the heat flux $\bar{q}$ is not established instantaneously, and it is characterized by a finite relaxation time τ/2 (Maxwell relaxation time), the generalized law of thermal conductivity in an isotropic elastic medium can be written as [37]:

$$\bar{q}+\frac{\tau}{2}\bar{q}_{,t}=-\lambda grad\theta \tag{7}$$

If τ = 0, then the heat flux vector is reduced to the ordinary Fourier law, i.e.:

$$\bar{q}=-\lambda grad\theta \tag{8}$$

The continuity equation for heat transfer has the form:

$$\rho c\theta_{,t}=-div\bar{q}+W \tag{9}$$

Differentiating expression (9) by time t and making mathematical transformations, we obtain a generalized equation of thermal conductivity in cylindrical coordinates (r, z):

$$\Delta\theta-\frac{1}{\alpha}(\theta_{бе}+\frac{\tau}{2}\theta_{бее})+\frac{1}{\lambda}(W+\frac{\tau}{2}W_{,t})=0 \tag{10}$$

If τ = 0, equation (10) will have the form of a normal Fourier equation (6):

$$\Delta\theta-\frac{1}{\alpha}\theta_{бе}+\frac{1}{\lambda}W=0 \tag{11}$$

When the temperature change $\theta=T-T_0$ does not depend on time, then the thermal conductivity equation will be following:

$$\Delta\theta+\frac{1}{\lambda}W=0 \tag{12}$$

In the case of a heat-insulated body, i.e. when the body does not exchange heat with the environment and in the absence of heat in the body ($W=0$), equation (10) is following:

$$\Delta\theta=0 \tag{13}$$

Generally, in the right member of an equation (10) there can be a term that depends on the external load, such as the volumetric centrifugal force F_r, through which the deformations also can be expressed. According to this it is possible to define dependence of deformations on temperature change θ . Thus, the equations of motion and the equations of thermal conductivity are indirectly related.

When considering the problem of transients for a cylinder or a rotating disk, it is necessary to solve the differential equations of rotation of a rigid body around a fixed axis. When acting on the body of a cylinder or disk of any moment of rotation, the differential equation of rotation is following:

$$J\omega(t)_{,t}=M \tag{14}$$

where J – the moment of inertia of the cylinder or disk about the axis of rotation Z, and $\omega_{(t)}$ is the projection of the its angular velocity on this axis.

Let the cylinder or disk be affected by the moment of rotation, which varies according to the following law:

$$M(t)=M_0-\beta\omega(t) \tag{15}$$

where M_0 is the constant torque at the initial time t = 0, and the angular velocity $\omega_{(t)}$ = 0, when t =0; M_0 and β – some positive arbitrariness, which characterize the engine of a cylinder or rotating disk (for example, the rotor of the gyroscope).

Next, it is necessary to determine the laws of change of angular velocity $\omega_{(t)}$ during the acceleration of the cylinder or disk, i.e. when turning on the torque M_0 at t = 0 and when turning it off (t = t_0). The friction forces are taken into account by arbitraries M_0 and β.

The differential equation of rotation (14) will be following:

$$J\omega(t)_{,t}=M_0-\beta\omega(t) \text{ or}$$
$$\omega(t)_{,t}+\frac{1}{\tau}\omega(t)=\frac{1}{\tau}\omega_0(t) \tag{16}$$

where $\tau=J/\beta$ – time of relaxation.

The solution of equation (16) with respect to ω(t):

$$\omega(t)=\frac{M_0}{\beta}(1-e^{-t/\tau}) \tag{17}$$

This equation defines the angular velocity defect law if $t\to\infty$

$$\omega_{def}=M_0/\beta=\omega_{(0)};\ \omega(t)|_{t=0}=0$$

The angular velocity of the cylinder or disk increases monotonically, striving for its limit value, which is established according to the process. $\omega_{def}=\omega_0$. Thuswise:

$$\omega(t)=\omega_0(1-e^{-t/\tau}) \tag{18}$$

The process of accelerating a rotating cylinder or disk is called a transient process. The transient process for most engines is ultimate when the angular velocity ω reaches 0.95 its limit value. The duration of the transient process or the acceleration time is easy to find from expression (18):

$$t=-\tau\ln(1-\frac{\omega}{\omega_0})\ \ (\tau=J/\beta)$$

If $\omega/\omega_0=0{,}95$, then $t=t_{tran}$

Therefore: $t_{tran}=\tau\ln 20\approx 3\tau$

The moment of inertia of the rotor or ring J and the characteristic β are selected according to the conditions in which the transient time was within the specified limits:

$t_{tarn}\approx 2-3c$, i.e. $J/\beta\approx 1$.

We terminate the action of the constant torque M_0 if $t=t_0$, by turning off the rotation $\omega/t=t_0=\omega_0$ It means that M_0 = 0. In this case, according to (16):

$$J\omega(t)_{,t}=-\beta\omega(t) \text{ or}$$
$$\omega(t)_{,t}=-\frac{1}{\tau}\omega(t) \tag{19}$$

After solving (19) we get the final formula of ω(t) after the rotation stops:

$$\omega(t)=\omega_0 e^{-\frac{t-t_0}{\tau}} \tag{20}$$

If $t=t_0$ then according to formula (20) $\omega=\omega_0$, if $t\to\infty$, $\omega\to 0$.

Graphs in transients have are the following – Fig. 3.

Let's consider the temperature change (3) with allowance for equation (18), meaning while accelerating the cylinder or disk:

$$\theta=-\frac{\nu}{4G\alpha}\rho\omega^2 z=-\frac{\nu}{4G\alpha}z^2\rho\omega^2(1-e^{-t/\tau}) \tag{21}$$

Then, from the equation of rotation (16) it is obtained that:

$$\omega_{,t}+\frac{1}{\tau}\omega=\frac{1}{\tau}\omega_0 \tag{22}$$

Multiplying (22) by $-\frac{\nu}{4G\alpha}z^2\rho$ and, taking into account expression (21), we obtain:

$$\frac{\tau}{\lambda}\theta_{,t}+\theta=\omega\omega_0(-\frac{\nu}{4G\alpha}z^2\rho) \tag{23}$$

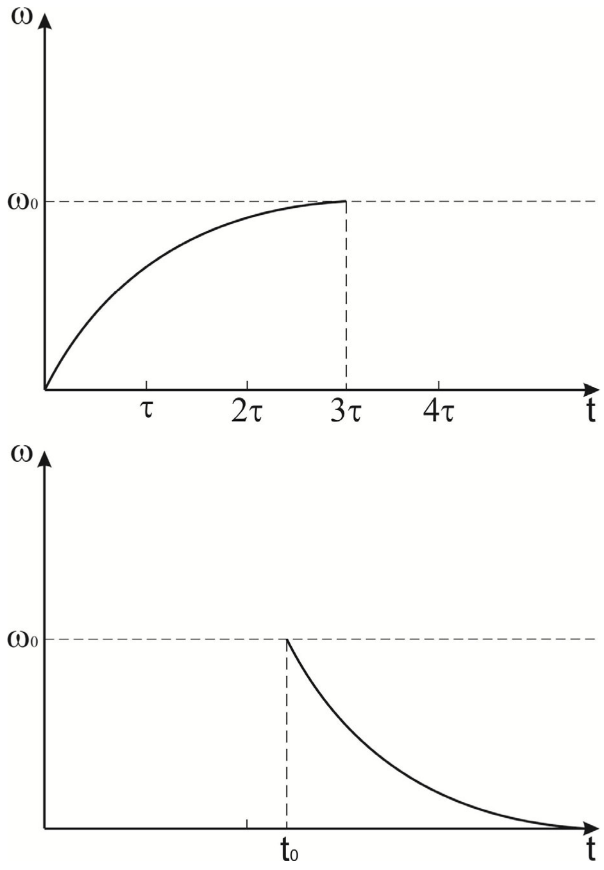

Fig. 3. Graphic of the dependence of ω on t in transients

Differentiating (23) with respect to *t*:

$$\frac{\theta}{\partial t}\left(\theta+\frac{\tau}{2}\frac{\partial\theta}{\partial t}\right)=\frac{\omega_0}{2\omega}\frac{\partial\theta}{\partial t} \tag{24}$$

Multiplying (24) by - 1 / a and adding the expression:

$$\Delta\theta+\frac{I}{\lambda}\left(W+\frac{\tau}{2}\frac{\partial W}{\partial t}\right)=0$$

The general equation of thermal conductivity of hyperbolic type when the cylinder or the rotating disk on exposure to centrifugal force (1) during acceleration is received:

$$\Delta\theta-\frac{I}{\alpha}\left(\frac{\partial\theta}{\partial t}+\frac{\tau}{2}\frac{\partial^2\theta}{\partial t^2}\right)+\frac{I}{\lambda}\left(W+\frac{\tau}{2}\frac{\partial W}{\partial t}\right)= =-\frac{I}{\alpha}\frac{\omega_0}{2\omega}\frac{\partial\theta}{\partial t} \tag{25}$$

Finding the terms of formula (25), and then solving the equation in W, the power of the heat flux during acceleration of the cylinder or rotating disk is obtained:

$$W=\frac{\lambda\nu}{2G\alpha}\rho\omega_0^2\left(1-4e^{-t/\tau}+\left(2^{t/\tau}+3\right)e^{-2t/\tau}\right) \tag{26}$$

The temperature change (3) is studied with allowance for equation (20) after the rotation stops:

$$\theta=\frac{\nu}{4G\alpha}\rho\omega^2z^2=\frac{\nu z^2}{4G\alpha}\rho\omega_0^{\frac{-2(t-t_0)}{\tau}} \tag{27}$$

From the equation of rotation (16) we obtain:

$$\tau\frac{d\omega}{dt}+\omega=\omega_0 \tag{28}$$

After formula manipulations similar to the stated above, we obtain the general equation of thermal conductivity of the hyperbolic type when a cylinder or a rotating disk on exposure to centrifugal force (1) stops its' rotational motion:

$$\Delta\theta-\frac{I}{\alpha}\left(\frac{\partial\theta}{\partial t}+\frac{\tau}{2}\frac{\partial^2\theta}{\partial t^2}\right)+\frac{I}{\lambda}\left(W+\frac{\tau}{2}\frac{\partial W}{\partial t}\right)=0 \tag{29}$$

Finding the terms of formula (29), and then solving the equation in W, we obtain the heat flux power when the cylinder or disk stops:

$$W=\frac{\lambda\nu}{2G\alpha}\rho\omega_0^2\left(\frac{2(t-t_0)}{\tau}+1\right)e^{\frac{-2(t-t_0)}{\tau}} \tag{30}$$

Substituting equation (18) in formulas (2), the stresses values during acceleration of the cylinder or disk are obtained:

$$\begin{aligned}\sigma_{11}&=\frac{3+\nu}{8}\rho\omega_0^2\left(1-e^{-t/\tau}\right)^2\left(\alpha_1^2+\alpha_2^2-\frac{\alpha_1^2\alpha_2^2}{r^2}-r^2\right)\\ \sigma_{22}&=\frac{3+\nu}{8}\rho\omega_0^2\left(1-e^{-t/\tau}\right)^2\left(\alpha_1^2+\alpha_2^2-\frac{\alpha_1^2\alpha_2^2}{r^2}-\frac{1+3\nu}{3+\nu}r^2\right)\end{aligned} \tag{31}$$

Taking into account equation (20), the following equation was obtained to determine the stresses (2) in the metallic material when the cylinder or disk stops:

$$\sigma_{11} = \frac{3+\nu}{8}\rho\omega_0^2\left(1-e^{\frac{t-t_0}{\tau}}\right)^2\left(\alpha_1^2+\alpha_2^2-\frac{\alpha_1^2\alpha_2^2}{r^2}-r^2\right)$$
$$\sigma_{22} = \frac{3+\nu}{8}\rho\omega_0^2\left(1-e^{\frac{t-t_0}{\tau}}\right)^2\left(\alpha_1^2+\alpha_2^2-\frac{\alpha_1^2\alpha_2^2}{r^2}-\frac{1+3\nu}{3+\nu}r^2\right) \quad (32)$$

These analytical dependencies provide a better understanding of thermomechanical transients and develop a theoretical basis for determining stresses and heat fluxes in solving problems of reliability and durability of disc brakes. In this research differences between base metal and corrosion products heat capacity and thermal fatigue defects (multiple cracks) were taken into account when modelling stress-strain state.

4. Conclusions

It is established that the transients in the mechanical system, meaning the brake disc, under impulse loads cause thermal effects that occur because of the action of external loads. The solution has practical importance, in contrast to the differential equation of thermal conductivity of the parabolic Fourier type, which is obtained based on the hyperbolic type differential equation of thermal conductivity. Also it does not allow an infinite velocity of propagation of temperature perturbations. Differential equations of rotation of a rigid body around a fixed axis are solved, and it is shown that the equations of motion and the equations of thermal conductivity are tangentially related. The use of indicative equations provides a better understanding of thermomechanical transients and develops a theoretical basis for solving problems of reliability and durability of disc brakes with possible further experimental studies to verify the model for specific cases.

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