

High-resolution Direction of Arrival Estimation Method Based on Sparse Arrays with Minimum Number of Elements

Jafar Ramadhan Mohammed

College of Electronics Engineering, Ninevah University, Mosul, Iraq

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Abstract—Regular fully filled antenna arrays have been widely used in direction of arrival (DOA) estimation. However, practical implementation of these arrays is rather complex and their resolutions are limited to the beamwidth of the array pattern. Therefore, higher resolution and simpler methods are desirable. In this paper, the compressed sensing method is first applied to an initial fully filled array to randomly select the most prominent and effective elements which are used to form the sparse array. To keep the dimension of the sparse array equal to that of the fully filled array, the first and the last elements were excluded from the sparseness process. In addition, some constraints on the sparse spectrum are applied to increase estimation accuracy. The optimization problem is then solved iteratively using the iterative reweighted l_1 norm. Finally, a simple searching algorithm is used to detect peaks in the spectrum solution that correspond to the directions of the arriving signals. Compared with the existing scanned beam methods, such as the minimum variance distortionless response (MVDR) technique, and with subspace approaches, such as multiple signal classification (MUSIC) and ESPRIT algorithms, the proposed sparse array method offers better performance even with a lower number of array elements and in severely noisy environments. Effectiveness of the proposed sparse array method is verified via computer simulations.

Keywords—compressed sensing, direction of arrival (DOA) estimation, sparse array.

1. Introduction

The performance of many modern communication systems depends directly on the precision of estimating the direction of arrival of the signals that impinge on the antenna arrays used [1]–[2]. High directional beamforming that is a feature of antenna arrays is important not only for good performance but also for achieving high-resolution direction of arrival (DOA) estimates. It is known that the angular resolution (i.e. the angular distance between the two closely spaced sources) of an aerial array is limited by its beamwidth which, in turn, is reversely proportional to the

array dimension or aperture size. This means higher resolutions may be obtained by increasing array dimensions (i.e. using a larger number of array elements) – an approach relied upon by current massive MIMO systems. However, high cost of implementation and fault diagnosis associated with such large arrays continues to remain the key practical constraint. To obtain high resolution DOA estimators, many methods have been proposed in the literature [2]–[6]. These methods may be divided into three basic categories, according to their mathematical formulations. The first category is based on the array beam scanning (or beamforming) concept, such as delay-and-sum (DS) beamformer [7] and the minimum variance distortionless response (MVDR) beamformer [8], where array elements may either be distributed uniformly along linear or planar forms, or may be non-uniformly spaced arrays. The second category is based on the subspace approach, such as MUSIC [9], ESPRIT [10] and their variants, where the observation space is decomposed into signal and noise subspaces. The third category is based on stochastic optimization algorithms, such as genetic algorithm (GA) [11], particle swarm optimization (PSO) [12] or maximum likelihood methods [13].

The second and the third types usually perform well, but their computational complexity is generally high, especially when dealing with a large number of array elements. Less attention has been attached to the first category, due to the main beam limitation. However, among these three types of DOA estimations, the array beamforming method enjoys many implementation-related advantages, such as simplicity, versatility, effectiveness and low costs when controlling only a part of array elements, instead of all of them, i.e. when optimizing only the most effective and prominent array elements, instead of optimizing all of them [14]–[17]. Thus, the array beamforming methods may be relied upon to achieve good and competitive solutions. However, their angular resolutions are limited by the arrays' physical apertures, meaning they are unable to distinguish between two spatial sources within beam widths of the array's radiation

patterns. Therefore, overcoming this limitation is currently an important research direction. In papers [18]–[20], the authors presented different methods based on compressed sparse arrays for DOA estimation.

In this paper, an antenna array-based beamforming method that utilizes a compressive sensing approach for DOA estimation is presented. The proposed method is applicable to both linear and planar array configurations. An initial regular antenna array with full density is first considered, and then only the most effective and prominent elements are chosen randomly to reconstruct the sparse array. To keep the array dimension fixed, the first and the last elements of the initial regular array were excluded from the sparseness process. Next, the problem is optimized iteratively to find the optimum sparse elements, which are used to reconstruct the required signals and to estimate their directions. The effect of the SNR and of the minimized number of the sparse elements on estimation performance of the proposed method is also presented and is compared with other existing methods. Furthermore, the resolution and the maximum allowable number of estimated directions are analyzed as well.

2. Sparse Array Method

Consider a fully filled linear array consisting of N elements that are distributed uniformly with a separation distance of d , receiving P signals from a far field region. For simplicity, mutual couplings between the array elements are ignored. The output signal $x(k) \in C^{N \times 1}$ is:

$$\mathbf{x}(k) = \mathbf{A}(\theta)\mathbf{s}(k) + \mathbf{n}(k), \quad (1)$$

where:

k is the discrete time which is equal to $k = 1, 2, \dots, L$, and L is the total number of snapshots,

$\mathbf{s}(k) \in C^{P \times 1}$ is the complex amplitude of signal $\mathbf{s}(k) = [s_1(k) \ s_2(k) \ \dots \ s_P(k)]^T$ which is a vector representing signals with size $P \times 1$,

P is the total number of signal sources that impinge on the array,

$\mathbf{n}(k) \in C^{N \times 1}$ is the complex vector of noise,

$\mathbf{A}(\theta) = [\mathbf{a}_1(\theta_1) \ \mathbf{a}_2(\theta_2) \ \dots \ \mathbf{a}_P(\theta_P)]$ is an $N \times P$ matrix of steering vectors with $\mathbf{a}(\theta) = \frac{1}{\sqrt{N}} [1 \ e^{-j\frac{2\pi d}{\lambda} \sin(\theta)} \ \dots \ e^{-j(N-1)\frac{2\pi d}{\lambda} \sin(\theta)}]^T$.

Here, the values of θ are between $-\pi/2$ and $\pi/2$. In general, the directions of the received signals, i.e. $\mathbf{s}(k)$, are unknown and need to be determined. In the array beamforming methods, the scanned beams are used to estimate the signals' DOA. This may be done simply by dividing the total scanning region into a certain number of grids or angles, e.g. G . By using steering vector $\mathbf{a}(\theta)$ for N values of θ , the discrete grid (or scan angle) matrix Ψ can be given by $\Psi(\theta) = [\mathbf{a}_1(\theta_1) \ \mathbf{a}_2(\theta_2) \ \dots \ \mathbf{a}_N(\theta_N)]$ with $N \times N$ dimension. The $\theta_1, \theta_2, \dots, \theta_N$ are the set of discrete points within

the scan region (or angles to be scanned). Let the signal received by the array elements be $\mathbf{r}_S(k) = [r_{s1} \ \dots \ r_{sN}]^T$. The received signal is now multiplied with the scan angle matrix Ψ as:

$$\mathbf{x}(k) = \Psi(\theta) \mathbf{r}_S(k) + \mathbf{n}(k). \quad (2)$$

As a result, the scanned beam can be obtained in which the DOAs of the source signals are visible. Then, the array beam is scanned for each angle within the spatial spectrum. The peak values indicate the DOAs of the received signals. Figure 1a shows the result of applying classic two dimensional DS beamformer array with 5×5 elements distributed uniformly at a distance $\lambda/2$ on a rectangular grid to estimate both azimuth and elevation angles of two signals that impinged on the array from directions $0, 0$ and $-10^\circ, 10^\circ$, while Fig. 1b shows the result of applying the two dimensional standard MVDR beamformer array for the same scenario as above. One may observe that the DS method fails to estimate the two closely-spaced signals due to its widened beamwidth pattern which is larger than the angular separation between the two impinged signals. On the other hand, the MVDR method offers better resolution and is capable of accurately estimating both signals provided that the positions of the array elements are perfectly determined and there no imperfection errors are present.

To increase the resolution of the arrays under consideration, the results shown in Fig. 1 are recalculated, as presented in Fig. 2, with an increased array dimension (i.e. an array with 10×10 elements instead of 5×5 elements). From these results, as expected, a general improvement in the resolution is observed, at the cost of higher computational complexity which is undesirable and may limit the range of practical implementations. This problem may be solved by compressing sparse arrays, as shown below.

The mathematical formulation of compressive sensing that takes into consideration signal $\mathbf{x}(k) \in C^{N \times 1}$, sparseness matrix $\Psi_s(\theta)$ with dimension $N \times N$, and P -sparse signal vector \mathbf{z} with dimension $N \times 1$, may be expressed as $\mathbf{x} = \Psi_s \mathbf{z}$, where P -sparse means that only $P < N$ entries in the vector are non-zero. The goal of the compressed sensing method is to recover the output signal $\mathbf{x}(k) \in C^{N \times 1}$ using a smaller set of measurements, say $M \times 1$ instead of $N \times 1$, where M is less than N . Thus, $\mathbf{x}(k) \in C^{N \times 1}$ will be changed to a new vector called measurement vector $\mathbf{y}(k) \in C^{M \times 1}$. Then, the system becomes underdetermined, as it consists of linear equations with numerous solutions, i.e. it does not have a unique solution as long as $M < N$. Measurement vector \mathbf{y} may be related to sensing matrix Φ of dimension $M \times N$ as $\mathbf{y} = \Phi \mathbf{x}$. In light of the above, the output of the sparse array $\mathbf{y}(k) \in C^{M \times 1}$ may be given by:

$$\mathbf{y}(k) = \Phi(\theta) \mathbf{x}(k) = \Phi(\theta) \Psi_s(\theta) \mathbf{z}(k) = \Theta(\theta) \mathbf{z}(k), \quad (3)$$

where $\Theta(\theta)$ is the observation matrix with dimension $M \times N$. P sources from only M measurements of $\mathbf{y}(k)$ are then found by applying compressed sensing. It should be

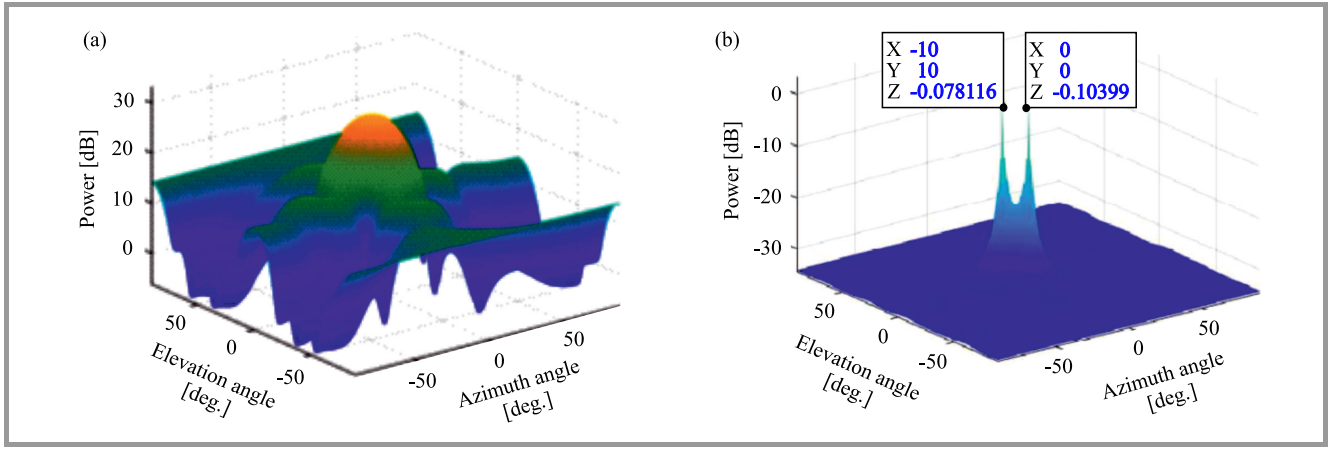


Fig. 1. Results for a 5×5 uniform planar array, classical DS method (a) and for standard MVDR method (b). (For color pictures see the digital version of the paper).

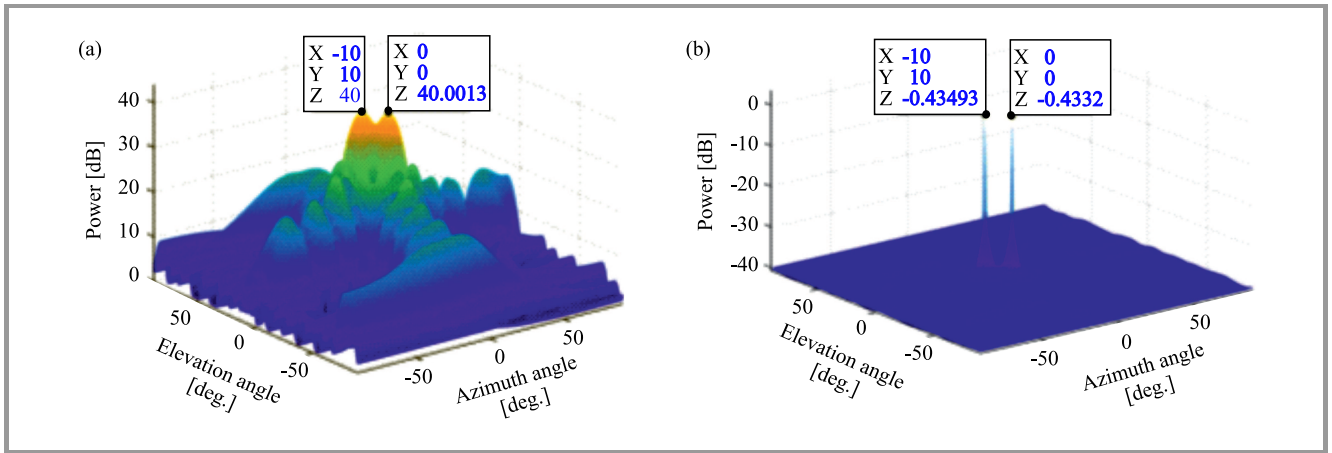


Fig. 2. Results for a 10×10 uniform planar array, classical DS method (a) and for standard MVDR method (b).

mentioned that the system in Eq. (1) may be solved by means of the least squares method:

$$\min \|\mathbf{s}\|_2 \text{ subject to } \mathbf{A}\mathbf{s} = \mathbf{x} , \quad (4)$$

and its solution is:

$$\mathbf{s}_{ls} = \mathbf{A}^T (\mathbf{A}\mathbf{A}^T)^{-1} \mathbf{x} . \quad (5)$$

In this paper, the author expects to find the sparse solution rather than the full solution using an iterative reweighted optimization algorithm. Therefore, \mathbf{s} is represented by $\mathbf{s} = \mathbf{W}\mathbf{q}$, where \mathbf{s} is the unknown source vector, \mathbf{W} is the weighting matrix with dimension $N \times N$, and \mathbf{q} may be found from:

$$\min \|\mathbf{q}\|_2^2 \text{ subject to } \mathbf{A}\mathbf{W}\mathbf{q} = \mathbf{x} . \quad (6)$$

Equation (6) is solved iteratively using the reweighted l_1 norm in conjunction with the algorithm that was presented in [21]. To detect the peaks in the spectrum solution that correspond to the directions of the arrived signals, a simple searching algorithm is applied to the final optimization solution. Note that only M out of N array elements are used

to reconstruct the signals and estimate their DOAs. Thus, computational complexity is greatly reduced.

3. Simulation Results

In this section, extensive simulation results are demonstrated to illustrate the effectiveness of the proposed method. First, performance in terms of mean squared errors (MSE), signal-to-noise ratio (SNR), resolution and computational complexity of such conventional methods as DS, MVDR, MUSIC, ESPRIT, and the proposed method are demonstrated to verify the superiority of the proposed method.

In all scenarios, a full dense (filled) antenna array with $N = 30$ identical elements is considered, and all received signals are of the narrow-band variety. For regular full dense arrays, the separation distance between their elements is set to $d = \lambda/2$. The number of snapshots is set to $L = 1$. The power of each signal source is set to 0 dBm and the power of noises is specified. To evaluate the estimation performance of the tested methods, MSE – representing

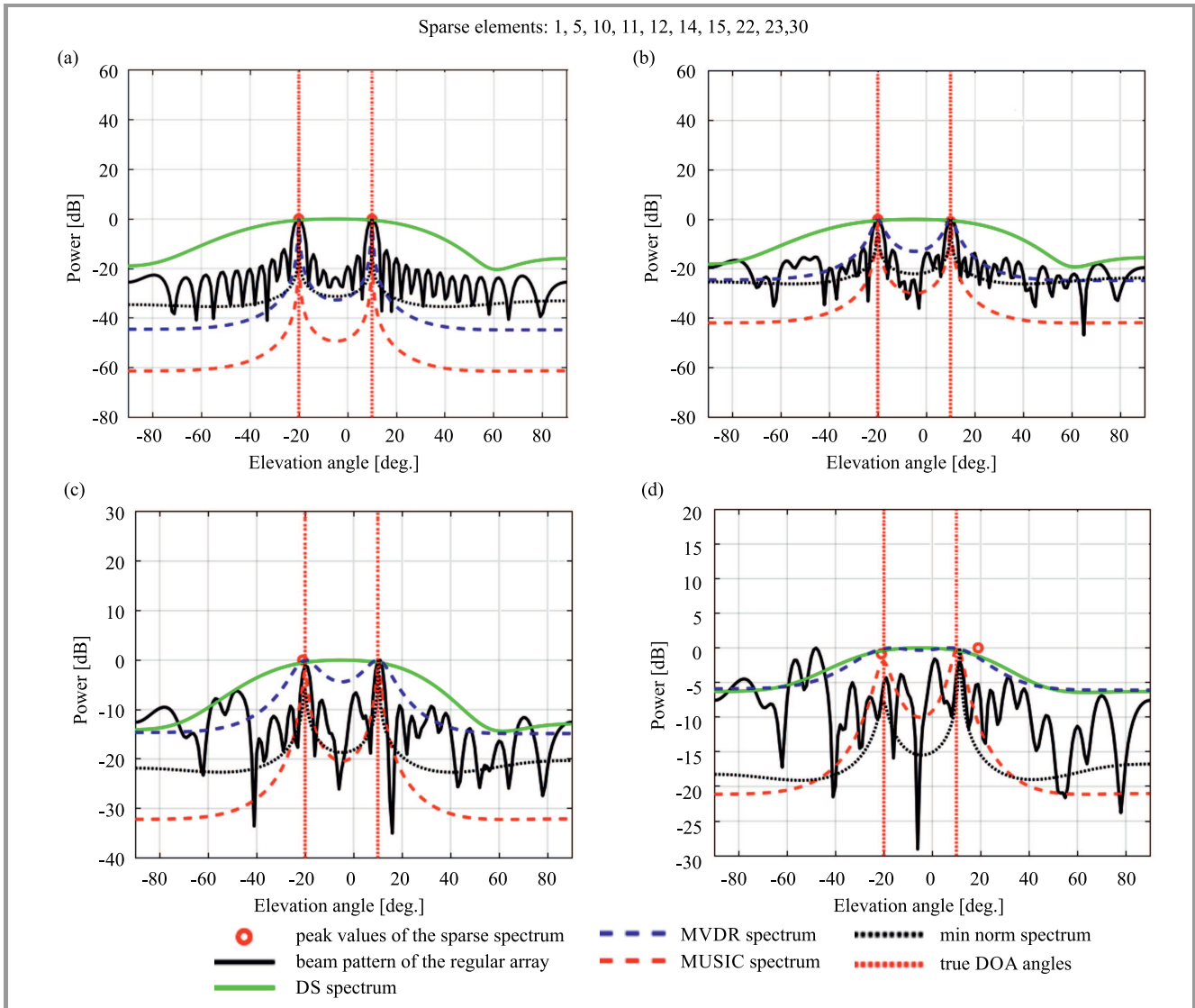


Fig. 3. Results for the tested methods for $N = 30$, $M = 10$ for two sources at $(-20^\circ, 10^\circ)$ and SNR: (a) 30 dB, (b) 10 dB, (c) 0 dB, (d) -10 dB.

the deviation between the estimated $\hat{\mathbf{x}}$ and the actual \mathbf{x}_0 DOA values – was calculated as:

$$\text{MSE} = E \frac{\|\hat{\mathbf{x}} - \mathbf{x}_0\|_F^2}{\|\mathbf{x}_0\|_F^2}, \quad (7)$$

where $\|\cdot\|_F$ represents the Frobenius norm. A lower MSE value means better estimation accuracy. To construct the sparse array, we assume that only 8 randomly elements out of $N = 30$ regular elements will remain in the resulting compressed array. As mentioned earlier, to maintain the array dimension unchanged, rows number 1 and 30 of the measurement matrix will always remain. Thus, the total number of the compressed array elements including the two end elements will be $M = 10$. Then, the beam width of the initial full dense array with $N = 30$ is equal to 3.38° and is same as that of the compressed array with $M = 10$, since the overall array dimension remained unchanged. The range

of the scanning region is chosen to be from -90° to 90° . Then, the total number of the angles that need to be scanned is equal to 181 and the angular separation between any two tested angles is set to be 1° , i.e. is lower than the beamwidth value, thus enabling to attain maximum resolution levels.

For the proposed method, first the sparse spectrum of the reconstructed signals is found by using the algorithm that was presented in [22]–[23]. Then, the peak values that correspond to the estimated DOAs are calculated by using a simple searching algorithm. Finally, the peak values are plotted and compared with other tested methods, as shown in the following scenarios.

In the first scenario, two uncorrelated sources located at $\theta_1 = -20^\circ$ and $\theta_2 = 10^\circ$ with four different SNRs: 30, 10, 0, and -10 dB, are considered. Figure 3 shows the results of applying the proposed sparse array and compares them with those of the regular fully filled array: DS, MVDR, MUSIC, and min norm methods. For the proposed sparse

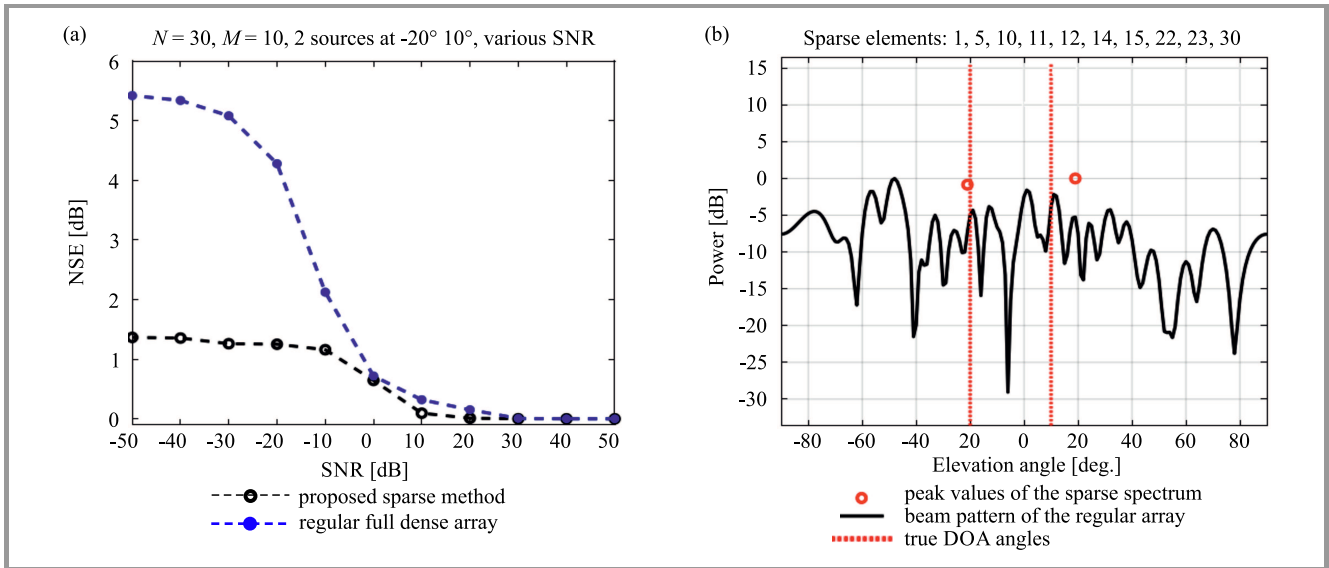


Fig. 4. MSE vs. SNR (a) and one sample result for SNR = -10 dB (b).

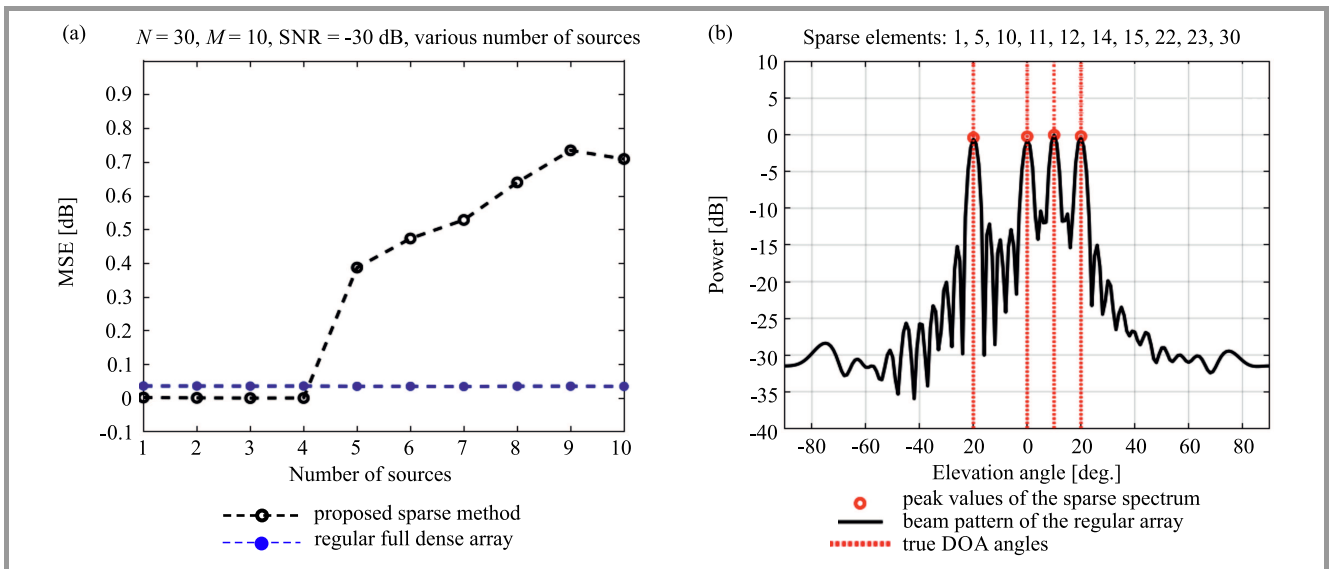


Fig. 5. MSE vs. the number of sources (a) and one sample result for four sources (b).

array method, the indices of the elements that remain after the sparseness process are also shown at the top of Fig. 3. It may be observed that all of the tested methods, including the proposed method with sparse elements of indices 1, 5, 10, 11, 12, 14, 15, 22, 23, and 30, perform very well as far as estimating the correct DOAs under high SNR is concerned. This estimation degrades for low SNR levels. For the proposed method and for each considered SNR value, the estimated DOAs were found to be $(-20^\circ, 10^\circ)$, $(-20^\circ, 10^\circ)$, $(-21^\circ, 11^\circ)$ and $(-21^\circ, 19^\circ)$, meaning they differ from the true DOA angles by the following MSE values: 0.001, 0.0941, 0.6443, and 1.1588, respectively. Although little deviations in the estimation of DOA exists for SNR of -10 dB, performance of the proposed method was considered to be satisfactory. Figure 3d clearly shows the superiority of the proposed method in

comparison to all other tested methods which fail to estimate the DOAs.

In the second scenario, the estimation performance in terms of MSE of the proposed sparse and regular fully filled (or dense) arrays under various SNR values is further investigated and highlighted, as shown in Fig. 4. Sample results at specific -10 dB SNR are shown as well. Again, superiority of the proposed sparse array is evident, especially for lower SNR values.

In the third scenario, MSE is investigated versus the maximum allowable number of sources (Fig. 5). It may be observed that the maximum detectable number of source directions is only 4 for the case of $M = 8$ sparse elements. The first and the last elements were not considered here, because they are not sparse elements. It should be noted that many other cases have been examined and, in general,

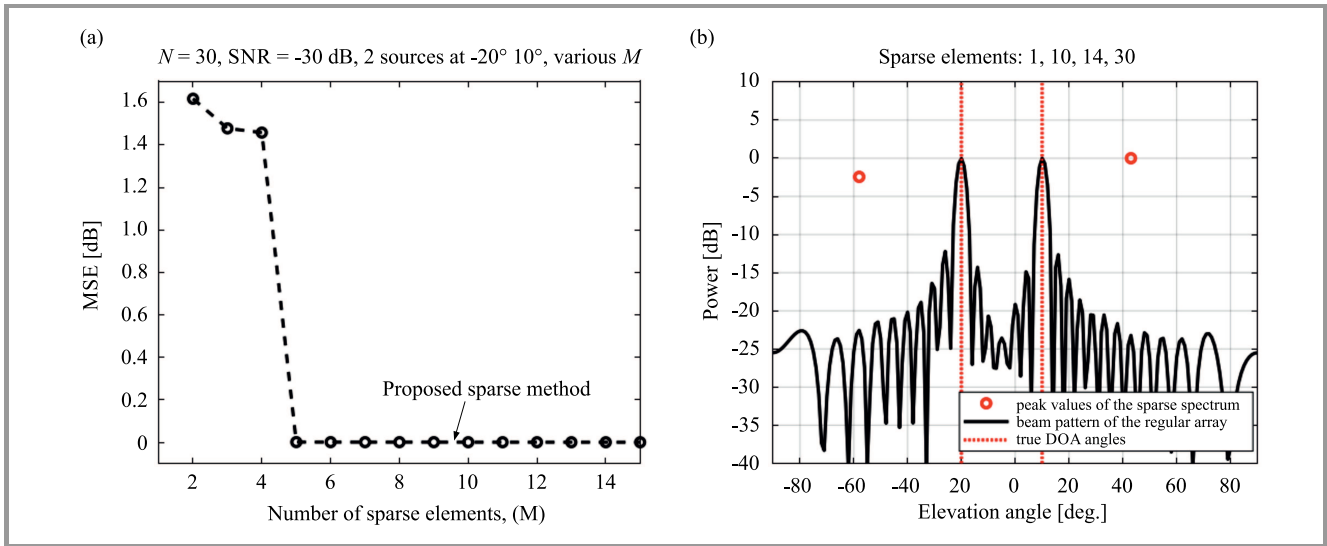


Fig. 6. MSE vs. the number of sparse elements (a) and one sample result for four sources for $M = 2$ (b).

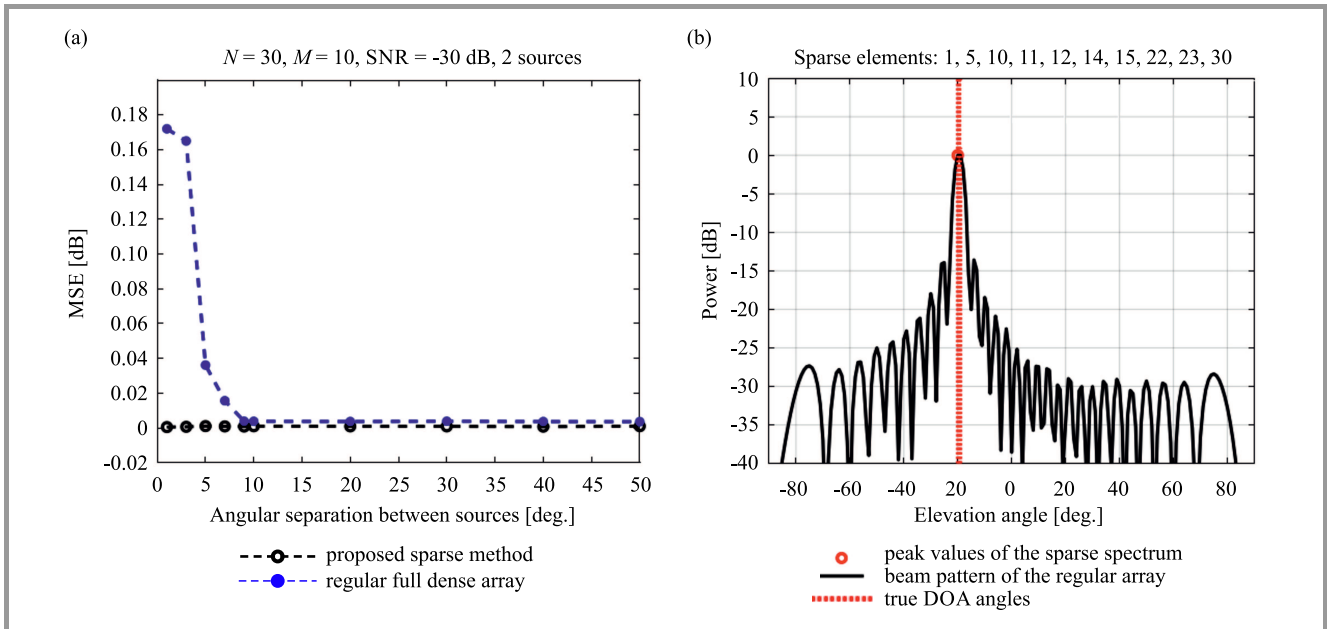


Fig. 7. MSE vs. angular separation between sources (a) and one sample result for 1° angular separation (b).

it is found that the maximum detectable number of source directions is directly proportional to the number of sparse elements. It may be expressed as $M/\log N$ which is equal to 5.415 for $M = 8$ and $N = 30$.

In the next scenario, the effect that the number of sparse elements exerts on estimation performance is studied, as shown in Fig. 6. It may be concluded that for two source directions and only two considered sparse elements, estimation performance is unsatisfactory and the directions are calculated incorrectly. To obtain correct directions, we need to set the value of M to equal at least 5 elements.

Finally, the resolution of the proposed sparse array under two closely spaced sources is investigated and shown in Fig. 7. Performance of the proposed array still remain better

than that of the regular full dense array, especially for very small angular distances, and this distinction vanishes for larger angular distances.

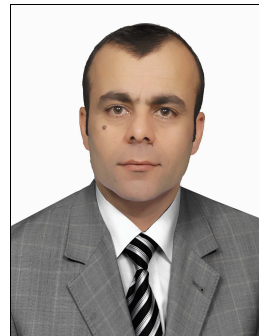
4. Conclusions

It has been shown that the proposed sparse array based compressed sensing method was effectively able to estimate the required DOAs. Its resolution was found to be accurate even under severe noisy environments. Moreover, the maximum allowable number of the detected sources was found to be proportional to the number of the sparse elements. In all tested scenarios, the output spatial spectrum

was plotted and compared. Unlike existing DOAs methods, the sparse spectrum of the proposed method had best spatial resolution.

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Jafar Ramadhan Mohammed

received his B.Sc. and M.Sc. degrees in Electronics and Communication Engineering in 1998 and 2001, respectively, and Ph.D. in Digital Communication Engineering from Panjab University, India in 2009. He was a Visiting Lecturer at the Faculty of Electronics and Computer Engineering of the

Malaysia Technical University Melaka (UTeM), Melaka, Malaysia in 2011 and at the Autonoma University of Madrid, Spain in 2013. He is currently a Professor and Vice Chancellor for Scientific Affairs at Ninevah University. His main research interests are in the area of digital signal processing and its applications, antennas, and adaptive arrays.

 <https://orcid.org/0000-0002-8278-6013>

E-mail: jafarram@yahoo.com

College of Electronics Engineering
Ninevah University
Mosul, Iraq