*European Journal of Management Issues*

*Volume 29(2), 2021, pp. 101-108*

*DOI: 10.15421/192110*

*Received: 28 May 2021*
*Revised: 07 June 2021; 08 June 2021*
*Accepted: 21 June 2021*
*Published: 25 June 2021*

**UDC classification: 336.71:368.03+330.4**

**JEL Classification: C52, G21, G22**

# Model Assessment of Interaction between Banking and Insurance Segments of the Financial Market

***R. Ivanov*** [‡]***,***
***N. Maksyshko*** [#]

**Purpose:** To develop a model of interaction between banks and insurance companies, allowing for a joint model assessment from the point of view of the possibility of sustainable operation and development.

**Findings:** The article develops the theory of dynamic systems and a numerical method for solving the problem of assessing the results of interaction between banks and insurance companies based on a system of differential equations. The additive and multiplicative types of interaction are considered. The results of interaction are analyzed for various variants of the influence of the activities of the studied subjects.

**Practical Implications:** The proposed approach is applicable to assess the effectiveness of the used model of interaction between banks and insurance companies.

**Originality/Value:** The author's model of interaction between banks and insurance companies is original. The model has no analogues in the scientific literature of the studied subject area.

**Research Limitations/Future Research:** The study proposes a generalized dynamic model of interaction without specifying the content of variables and the procedure for determining the input parameters. Its description and application determine potential areas for further research.

**Paper type:** Theoretical.

***Keywords:*** financial services sector, bank, insurance company, bankinsurance, additive and multiplicative interaction.

[‡]*Roman V. Ivanov,*
*Doctor of Economic Sciences, the associated professor,*
*Head of the Department of Economic Cybernetics*
*Oles Honchar Dnipro National University,*
*Ukraine,*
*e-mail: romanivanov1926@gmail.com,*
https://orcid.org/0000-0003-2086-5004

[#]*Nataliia K. Maksyshko,*
*Doctor of Economic Sciences, the associated professor,*
*Head of the Department of Economic Cybernetics*
*Zaporizhzhya National University,*
*Ukraine,*
*e-mail: maxishko@ukr.net,*
https://orcid.org/0000-0002-0473-7195

**Reference** to this paper should be made as follows:
Ivanov, R., & Maksyshko, N. (2021). Model Assessment of Interaction between Banking and Insurance Segments of the Financial Market. *European Journal of Management Issues*, 29(2), 101-108. doi:*10.15421/192110*.

# Модельна оцінка взаємодії банківського і страхового сегментів фінансового ринку

***Роман В. Іванов*[‡], *Наталія К. Максишко*[#]**

[‡]*Дніпровський національний університет імені Олеся Гончара, Україна*
[#]*Запорізький національний університет, Україна*

**Мета роботи:** Розробити модель взаємодії між банками та страховими компаніями, що дозволяє проводити спільну оцінку з точки зору можливості стійкого функціонування та розвитку.

**Результати дослідження:** У статті розвинено теорію динамічних систем та на основі системи диференціальних рівнянь розроблено методику вирішення проблеми оцінки результатів взаємодії банків та страхових компаній. Розглянуто адитивний та мультиплікативний типи взаємодії. Результати взаємодії аналізуються при різних варіантах впливу діяльності досліджуваних об'єктів.

**Практичне значення дослідження:** Запропонований підхід може бути використаний для оцінки ефективності використаної моделі взаємодії між банками та страховими компаніями.

**Оригінальність/Цінність дослідження:** Авторська модель взаємодії банків та страхових компаній оригінальна. Модель не має аналогів у науковій літературі досліджуваної предметної області.

**Обмеження досліджень/Перспективи подальших досліджень:** Дослідження пропонує узагальнену динамічну модель взаємодії без зазначення змісту змінних та процедури визначення вхідних параметрів. Його опис та застосування визначають потенційні напрямки подальших досліджень.

**Тип статті:** Теоретичний.

***Ключові слова:*** сектор фінансових послуг, банк, страхова компанія, банкострахування, адитивна та мультиплікативна взаємодія.

# Модельная оценка взаимодействия банковского и страхового сегментов финансового рынка

***Роман В. Иванов*[‡], *Наталья К. Максишко*[#]**

[‡]*Днепровский национальный университет имени Олеся Гончара, Украина*
[#]*Запорожский национальный университет, Украина*

**Цель работы:** Разработать модель взаимодействия между банками и страховыми компаниями, позволяющую проводить совместную оценку с точки зрения возможности устойчивого функционирования и развития.

**Результаты исследования:** В статье развивается теория динамических систем и на основе системы дифференциальных уравнений разработана методика решения проблемы оценки результатов взаимодействия банков и страховых компаний. Рассмотрены аддитивный и мультипликативный типы взаимодействия. Результаты взаимодействия анализируются при различных вариантах влияния деятельности исследуемых субъектов.

**Практическое значение исследования:** Предложенный подход может быть использован для оценки эффективности используемой модели взаимодействия между банками и страховыми компаниями.

**Оригинальность/Ценность:** Авторская модель взаимодействия банков и страховых компаний оригинальна. Модель не имеет аналогов в научной литературе исследуемой предметной области.

**Ограничения исследований/Перспективы дальнейших исследований:** Исследование предлагает обобщенную динамическую модель взаимодействия без указания содержания переменных и процедуры определения входных параметров. Его описание и применение определяют потенциальные направления дальнейших исследований.

**Тип статьи:** Теоретический.

***Ключевые слова:*** сектор финансовых услуг, банк, страховая компания, банкострахование, аддитивное и мультипликативное взаимодействие.

## 1. Introduction

The constant development of the world economy and financial markets, as well as increasing competition in all areas of trade makes new demands on the activities of monetary institutions. Banks are an integral part of the modern monetary economy; their activities are strongly linked to the needs of reproduction. They will always be at the center of economic life, serving the interests of producers, linking a cash flow to the industry and world trade, agriculture and population. All over the world, banks have both significant power and influence, they have a huge amount of money that comes from businesses, firms, the state, individuals and etc. In fact, banking institutions are the heart of the economic mechanism of each country *(Melnychuk, 2015)*.

As long ago as in the 1990s, a number of trends and development directions emerged on the financial services market, which significantly affected the quality of services and specific financial institutions. In particular, they included the dominance of the banking sector over all others on the financial market *(Barras, 1990)*.

The non-banking financial sector is still far behind the banking sector in terms of both total assets and growth rates. The leading position in the non-banking financial sector is occupied by the insurance market. Both the banking and insurance markets in Ukraine are becoming increasingly competitive, so it is extremely important to position new services. In addition, domestic entities have more confidence in the banking market and are more likely to buy an insurance product from a bank. The combination of banking and insurance services is also one of the ways to maintain relationships with customers, as the availability of a wider range of products can deter the customer from moving to another organization that provides financial services *(Makarenko, & Pokotylo, 2014)*.

The purpose of this research is to study the forms of interaction between banks and insurance companies, as well as to develop a methodology for evaluating the results of such interaction. After a short introduction, the study will focus on creating and testing a model of interaction between banks and insurance companies, which allows you to assess the possibility of joint operation and development.

## 2. Theoretical background

The article *(Makarenko, & Pokotylo, 2014)* considers a wide range of theoretical aspects of the interaction of banks and insurance companies. The analysis of forms of the interaction between banks and insurance companies is carried out, the risks arising from such interaction are defined, changes in the relations between bank and insurance companies at various forms of interaction are considered. A set of criteria for determining the feasibility of cooperation has been identified. The sources of information on the basis of which the financial condition of the insurance company is checked for decision-making on further interaction with the insurance company and what stages the insurance company goes through during the inspection by the bank are given.

The study *(Agres, & Tymkiv, 2019)* is devoted to the substantiation of the concept of «bank insurance» as a process of the interaction between banks and insurance companies. The results of this cooperation are illustrated as the sale of insurance policies together with the products offered by banks. An important factor in the formation of banking and insurance cooperation is new distribution channels of insurance companies.

A generalization of the advantages and disadvantages of the interaction between banks and insurance companies is devoted to the study *(Dichevska, Karadjova, & Jolevski, 2018)*, which emphasizes that the problem of banking insurance is a different philosophy of sales of banking and insurance products.

The article *(Prikazyuk, & Oliynik, 2017)* reveals the impact of the interaction between insurance companies and banks on the insurance system. The description of the main models of bank insurance in the context of their impact on the insurance system is given and their main advantages and risks are identified.

At the same time, there is some ambiguity in the interpretation of certain issues, and in most of the available studies, the assessment of the effectiveness of the interaction between banks and insurance companies, carried out from the operational, marketing and financial aspects and presented in the form of charts and tables, is empirical.

Finding the most preferred alliance structure between banks and insurance companies is a multi-criteria task (MCDM), to solve which individual authors *(Korhonen, & Voutilainen, 2006)* have used expert methods and analytical hierarchy processes (AHP).

Only in some cases *(Vygovska, Polchanov, & Vygovskyi, 2018)* studies of the cooperation of insurance companies and banking institutions or individual segments of the financial market *(Prymostka et al., 2020)* are based on the mathematical modeling of the dependence of the insurance market on banking in the form of correlation regression models, which provides sufficient arrays of statistics for each individual case.

Thus, it is important not only to systematically study the mechanisms of the cooperation between banks and insurance companies, which is very promising and important for banks, or to assess the feasibility of their interaction and customer satisfaction, but also to involve modern economic and mathematical modeling tools to solve this scientific and practical problem, which is able to reflect the processes of emergence and development of the synergetic effect as a result of combining these entities for a common goal: to make a profit and meet customer needs.

## 3. Problem statement

The purpose of the study is to assess the effectiveness of the interaction between banks and insurance companies in the financial services market. To achieve this goal, it is necessary to solve the following tasks: to standardize the forms of the interaction between banks and insurance companies in the modern financial services market; to build formalized in the form of economic and mathematical models of representation of the processes of the interaction between banks and insurance companies; to conduct a model assessment of the effectiveness of various forms of the interaction between banks and insurance companies in terms of their sustainable development. We suggest that the most effective form of the interaction between the banking and insurance sectors in the financial market is banking insurance, which, due to multiplicative links between entities, is able to create conditions for their sustainable operation.

## 4. Methodology

Speaking of achieving a synergy effect as a result of merging banks and insurance companies to achieve a common goal: to make a profit and meet customer needs, it should be noted that the development of synergy as an interdisciplinary field of research has led to changes in understanding the system and its structure.

In particular, the synergetic system is primarily dynamic, the understanding of which is a consequence of a certain idealization, which despises the influence of random perturbations, inevitably present in any real system *(Maksishko, & Glazova, 2016)*.

The article *(Malinetskiy, 2005)* establishes an isomorphic relationship between the concept of a «dynamic system» and «autonomous system of ordinary differential equations».

The main mathematical apparatus in this case is a qualitative theory of differential equations, and the model is presented by a system of autonomous differential equations in the formula (1):

$$\dot{x}_i = F_i(x_1, x_2 \dots x_n), (i = \overline{1,n}), \quad (1)$$

where $x_i$ is some quantitative characteristics of the socio-economic agent involved in the process.

The main difference between qualitative methods in mathematics is that they are not aimed at obtaining a numerical result, but to identify the characteristics of the phenomenon as a whole, to predict the development of a phenomenon or process. In this case, sometimes it is enough to establish areas of stability and their interpretation *(Milovanov, 2001)*.

The foundations of the qualitative theory of the study of systems of ordinary differential equations of the type were laid by A. Poincare at the turn of the XIX-XX centuries, and the theory itself has developed to such an extent that it is a generally accepted apparatus of nonlinear dynamics, in particular self-organizing systems.

The authors of the article *(Mangan, Kutz, Brunton, & Proctor, 2017)* emphasize that the theory of dynamical systems allows obtaining basic characteristics and understanding physical, technical, biological and socio-economic processes. Their unified models are created taking into account a specific subject area and allow studying chaotic, oscillatory or bifurcation modes.

Thus, in the article *(Gabrin, Meshkova, & Rumyantsev, 2015)* the interaction of socio-economic and ecological systems is modeled, and in the articles *(Ivanov, 2015; Ivanov, & Ivanova, 2018)*, two-factor models of household income behavior are built and studied.

The article *(Mangan, Brunton, Proctor, & Kutz, 2016)* is devoted to the study of nonlinear dynamics within biological systems, using the system of equations as a model of processes (1).

The classical model of nonlinear dynamics is the system of Volterra-Lotka equations allowed the author *(Kolyada, Kmytiuk, & Shatarska, 2020)* to obtain analytical formulas for estimating the quantitative measure of the risk of decision-maker behavior.

In the study *(Vitlinskyi, Kolyada, & Kharlamov, 2012)* an analytical, qualitative and quantitative study of the three-factor model of the bank, built in the form of a system of ordinary nonlinear differential equations, which allowed investigating the dynamics of different indicators of efficiency of loan and deposit portfolios, was carried out.

Therefore, taking into account formula (1), we assume that $x_1$ is a quantitative characteristic of the bank (banking system); $x_2$ is quantitative characteristics of the insurance company (insurance system).

Then, in terms of the interaction between banks and insurance companies, in which the insurance company provides services to the banking institution, and the banking institution provides services to insurance companies, a generalized two-factor model of the interaction can be represented as:

$$\begin{cases} \frac{dx_1}{dt} = f_1(x_1; x_2), \\ \frac{dx_2}{dt} = f_2(x_1; x_2). \end{cases} \quad (2)$$

The choice of the formula of the functions $f_1(x_1; x_2)$ and $f_2(x_1; x_2)$ and the dynamics of model (2) directly depend on the initial conditions, the nature of interaction and mutual influence of banks and insurance companies.

## 5. Results and Discussion

The article *(Makarenko, & Pokotylo,2014)* shows that insurance companies and banks interact in several areas. Forms of the interaction differ in the level of integration, as well as in the volume of operations.

Thus, possible insurance by the bank's insurance company provides two main forms – retail and comprehensive insurance. Banking services for the insurance company include the ability to service the accounts of the insurance company or keep the technical reserves of the insurer. Under such conditions, the interaction between the insurance company and the bank should help to improve performance.

We propose to call this form of the interaction, which does not involve strong integration and, accordingly, specific legal design, additive. The economic and mathematical model (2) of such interaction can be represented as a system:

$$\begin{cases} \frac{dx_1}{dt} = ax_1 + bx_2; \\ \frac{dx_2}{dt} = cx_1 + dx_2, \end{cases} \quad (3)$$

where *a* is the coefficient of proportionality, which characterizes the growth rate of the quantitative characteristics of the bank due to its own activities; *b* is the coefficient of proportionality, which characterizes the growth rate of the quantitative characteristics of the bank due to the interaction with the insurance company; *d* is the coefficient of proportionality, which characterizes the growth rate of the quantitative characteristics of the insurance company due to its own activities; *c* is the coefficient of proportionality, which characterizes the growth rate of the quantitative characteristics of the insurance company through the interaction with the banking institution. In connection with their meanings, the parameters *a, d* are called endogenous, and *b, c* are called exogenous.

Applying to system (3) of a qualitative theory of ordinary differential equations *(Schaeffer, & Cain, 2016)*, we can determine that a single steady state is determined from the condition

$$\begin{cases} \frac{dx_1}{dt} = ax_1 + bx_2 = 0; \\ \frac{dx_2}{dt} = cx_1 + dx_2 = 0 \end{cases} \quad (4)$$

and corresponds to the phase space point $x_1 = x_2 = 0$. The nature of a particular point depends on the ratio of parameters $a, b, c, d$.

If we consider a situation in which banks and insurance companies receive a positive result both through their own activities and as a result of the interaction according to the constructed model (5), then $a; b; c; d > 0$. A special point $x_1 = x_2 = 0$ is unstable (Unstable Node) in this case, which is expressed in the unlimited growth of quantitative indicators of banks and insurance companies (*Fig. 1*, *t* here and hereafter denotes the time measured in months).

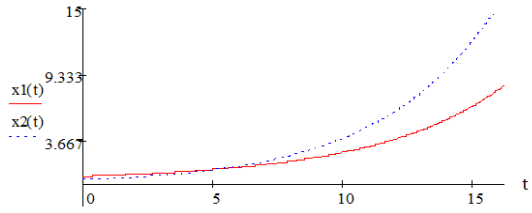


**Figure 1:** Dynamics of quantitative indicators of banks and insurance companies according to the model of interaction (3) at a=0,1; b=0,05; d=0,2; c=0,05 and initial conditions: $x_1(0)$=0.6; $x_2(0)$=0.3

***Source:*** developed by the authors.

It is known that the interaction of banks and insurance companies, in addition to advantages, also has disadvantages, which, among other things, may arise due to the risk of occurrence and transfer of risks from the banking sector to the insurance sector and vice versa *(Dichevska, Karadjova, & Jolevski, 2018)*.

In this case, in the model of additive interaction (3), the exogenous coefficients *b, c* acquire negative values.

Let the rate of growth of the quantitative characteristics of the insurance company due to the interaction with the banking institution decrease as a result of the additive interaction ($c < 0$). In this case, the special point at up to a certain limit of the value of the parameter $c < 0$ retains its character, Unstable Node then moves to the type Unstable Focus, for which in the short or medium term the values of indicators $x_1$ and $x_2$, after a certain increase, decrease rapidly in a negative direction (*Fig. 2*).

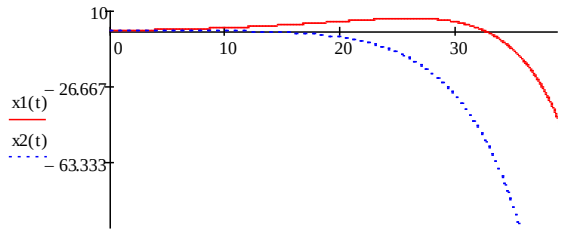


**Figure 2:** Dynamics of quantitative indicators of banks and insurance companies according to the model of interaction (3) at a=0,1; b=0,05; d=0,2; c=-0,06 and initial conditions: $x_1(0)=0.6$; $x_2(0)=0.3$

***Source:*** developed by the authors.

The only variant of the nature of the studied special point, which has a neutral stability is Linearization Fails, for which the variables $x_1, x_2$ are characterized by oscillatory dynamics (*Fig. 3*).

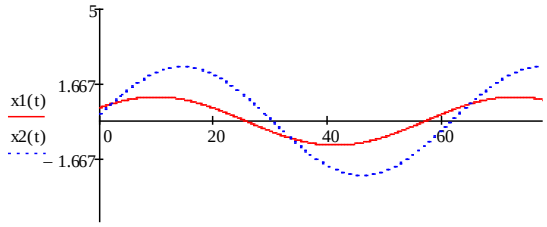


**Figure 3:** Dynamics of quantitative indicators of banks and insurance companies according to model of interaction (3) at a=0,2; b=-0,1; d=-0,2; c=0,5 and initial conditions: $x_1(0)=0.6$; $x_2(0)=0.3$

***Source:*** developed by the authors.

The proposed example corresponds to the situation of negative development of the insurance market and its negative impact on the banking sector ( $b;\ d < 0$). At the same time, maintaining a stable fluctuating nature of the process involves significant assistance from banks, which in such a situation and the proposed additive model of the interaction should be considered impossible.

The stability of the state $x_1 = x_2 = 0$ is an undesirable result, and the conditions for its occurrence correspond to the opposite in magnitude and different in absolute value indicators of the efficiency of banks and insurance companies (*a* and *d*, respectively), which is typical of the financial market with a significant negative imbalance in the performance of individual segments or asynchronous fluctuations of these indicators *(Adarov, 2019)*, which leads to an imbalance. This situation is accompanied by the corresponding, commensurate in magnitude with the endogenous parameters, exogenous indicators. *Fig. 4* shows the dynamics corresponding to a particular point of the Stable Focus type.

Nowadays, the interaction of banks and insurance companies is gathering pace, moving from a simple mutual service of their institutions to comprehensive intersectoral and intermarket interaction, due to the factors that are mutually beneficial from the partnership *(Dichevska, Karadjova, & Jolevski, 2018)* and aimed at achieving mutual stability and reliability.

We are talking about Bansassurance – a set of financial services that can meet both banking and insurance needs of a client at the same time.

Bancassurance is used to describe the partnership between a bank and an insurance company through which an insurance company uses a bank sales channel to sell insurance products *(Dicevska, & Karadjova, 2016)*.

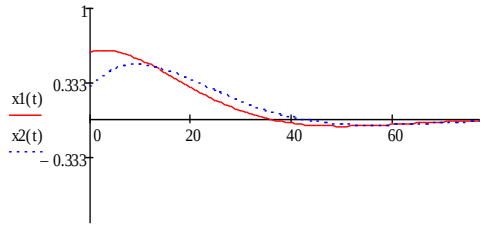


**Figure 4:** Dynamics of quantitative indicators of banks and insurance companies according to model of interaction (3) at a=0,1; b=-0,15; d=-0,2; c=0,18 and initial conditions: $x_1(0)=0.6$; $x_2(0)=0.3$

***Source:*** developed by the authors.

Different types of Bancassurance models are classified according to the specifics of interaction and dependence of participants:

- integrated models (Bancassurance activities are closely related to the main activities of the bank);
- models based on consultations, which are characterized by less integration, and sales of which are carried out with the help of professional insurance consultants (non-integrated models);
- models of open architecture.

Thus, non-integrated models of bank insurance are often characterized by countries with underdeveloped financial markets *(Devi, 2019)*, for whom such a distribution channel is relatively new.

This form of interaction is called additive-multiplicative, and its mathematical model is represented as a system

$$\begin{cases} \frac{dx_1}{dt} = ax_1 + bx_1x_2; \\ \frac{dx_2}{dt} = cx_1x_2 + dx_2. \end{cases} \quad (5)$$

System (5) is nonlinear, and its qualitative analysis *(Hirsch, & Smale, 1974)* shows the presence of two singular points: $x_1 = x_2 = 0$ ; $x_1 = -\frac{d}{c}$; $x_2 = -\frac{a}{b}$.

The characteristic equation for $x_1 = x_2 = 0$ has the formula $(a-\lambda)(d-\lambda) = 0$, and its roots are $\lambda_1 = a$; $\lambda_2 = b$ indicating that the studied state is stable with the negative values of endogenous parameters even in cases when the results of the introduction of bank insurance bring tangible results at least for one of the parties (*Fig. 5*). Obviously, they have only a short-term positive impact on the development of the process.

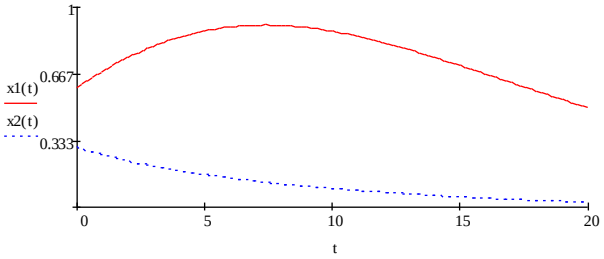


**Figure 5:** Dynamics of quantitative indicators of banks and insurance companies according to model of interaction (5) at a=-0,1; b=0,8; d=-0,2; c=0,1 and initial conditions: $x_1(0)=0.6$; $x_2(0)=0.3$

***Source:*** developed by the authors.

The characteristic equation for the singular point $x_1 = -\frac{d}{c}$; $x_2 = -\frac{a}{b}$ has the formula $\lambda^2 = ad$, and its conjugate complex roots with different-sign values of endogenous parameters provide the dynamics of the process in its vicinity, typical of Linearization Fails (*Fig. 6*, *Fig.7*).

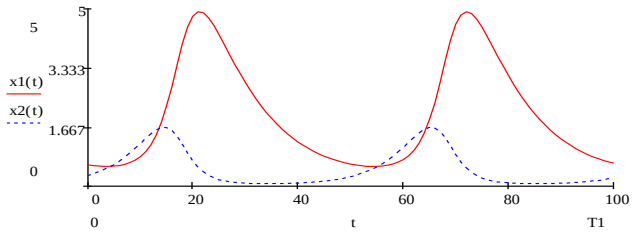


**Figure 6:** Dynamics of quantitative indicators of banks and insurance companies according to model of interaction (5) at a=-0,1; b=0,2; d=0,2; c=-0,1 and initial conditions: $x_1(0)=0.6$; $x_2(0)=0.3$

***Source:*** developed by the authors.

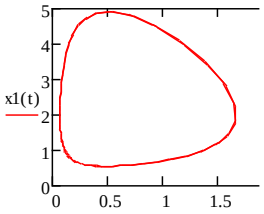


**Figure 7:** Dynamic system phase plane (5) at a=-0,1; b=0,2; d=0,2; c=-0,1 and initial conditions: $x_1(0)=0.6$; $x_2(0)=0.3$

***Source:*** developed by the authors.

It is interesting that under the conditions of the additive-multiplicative interaction described by model (5), shown in *Fig. 6*, *Fig. 7*, the dynamics arises as a result of asynchronous fluctuations in the performance of banks and insurance companies, when, to compensate for its own losses, one of the subjects of interaction uses the results of their joint activities in conditions when the other one needs to make some losses for this.

The studied situation is completely identical to the dynamics of the classical interaction model which is known as Predator-Prey Model or Lotka-Volterra system *(Kunwar, 2019)*.

Strengthening the integration processes of the interaction between banks and insurance companies can be displayed in the multiplicative formula of the model

$$\begin{cases} \frac{dx_1}{dt} = ax_1x_2; \\ \frac{dx_2}{dt} = bx_1x_2, \end{cases} \quad (6)$$

in which *a* is a coefficient that characterizes the impact on the bank of the interaction between it and the insurance company; *b* is a coefficient that characterizes the impact on the insurance company of the interaction between it and the bank.

Let us assume that the coefficients a; b can change over time, they are a = a(t); b = b(t). Their formula can be set taking into account the fact that one of the conditions for interaction is internal competition between banks *(Fuior, & Zavațki, 2020)* and in the segment of insurance services *(Mangra, Stanciu, & Mangra, 2019)*, which is typical of bank insurance models with an open architecture.

Let us make the dimensionless and assume that $x_1$ is a part of the banking system involved in bank insurance, $x_1 \in [0; 1]$; $x_2$ is a part of the insurance system involved in bank insurance, $x_2 \in [0; 1]$.

We represent $a = \tilde{a}(1 - x_1)$; $b = \tilde{b}(1 - x_2)$. System (6) is converted to the formula:

$$\begin{cases} \frac{dx_1}{dt} = \tilde{a}x_1(1 - x_1)x_2; \\ \frac{dx_2}{dt} = \tilde{b}x_1x_2(1 - x_2). \end{cases} \quad (7)$$

Note that the factors $\tilde{a}x_1(1 - x_1)$ and $\tilde{b}x_2(1 - x_2)$ in the corresponding equations of model (7) reflect the relationships inherent in Verrhulst models for population dynamics *(Brilhante, Gomes, & Pestana, 2019)*.

Applying to system (7) of elements of the theory of ordinary differential equations *(Hirsch, & Smale, 1974)*, steady states are determined from the condition

$$\begin{cases} \frac{dx_1}{dt} = \tilde{a}x_1(1 - x_1)x_2 = 0; \\ \frac{dx_2}{dt} = \tilde{b}x_1x_2(1 - x_2) = 0. \end{cases} \quad (8)$$

The special points of the phase space are $x_1 = x_2 = 0$ and $x_1 = x_2 = 1$, among which the point $x_1 = x_2 = 0$ is not stable, and the point $x_1 = x_2 = 1$ at positive values of the parameters *a, b* has a stable character (*Fig. 8*).

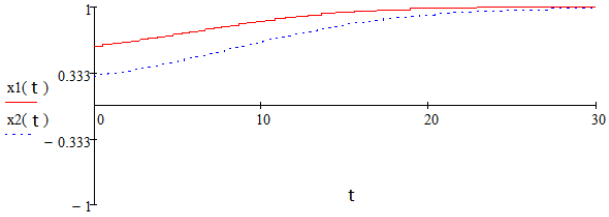


**Figure 8:** Dynamics of quantitative indicators of banks and insurance companies according to model of interaction (7) at a=0,3; b=0,2

***Source:*** developed by the authors.

It is clear that increasing the efficiency of the integration of banks and insurance companies in the form of bank insurance to achieve the desired level of ownership of potential markets is accelerated.

To take into account the additional conditions of operation of banks and insurance companies, model (7) should be supplemented accordingly. Thus, when there are the factors causing the fall of the banking sector, model (7) has the formula

$$\begin{cases} \frac{dx_1}{dt} = ax_1x_2(1 - x_1) - cx_1; \\ \frac{dx_2}{dt} = bx_1x_2(1 - x_2), \end{cases} \quad (9)$$

in which *c* is the coefficient of reduction of the functioning of the banking component.

In this case, maintaining (increasing) the level of integration allows, at a certain level, compensating for this negative impact on the banking sector.

Thus, one of the special points of system (9) will be: $x_1 = \frac{a-c}{a}$, $x_2 = 1$ (*Fig. 9*).

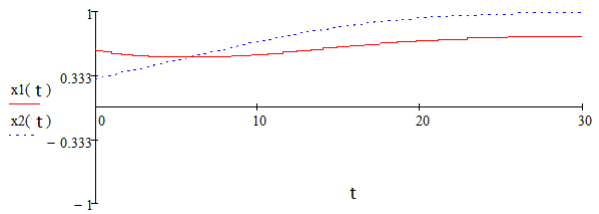


**Figure 9:** Dynamics of quantitative indicators of banks and insurance companies according to model of interaction (9) at a=0,4; b=0,3; c=0,1

***Source:*** developed by the authors.

Thus, the interaction of banks and insurance companies on the principles of bank insurance allows achieving stable states in the conditions of negative exogenous influence. The factors of these states are based on the existing, in this case, synergistic effect.

## 6. Conclusions

Between all system elements, their aggregates and separate systems, there is always a direct or indirect interaction, the system-forming character of which should provide their influence, participation in system-wide processes.

The presented work proposes a methodology for assessing the results of the interaction between the banking and insurance segments of the financial market, which is based on the assumption of the existence of a self-organizing nature of this process. The economic-mathematical model is built in the form of a system of differential equations, which is consistent with the theory of dynamical systems.

It is proposed to call the form of the interaction, which does not provide any strong integration and specific legal design, additive. It is shown that this form of the interaction is not effective enough. After all, in the conditions of negative development of one of the studied subjects, the support of the stable (oscillating) nature of the process provides significant assistance from the other subject, which in such a situation and the proposed additive model of the interaction should be considered impossible.

Such a topical organizational form of the partnership between banks and insurance companies as bank insurance, which provides the ability to meet the banking and insurance needs of the client at the same time, has been studied in detail.

Different types of bancassurance models have been analyzed, which are classified according to the specifics of interaction. It is proposed to call non-integrated models additive-multiplicative. The results of the model experiment show that under such conditions of the interaction to compensate for their own losses, one of the subjects uses the results of mixed activities, which leads to stable asynchronous fluctuations of an economic activity. A possible form of interaction in this case could be a joint venture or a strategic alliance.

Strengthening the integration processes of the interaction between banks and insurance companies is reflected in the multiplicative form of an interaction model. It is characterized by high efficiency of joint sustainable development not only in the conditions of market competition, but also in situations when one of the subjects is exposed to external negative influences. This can be explained by the emergence of a synergistic effect.

The article proposes a new direction of research and evaluation of the effectiveness of the interaction between banks and insurance companies in the financial market. The authors plan to conduct the research that clarifies the content of the variables included in the economic and mathematical model and the procedure for determining the input parameters.

## 7. Funding

This study received no specific financial support.

## 8. Competing interests

The authors declare that they have no competing interests.

## References

Adarov, A. (2019). *Financial cycles in Europe: Dynamics, synchronicity and implications for business cycles and macroeconomic imbalances* (No. 166). wiiw Working Paper. Retrieved from: https://www.econstor.eu/handle/10419/223071.

Agres, O. H., & Tymkiv, I. M. (2019). Economic essence of cooperation with banks and insurance companies. *Economic Bulletin of the National Mining University*, 67, 90–98. doi:10.33271/ev/67.090.

Barras, R. (1990). Interactive innovation in financial and business services: The vanguard of the service revolution. *Research Policy, 19*(3), 215–237. doi:10.1016/0048-7333(90)90037-7.

Brilhante, M. F., Gomes, M. I., & Pestana, D. D. (2019). Extreme stability in generalized Verhulst models for population dynamics. *Statistical Journal*, 17, 145-162.

Devi, P. P. (2019). Bancassurance: a marketing Perspective. *International Journal of Civil Engineering and Technology (IJCIET), 10*(3), 2093-2102.

Dicevska, S., & Karadjova, V. (2016). Bankinsurance-an opportunity for development and improvement of financial market. *10.20544/HORIZONS. A. 20.1. 17*. P24. Retrieved from: http://eprints.uklo.edu.mk/912/.

Dichevska, S., Karadjova, V., & Jolevski, L. (2018). Advantages and disadvantages of cooperation between banks and insurance companies. *Recent Advances in IT, Tourism, Economics, Management and Agriculture*. (pp. 646-651). doi:10.31410/itema.2018.646.

Fuior, E., & Zavaţki, T. (2020). *Competition in the banking sector: theoretical aspects, essence, specificity*. In Dezvoltarea economico-socială durabilă a euroregiunilor şi a zonelor transfrontaliere. 37, 171-185. Retrieved from: https://ibn.idsi.md/vizualizare_articol/113672.

Gabrin, K., Meshkova, T., & Rumyantsev, Y. (2015). Mathematical model of socio-economic and ecological systems interaction. *Bulletin of South Ural State University Series «Economics and Management», 9*(4), 51–56. doi:10.14529/em090408.

Hirsch, M. W., & Smale, S. (1974). *Difierential Equations. Dynamical Systems, and Linear Algebra.* San Diego: Academic Press.

Ivanov, R. V. (2015). Dvofaktorna model ratsionalnoi dokhidnoi povedinky domohospodarstva [Two-factor model of rational household income behavior]. *Business Inform*, 2, 108-112. [in Ukrainian].

Ivanov, R. V., & Ivanova, T. A. (2018). Modyfikovana dvofaktorna model' dokhidnoyi povedinky domohospodarstv [Modified two-factor model of household income behavior]. *Business Inform, 11*(490), 180-186. [in Ukrainian].

Kolyada, Y., Kmytiuk, T., & Shatarska, I. (2020). System of methods and models for estimating nonlinear dynamics of the numerical measure of economic risk. *Efektyvna Ekonomika*, (7). doi:10.32702/2307-2105-2020.7.60. [in Ukrainian].

Korhonen, P., & Voutilainen, R. (2006). Finding the most preferred alliance structure between banks and insurance companies. *European Journal of Operational Research, 175*(2), 1285–1299. doi:10.1016/j.ejor.2005.04.046.

Kunwar, L. B. (2019). Analyzing Predator-Prey Model. *Mathematics Education Forum Chitwan,* 4(4), 79–87. doi:10.3126/mefc.v4i4.26361.

Makarenko, Y., & Pokotylo, K. (2014). Teoretychni aspekty vzayemodiyi bankiv ta strakhovykh kompaniy [Theoretical aspects of cooperation between banks and insurance companies]. *Ekonomika ta derzhava*, 9, 67–70. Retrieved from: http://www.economy.in.ua/?op=1&z=2914&i=14 [in Ukrainian].

Maksishko, N.K., & Glazova, Y.V (2016). Modeli dynamiky protsesiv samoorhanizatsiyi v terytorial'niy hromadi mista [Models of the dynamics of self-organization in the local community]. *Scientific Bulletin of Kherson State University, 17*(1), 150-154. Retrieved from:

http://www.ej.kherson.ua/journal/economic_17/1/41.pdf [in Ukrainian].

Malinetskiy, G. G. (2005). Matematicheskiye osnovy sinergetiki: khaos, struktury, vychislitel'nyy eksperiment *[Mathematical foundations of synergetics: chaos, structures, computational experiment]*. Moscow: Kom-Kniga [in Russian].

Mangan, N. M., Brunton, S. L., Proctor, J. L., & Kutz, J. N. (2016). Inferring Biological Networks by Sparse Identification of Nonlinear Dynamics. *IEEE Transactions on Molecular, Biological and Multi-Scale Communications,* 2(1), 52–63. doi:10.1109/tmbmc.2016.2633265.

Mangan, N. M., Kutz, J. N., Brunton, S. L., & Proctor, J. L. (2017). Model selection for dynamical systems via sparse regression and information criteria. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences, 473*(2204), 20170009. doi:10.1098/rspa.2017.0009.

Mangra, M. G., Stanciu, M., & Mangra, G. I. (2019). Insurance market: competition and opportunities. *Annals of the University of Craiova, Economic Sciences Series,* 2(48). 118-125. Retrieved from: https://ideas.repec.org/a/aio/aucsse/v2y2019i47p135-142.html.

Melnychuk, N. Yu. (2015). Teoretychni pidkhody rozuminnya sutnosti definitsiyi «bank» y vyznachennya yoho roli v ekonomichniy sferi [Theoretical approaches to understanding the essence of the definition of «bank» and defining its role in the economic sphere]. *Global and national economic problems*, 4, 856-860. Retrieved from: http://global-national.in.ua/archive/4-2015/182.pdf [in Ukrainian].

Milovanov, V.P. (2001). *Neravnovesnyye sotsial'no-ekonomicheskiye sistemy: sinergetika i samoorganizatsiya* [*Unbalanced socio-economic systems: synergy and self-organization]*. Moscow: Editorial URSS [in Russian].

Prikazyuk, N., & Oliynik, G. (2017). The impact of cooperation between insurers and banks on the development of the insurance system. *Baltic Journal of Economic Studies, 3*(2), 121–127. doi:10.30525/2256-0742/2017-3-2-121-127.

Prymostka, L., Krasnova, I., Kulish, G., Nikitin, A., & Shevaldina, V. (2020). Modeling the segment interactions of Ukraine's financial market. *Investment Management and Financial Innovations, 17*(2), 101–112. doi:10.21511/imfi.17(2).2020.09.

Schaeffer, D. G., & Cain, J. W. (2016). Linear Systems with Constant Coefficients. *Texts in Applied Mathematics*, 41–78. doi:10.1007/978-1-4939-6389-8_2

Vitlinskyi, V., Kolyada, Yu., & Kharlamov, A. (2012). Neliniyna dynamika protsesu funktsionuvannya komertsiynoho banku na pidgrunti neperervnoyi matematychnoyi modeli [Nonlinear dynamics of the process of functioning of a commercial bank based on a continuous mathematical model]. *Business Inform*, 3, 29-34. Retrieved from: https://www.business-inform.net/export_pdf/business-inform-2012-3_0-pages-29_34.pdf [in Ukrainian].

Vygovska, N., Polchanov, A., & Vygovskyi, V. (2018). Modelyuvannya vplyvu bankivs'kykh ustanov na rozvytok strakhovykh kompaniy [Modeling of the influence of banking insnitutions on development of insurance companies]. *Black sea economic studies,* 28(2), 97-100. Retrieved from: http://bses.in.ua/journals/2018/28_2_2018/22.pdf [in Ukrainian].