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UNIVERSAL CRITICAL QUANTUM PROPERTIES OF CUPRATE SUPERCONDUCTORS

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Using the scaling theory of quantum critical phenomena we explore the occurrence of universal critical behavior at the insulator-to-superconductor and superconductor-to-normal state transitions at zero temperature. Experimentally, these phase transitions are driven by doping and correspond to critical end points of the phase transition line in the temperature–hole concentration plane. Provided that the order parameter is a complex scalar in two dimensions, and that the London relation between superfluid number density and magnetic penetration depth holds, the scaling theory predicts universal behavior close to the insulator-to-superconductor transition. In particular, transition temperature and zero temperature penetration depth are universally related and the sheet resistance adopts a universal value. These predictions agree remarkably well with available experimental data and provide useful constraints for a microscopic theory.

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Many physical properties of cuprate superconductors depend on hole doping. The generic doping dependence of the transition temperature T_c is depicted in Fig. 1. At a certain doping level, the so-called underdoped limit, these materials undergo at the hole concentration x_u and $T > 0$ a transition from insulator to anomalous metal, and at $T = 0$ a transition from insulator to superconductor. As x is increased, T_c rises and reaches its maximum at x_m. This behavior is shared by many cuprates, including $YBa_2Cu_3O_{7-\delta}$ [1], $La_{2-x}Sr_xCuO_{4-\delta}$ [2], $Y_2Ba_4Cu_7O_{15+\delta}$ [3], $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ [4], $Tl_2Ba_2CuO_{6+\delta}$ [5], $Bi_2Sr_2CuO_{6+\delta}$, $Bi_2Sr_2CaCu_2O_{8+\delta}$ [6], and $HgBa_2CuO_{4+\delta}$ [7]. In some compounds a further increase in the doping level leads to more metallic normal-state properties [8, 9], but T_c decreases and vanishes in the overdoped limit x_o. Here, these materials undergo at $T = 0$ a superconductor-to-normal-state transition. A prominent example is $La_{2-x}Sr_xCuO_{4-\delta}$ [2].

Another essential and doping-dependent property is the effective mass anisotropy, characterized by $\gamma = \sqrt{M_\perp/M_\parallel}$. In optimally doped ($x = x_m$) $YBa_2Cu_3O_{6.97}$ [10], $La_{2-x}Sr_xCuO_4$ [11, 12], and $Bi_2Sr_2CaCu_2O_8$ [13], γ adopts

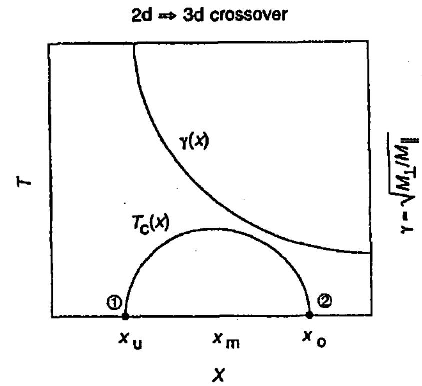

Fig. 1. Schematic phase diagram of cuprate superconductors. At $T = 0$ and with increasing hole concentration x the materials undergo a $2d$ to $3d$ crossover, at x_u an insulator–superconductor and at x_o a superconductor-to-normal state transition. End point 1 denotes the $2d$ quantum-critical point of the insulator-to-superconductor transition and 2 — the anisotropic $3d$ quantum-critical point of the superconductor-to-normal state transition. 1 and 2 are the end points of the phase transition line $T_c(x)$. $\gamma = \sqrt{M_\perp/M_\parallel}$ is the effective mass anisotropy.

the values 5, 20, and > 150, respectively. Moreover, in both $La_{2-x}Sr_xCuO_4$ [12] and $YBa_2Cu_3O_{6.97}$ [10], γ becomes very large by approaching the underdoped limit. Noting then that $\gamma \to \infty$ represents the two-dimensional ($2d$) limit, it becomes evident that $2d$ fluctuations dominate in the underdoped limit, whereas with increasing doping level a $2d$–$3d$ crossover occurs. Nevertheless, for sufficiently large γ values, the regime where $3d$ fluctuations dominate might be experimentally inaccessible.

As there is a phase transition line with two end points at $T = 0$ (1 and 2), a $2d$ to $3d$ crossover, and strong evidence of the importance of critical fluctuations at finite temperature [14, 15], it seems natural to explore the problem of a consistent description of the doping dependence of various superconducting properties with the scaling theory of quantum critical phenomena [16]. Using this approach we show that at the critical end point 1 the relations

$$T_c(\delta) = \frac{\Phi_0^2}{Q_0 16\pi^3 k_B} \frac{d_s}{\lambda_\parallel^2(\delta, T=0)}, \quad \tilde{\rho} = \frac{\rho}{d_s} = \frac{1}{\sigma_0}\frac{h}{4e^2} = \frac{1}{\sigma_0} 6.5\,\mathrm{k\Omega} \tag{1}$$

are universal, provided that the order parameter is a complex scalar in $d = 2$ and the London relation between superfluid number density and magnetic penetration depth holds. The parameter $\delta = x - x_u$ measures the distance from the critical point 1, d_s is the thickness of the $d = 2$ superconducting unit, $\lambda_\parallel(T = 0)$ — the magnetic penetration depth parallel to the layers, and ρ — the sheet resistance. Q_0 and σ_0 are constants which adopt model-dependent universal values. The finite-temperature analog of the relation between $T_c(\delta)$ and $\lambda_\parallel^{-2}(\delta, T = 0)$ is $k_B T_{KT} = (\Phi_0^2/8\pi^2) d_s \lambda_\parallel^{-2}(T_{KT})$, yielding the universal Nelson–Kosterlitz jump [17] for $\lambda_\parallel^{-2}(T)$ at the Kosterlitz–Thouless transition temperature

T_{KT} in $d = 2$. In Fig. 2 we depicted μSR data and magnetic measurements in terms of T_c versus $1/\lambda_{\parallel}^2(T \to 0)$ of $La_{2-x}Sr_xCuO_4$ [18, 19], $YBa_2Cu_3O_{7-\delta}$ [1], and $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ [4]. As T_c approaches the critical end point 1 (Fig. 1), the data merges into the single line

$$T_c \approx 3.57 \times 10^8 \frac{1}{\lambda_{\parallel}^2(T=0)} \tag{2}$$

with T_c in units of K and $\lambda_{\parallel}(0)$ in Å. Thus, as far as superconductivity is concerned, the bulk behaves in the limit $x \to x_u$ like a stack of uncoupled $2d$ units of thickness $d_s \approx Q_0 \times 5.7$ Å.

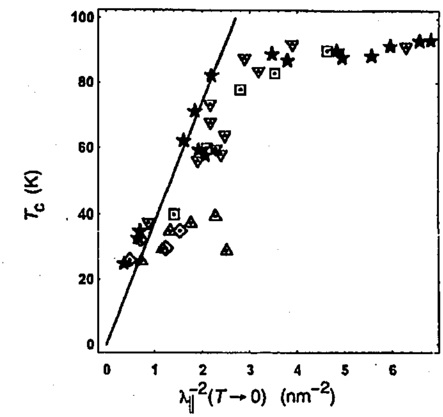

Fig. 2. T_c versus $\lambda_{\parallel}^{-2}(T \to 0)$ as obtained from μSR and magnetic measurements: $La_{2-x}Sr_xCuO_4$: $\diamond$, mag, Δ, μSR [14, 18, 19]; $YBa_2Cu_3O_{7-\delta}$: ∇, mag, $*$, μSR [1, 14]; $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$: $\Box$, μSR [4].

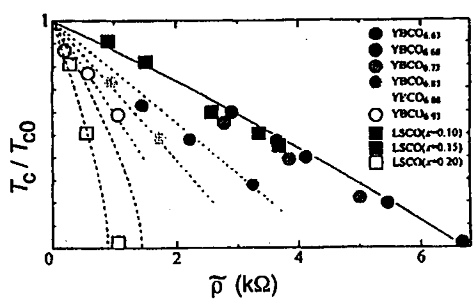

Fig. 3. Normalized critical temperature versus in-plane $2d$ residual resistance taken from Ref. [20]. The solid curve is a line at which all the data in the underdoped regime merge. T_{c0} denotes the transition temperature of the Zn-free compound.

There is also considerable experimental evidence that the sheet resistance tends to a universal value at the insulator-to-superconductor transition 1. Figure 3 shows T_c plotted versus the in-plane $2d$ resistance per CuO_2 plane for Zn-substituted $YBa_2Cu_3O_{7-\delta}$ and $La_{2-x}Sr_xCuO_4$ [20]. For the underdoped materials the data collapses onto the solid line, which tends in the limit $T_c \to 0$ to 6.5 kΩ. Thus, at the insulator-to-superconductor transition 1 the underdoped compounds exhibit, in agreement with Eq. (1), true metallic and universal resistance. In contrast, the higher-doped cuprates show quite distinct T_c versus ρ curves, which are strongly dependent on the doped-hole concentration. Closely related behavior has been found in $YBa_2Cu_3O_{7-\delta}$ as the film was thinned down by ion bombardment [21] and in ultrathin $DyBa_2Cu_3O_{7-\delta}$ films [22]. Superconductivity was found to disappear roughly at the sheet resistance 6.5 kΩ.

We have seen that the predictions of the scaling theory of quantum critical phenomena in $d = 2$ (Eq. (1)) agree remarkably well with available experimental data. Noting that this formalism, supplemented by the critical properties of end points 1 and 2, also describes the asymptotic doping dependence, one expects this approach to provide a coherent description of the doping dependence of the superconducting properties. To substantiate this conjecture we begin, following Ref. [16], with a short sketch of the scaling theory of quantum critical phenomena. This also serves to derive Eq. (1) and relations used later on.

At $T = 0$ a phase transition is driven by quantum fluctuations through parameters such as the chemical potential. Using this control parameter, the distance from the critical point is measured in terms of δ. At $T = 0$ and small δ, one defines two correlation lengths, via the rate of decay of the Matsubara–Green function: the usual spatial correlation length in the disordered phase

$$\xi = \xi_0^{+}|\delta|^{-\bar{\nu}} \tag{3}$$

and the temporal correlation length

$$\xi_\tau = \xi_{\tau 0}^{\pm}|\delta|^{-\nu_\tau}. \tag{4}$$

The dynamic critical exponent z is defined by the ratio

$$z = \frac{\nu_\tau}{\bar{\nu}}. \tag{5}$$

Indeed, the inherent quantum dynamics determines the temporal evolution, which is generally different from the spatial evolution. The characteristic frequency scale Ω is determined from the dynamics as $\Omega \propto |\xi|^{-z} \propto |\delta|^{-z\bar{\nu}}$. Analogous to the classical hyperscaling expression for the free-energy density, one requires that the singular part of the free-energy density f_s in the correlation volume, $f_s \prod_{i=1}^{d} \xi_i \xi_\tau$, is universal when $|\delta| \to 0$. The parameter ξ_i denotes the spatial correlation length in direction $(i = (||, \perp))$. Accordingly

$$f_s \propto \prod_{i=1}^{d} \xi_i^{-1}\xi_\tau^{-1} \propto |\delta|^{2-\bar{\alpha}} = |\delta|^{\bar{\nu}(d+z)}. \tag{6}$$

This yields the generalized hyperscaling relation

$$2 - \bar{\alpha} = \bar{\nu}(d + z). \tag{7}$$

Note that $\bar{\alpha}$ is defined via $-\partial^2 f_s/\partial\delta^2 = (A_\pm/\bar{\alpha})|\delta|^{-\bar{\alpha}}$. The occurrence of superfluidity is conveniently described in terms of the free energy density in the presence of an imposed order-parameter twist with wave vector k_i. The extension of Eq. (6) is

$$f_s \propto |\delta|^{2-\bar{\alpha}}\Phi^{-}\left(C_i k_i |\delta|^{-\bar{\nu}}, \ldots\right) \tag{8}$$

from which one derives for the helicity modulus

$$Y_i = \left.\frac{\partial^2 f_s}{\partial k_i^2}\right|_{k_i=0} \propto |\delta|^{2-\bar{\alpha}-2\bar{\nu}}. \tag{9}$$

The related transverse correlation length is then defined as

$$\xi_i{}^{\mathrm{T}} = (Y_i \xi_\tau)^{1/(2-d)} = \xi_{i0}^{\mathrm{T}} \delta^{-\bar{\nu}}. \tag{10}$$

Equation (8) can be extended to finite temperatures and finite frequency. The temperature and frequency-dependent helicity modulus is then defined in terms of the temporal Fourier transform of the current–current correlation function. The resulting scaling form is

$$Y_i(\delta, T, \omega) \propto |\delta|^{2-\bar{\alpha}-2\bar{\nu}} Y_i \left(C_i \hbar\omega |\delta|^{-z\bar{\nu}}, D_i \beta^{-1} |\delta|^{-z\bar{\nu}}\right). \tag{11}$$

We now assume that there is a line of finite temperature phase transitions $T_c(\delta)$ ending at $T = 0$ and $\delta = 0$. The scaling form (Eq. (11)) taken at $\omega = 0$ then reveals that

$$k_B T_c \propto D|\delta|^{z\bar{\nu}}. \tag{12}$$

Moreover, combining Eqs. (11) and (12), it follows that

$$\lim_{\delta \to 0} \frac{Y_i(\delta)}{k_B T_c(\delta)} \propto |\delta|^{2-\bar{\alpha}-2\bar{\nu}-z\bar{\nu}} \propto \xi_i^{2-d} \tag{13}$$

is a universal number in $d = 2$, namely

$$\lim_{\delta \to 0} \frac{Y_i(\delta)}{k_B T_c(\delta)} = Q_0. \tag{14}$$

Clearly, Q_0 will adopt a characteristic value within a universality class. Using the relation between the helicity modulus and the superfluid aerial density in $d = 2$ (pairs per unit area), as well as the relation between aerial superfluid density and bulk penetration depth the helicity modulus reduces to

$$Y_{\|}(\delta, T = 0) = \frac{\hbar^2 \tilde{n}_s}{M_{\|}} = \frac{\Phi_0^2}{16\pi^3} \frac{d_s}{\lambda_{\|}^2(\delta, T = 0)}, \tag{15}$$

where d_s is the thickness of the superconducting units in $d = 2$. Combining then Eqs. (14) and (15) we recover the universal relation given in Eq. (1). For $d \neq 2$, however, Eqs. (12) and (13) lead to

$$Y_i(\delta, T = 0) \propto T_c(\delta)^{(\delta+z-2)/z} \propto \frac{\Phi_0^2}{16\pi^3} \frac{1}{\lambda_{\|}^2(\delta, T = 0)} \tag{16}$$

with a nonuniversal coefficient of proportionality. It is important to emphasize that relation (14) holds generally for a system where the order parameter is a complex scalar, whereas relations (1) and (16) require that the London relation between the superfluid number density and the magnetic penetration depth holds.

The frequency-dependent conductivity is given by

$$\sigma(\delta, T, \omega) = \frac{4e^2}{h} \frac{Y(\delta, T, -\mathrm{i}\omega)}{-\mathrm{i}\hbar\omega}, \tag{17}$$

where $2e$ is the charge of the pair. Using Eq. (11) and approaching the critical point at $T = 0$, $\omega = 0$, and $\delta = 0$ along a path in such a way that $C_i \hbar\omega |\delta|^{-z\bar{\nu}} =$

x, $y = D_i\beta^{-1}|\delta|^{-z\bar{\nu}}$ approach some fixed value x_0 and y_0, one finds with the aid of the hyperscaling relation (7)

$$\sigma_{\text{sing}} = \frac{4e^2}{h}(\xi)^{-(d-2)}\sigma_0 \tag{18}$$

and the resistance should satisfy the scaling relation

$$\rho(\delta, T) = (\xi)^{d-2}\frac{h}{4e^2}Q\left(\frac{b\delta}{T^{1/z\nu}}\right) \tag{19}$$

yielding for $d = 2$ the limiting universal resistivity given by Eq. (1).

We have seen that the scaling theory yields useful relations between critical exponents and discloses universal relations between critical amplitudes. It does not provide, however, the values of the critical exponents and the universal constants. This is because the universal scaling functions, the critical exponent relation, etc. are satisfied regardless of the particular universality class. A characteristic feature of the universality class are the critical exponents. Assuming that the distance from the critical point 1 is measured by $\delta = x - x_{\text{u}}$ we obtain from Eqs. (7), (12), (13) and (15) in $d = 2$ for the asymptotic behavior

$$\lambda_{\|}^2(T=0) = \lambda_{\|\text{u}}^2(T=0)(x - x_{\text{u}})^{-z\bar{\nu}}, \qquad T_{\text{c}} = r_{\text{u}}(x - x_{\text{u}})^{z\bar{\nu}}. \tag{20}$$

Equations (14) and (15) require that the critical amplitude combination

$$\frac{r_{\text{u}}\lambda_{\|\text{u}}^2}{d_{\text{s}}} = \frac{\Phi_0^2}{16\pi^3 Q_0} \tag{21}$$

be universal. The available experimental data for the doping dependence of T_{c} [2, 20] and $\lambda_{\|}(T = 0)$ [18, 19] is shown in Figs. 4 and 5 for $La_{2-x}Sr_xCuO_4$. It is too sparse, however, to estimate $z\bar{\nu}$, the critical amplitudes ($\lambda_{\|\text{u}}, r_{\text{u}}$), and the location of the critical point (x_{u}). For this reason we explore the consistency with the generic insulator-to-superconductor transition of preformed pairs, where $z\bar{\nu} = 1$, $z = 2$, and $\bar{\nu} = 1/2$ in all dimensions [23]. For $x_{\text{u}} = 0.05$ the dashed lines in Figs. 4 and 5 correspond to the critical amplitudes $r_{\text{u}} \approx 880$ K and $\lambda_{\|\text{u}}^2 \approx 3.84 \times 10^5$ Å^2, yielding for the universal prefactor in Eq. (1) the estimate 3.38×10^8, which is close to the value derived from μSR data (Eq. (2)). To strengthen this analysis of the rather sparse experimental data, we note that the exponents $z\bar{\nu} = 1$, $z = 2$, and $\bar{\nu} = 1/2$ describe the insulator-to-superconductor transition of preformed pairs. A characteristic feature of this transition is that at finite temperature, close to x_{u} and below a mean-field transition temperature, the electrons form pairs, but the true transition temperature T_{c}^{MF} is lowered by the enhanced phase fluctuations in $2d$. Consequently, pairs are expected to occur for $T < T_{\text{c}}^{\text{MF}}$ and the effects of pairing should then manifest themselves as a suppression of low-energy excitations, that is, in terms of a gap. The existence of such a gap in underdoped cuprates, opening below T_{c}^{MF} is well confirmed by a variety of measurements, including NMR [24], optical conductivity [25], and angular resolved photoemission [26].

To present our final application we speculate briefly about applying the scaling theory of quantum critical phenomena to the superconductor-to-normal-state transition 2 (Fig. 1). The experimental evidence of more isotropic and metallic normal-state properties in the overdoped regime [8, 9, 12] indicates that in this doping regime the two aforementioned energy scales, the binding energy of the

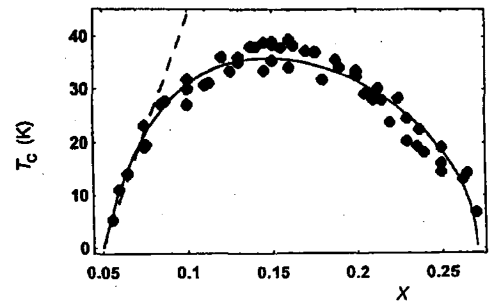

Fig. 4. T_c versus x for $La_{2-x}Sr_xCuO_4$ taken from Refs. [2, 12, 20]. The dashed line is a fit to Eq. (20) yielding for the critical amplitude the estimate $r_u \approx 880$ K. The solid curve is a guide to the eye.

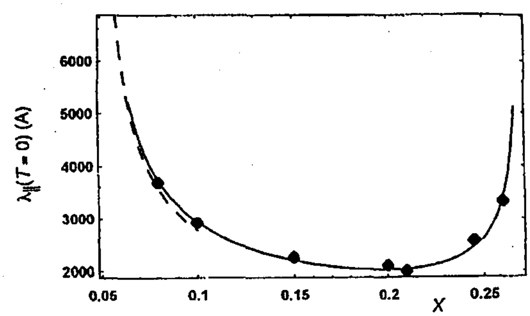

Fig. 5. $\lambda_{\|}(T=0)$ versus x for $La_{2-x}Sr_xCuO_4$ derived from μSR data of Refs. [18, 19]. The dashed line is a fit to Eq. (20) yielding for the critical amplitude the estimate $\lambda_{\|u}^2 \approx 3.84 \times 10^5$ Å^2. The solid curve is a guide to the eye.

pairs and the condensation energy approach one another. For these reasons a BCS–Eliashberg type of superconductivity might be expected. It appears nonetheless likely that the asymptotic critical properties of transition 2 are insensitive to the fermionic degrees of freedom. Indeed, the critical points along the phase transition line $T_c(x)$ and Bose condensation appear to belong to the same universality class $(3d - xy)$ [14, 15], the difference between the normal bosonic and fermionic phases notwithstanding. Thus, we expect that the critical properties along the phase transition line $T_c(x)$and at the $3d$ critical end point can be properly described in terms of neutral bosons. Relation (16) then reduces to

$$T_c(x_o - x) \propto \left[\frac{1}{\lambda_{\|}^2(x_o - x, T = 0)}\right]^{z/(1+z)} \tag{22}$$

with a nonuniversal coefficient of proportionality. Moreover, analogous to Eq. (20), the asymptotic doping dependence of T_c and the zero temperature penetration depth $\lambda_{\parallel}(T=0)$ obey

$$\lambda_{\parallel}^{2}(T=0)=\lambda_{\parallel 0}^{2}(T=0)(x_{\mathrm{o}}-x)^{-z\bar{\nu}}, \qquad T_{\mathrm{c}}=r_{\mathrm{o}}(x_{\mathrm{o}}-x)^{z\bar{\nu}}. \tag{23}$$

Unfortunately, even for $La_{2-x}Sr_xCuO_4$ (Figs. 4 and 5) the available experimental data is sparse, and reliable estimates for the location of the critical end point as well as for the associated exponents and amplitudes cannot be extracted. From Fig. 6, which shows T_c versus $\sigma(T=0)\propto\lambda_{\parallel}^{-2}(T=0)$ for $La_{2-x}Sr_xCuO_{4-\delta}$, $HgBa_2CuO_{4+\delta}$, and $Tl_2Ba_2CuO_{6+\delta}$ [27,28], where the overdoped regime is accessible, there is some evidence of the nonuniversal behavior predicted by Eq. (22). Indeed, by approaching the critical end point 1, the data of $La_{2-x}Sr_xCuO_{4-\delta}$ and $HgBa_2CuO_{4+\delta}$ merge on the line describing the limiting universal behavior close to the $d=2$ insulator-to-superconductor transition, whereas the data for overdoped materials indicates that the approach to end point 2 is nonuniversal.

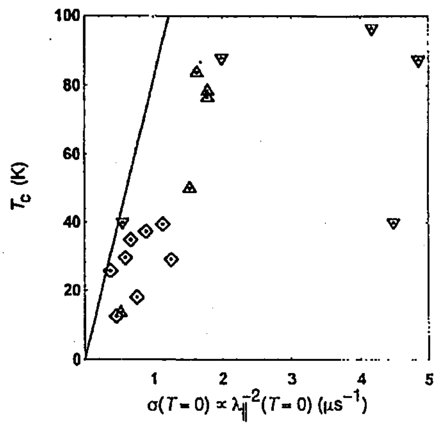

Fig. 6. T_c versus $\sigma(T=0)\propto\lambda_{\parallel}^{-2}(T=0)$ for $La_{2-x}Sr_xCuO_{4-\delta}$ [18, 19] : $\diamond$, $HgBa_2CuO_{4+\delta}$ [7] : ∇ and $Tl_2Ba_2CuO_{6+\delta}$ [27, 28]: Δ. The solid line corresponds to the limiting universal behavior of Eq. (1) at the insulator-to-superconductor transition.

The considerations presented here are macroscopic and independent of the underlying pairing mechanism. However, some constraints on a microscopic theory emerge. (1) The order parameter of the superconducting state appears to be well described by a complex order parameter with an amplitude and phase. (2) The agreement between experimental data and the universal relations (1) clearly reveal that phase fluctuations are essential and predominant close to the insulator-to--superconductor transition 1. For this reason the BCS and Eliashberg theories, where the phase of the order parameter is unimportant for determining the value of T_c, are not applicable. (3) The consistency with the bosonic insulator-to-superfluid transition at end point 1 to preformed pairs in the underdoped regime and pairing is then manifested in terms of a gap below T_c^{MF} which is well confirmed by a variety of measurements. (4) As the doping level rises, the materials undergo a $2d$- to anisotropic $3d$ crossover. (5) The experimental evidence for metallic normal-state properties in the overdoped regime requires that T_c and T_c^{MF} approach one another. For these reasons phase fluctuations are no longer predominant and the BCS–Eliashberg mean-field theory might apply. In any case, close to the critical end point 2 the materials are anisotropic but 3*d*, which renders the relation between T_c, and the zero temperature penetration depth nonuniversal.

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