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Interval Type-2 fuzzy Exponentially Weighted Moving Average Control Chart

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ABSTRACT

Some industrial data often come with uncertainty, which in some cases depends on the decision of those responsible for taking the measurement in the production process. While the fuzzy approach helps to tackle the ambiguity that arises in the measurement, an interval type-2 fuzzy set deals with such uncertainty better due to its flexibility over the control limits of its control chart. This paper aims to develop an Interval Type-2 fuzzy Exponentially Weighted Moving Average Control Chart (IT2FEWMA) under the fuzzy type-2 condition. This development will facilitate monitoring small and moderate shifts in the production process in conditions of uncertainty.

Key words: Exponentially weighted moving average control chart, Fuzzy control chart, Fuzzy sets, Interval Type-2 fuzzy sets, Interval Type-2 fuzzy Exponentially Weighted Moving Average Control Chart, Statistical process control.

1. Introduction

The control chart scheme is made up of variable and attribute control charts, the former of which constitutes observations with continuous random variable while the later constitutes observations of a discrete random variable. The exponentially weighted moving average (EWMA) control chart was introduced by Roberts, (1959). Many studies have then been conducted to extend this methodology. The EWMA chart is known for its sensitivity on a small shift in the process mean. In classical statistical process control chart, data are defined in crisp value. However, gage,

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environment conditions, operator's discretion, which are the measurement determinants, can collect the measurement with "vagueness" or "uncertain". These uncertainties about the measurements lead to difficult challenges while attempting to obtain a crisp result from the process. Human discretion is one of the significant factors used in defining the quality characteristics. However, data from such discretion are bound with uncertainty and direct application of the classical control charts on such data may require further information and transformation of the classical control chart limits and the data points. In such a situation, evaluation of fuzzy data is best achieved using fuzzy control charts as a tool. The fuzzy set theory was introduced by Zadeh (1965). Observations with uncertainty are handled mathematically with the fuzzy logic. A number of publications with many applications in Statistical Process Control (SPC) integrated with the fuzzy system has gained more efforts from many authors. The amalgamation of different SPC tools with the type-1 fuzzy theory, intuitionistic fuzzy theory, hesitant fuzzy theory, neurosophic fuzzy theory, type-2 fuzzy theory, and recently, the interval type-2 fuzzy theories have come into existence in recent publications. However, no publication exists in Interval Type-2 Exponentially Weighted Moving Average (IT2FEWMA) control chart. This paper is designed in a fuzzy environment for three-dimensional data, that is, the upper membership value, lower membership value and their respective representative value. The principal contribution of this research is to develop the theoretical foundation of the Interval Type-2 fuzzy Exponentially Weighted Moving Average Control Charts (IT2FEWMA) and their application.

2. Literature review

In a real life situation, vagueness and uncertainty occur as a result of human error usually due to judgmental decision based on qualitative measures, such as the weather is hot or too hot or cold is based on qualitative measurement which might not be presented with exact value. In most cases, one tends to ask what degree of the hotness or coldness of the weather is. Zadeh (1965) proposed the conceptual idea of the fuzzy set theory. He proposed the type-1 fuzzy sets, with a degree of membership called crisp membership value, whose values are over the range 0 to 1. He also proposed the type-2 fuzzy sets, which is an extension of the type-1 fuzzy sets. This type-2 fuzzy set is three dimensional, that is, it comprises two membership functions, thus, the upper membership function and the lower membership function and the representative values. The conception of the idea of the fuzzy control chart and application was firstly documented by Raz and Wang (1990), and Wang and Raz (1990). A considerable number of authors made various contributions to the extensional development in this area of research, which include but are not limited to Kanagawa *et al.* (1993), El-shal and Moris (2000), Rowlands and Wang (2000), Gulbay *et al.* (2004), Karnik and Mendel (2001), Mendel and John (2002), Cheng (2005), Gulbay and Kahraman (2006a), Mendel *et al.* (2006), Erginel (2008), Senturk and Erginel (2009), Senturk (2010), Senturk *et al.* (2010), Kaya and Kahraman (2011), Erginel *et al.* (2011), Senturk *et al.* (2014), Erginel (2014), Poongodi and Muthulakshmi (2015), Cervantes and Castillo (2015), Wang and Hyrniewicz (2015), Edmundas *et al.* (2015), Castillo *et al.* (2016), Chen and Huang (2016), Castillo *et al.* (2016), Hou *et al.* (2016), Kaya *et al.* (2017), Senturk and Antucheviciene (2017), Erginel *et al.* (2018), Ontiveros-Robles *et al.* (2018), Adepoju (2018), Ercan and Anagun (2018), Adepoju *et al.* (2019a), Adepoju *et al.* (2019c), and Adepoju *et al.* (2019b).

3. Methodology

3.1. Type-2 fuzzy sets and Interval type-2 fuzzy sets

Definition 1. A type-2 fuzzy set (T2 FS) denoted by $\tilde{\tilde{A}}$ in a universe of discourse is characterized by a type-2 membership function given as $\mu_{\tilde{\tilde{A}}}(x,u)$, where $x \in X$ and $u \in J_x \subseteq [0,1]$. Mathematically, this can be expressed as

$$\tilde{\tilde{A}} = \left\{ \left((x,u), \mu_{\tilde{\tilde{A}}}(x,u) \right) \middle| \forall x \in X, \forall u \in J_x \left[0,1\right], 0 \le \mu_{\tilde{\tilde{A}}}(x,u) \le 1 \right\}$$

where J_x denotes an interval $\left[0,1\right]$. This type-2 fuzzy set can be expressed as

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{\tilde{A}}}(x,u) / (x,u),$$

where $\int\int$ denotes union over all admissible x and u as given by Mendel *et al.* (2006) as well as Kahraman (2014).

3.2. Interval type-2 fuzzy sets

An interval type-2 fuzzy sets (IT2 FS) also known as closed interval type-2 fuzzy set (CIT2 FS) can be defined as a special case of type-2 fuzzy set $\tilde{\tilde{A}}$ represented by the type-2 membership function $\mu \approx (x,u)$. If all $\mu \approx (x,u) = 1$. It follows that the interval type-2 fuzzy set is expressed as

$$\tilde{\tilde{A}} = \int_{x \in X} \int_{u \in J_x} 1 / (x,u)$$

where $J_x \subseteq [0,1]$, Ghorabaee *et al.* (2016),

4. The proposed Interval Type-2 fuzzy Exponentially Weighted Moving Average Control Chart.

4.1. Exponentially Weighted Moving Average Control Chart (EWMA)

Exponentially Weighted Moving Average Control Chart (EWMA) was introduced by Roberts (1959). It is a better option to Shewhart control chart based on its sensitivity to a small shift in the process mean. It uses both current and historical data to monitor any slight shift in the mean of the process and its statistic is expressed as

$$z_i = \lambda \bar{X}_i + (1-\lambda) z_{i-1}$$

where z_i and $\bar{X}_i$ denote the ith exponentially weighted moving average and ith sample average respectively, and $i = 1, 2, 3, \ldots, k$, λ is the smoothing constant and it is given as $0 < \lambda < 1$. The starting value being the first sample mean is the process target such that $z_0 = \bar{\bar{X}}$, where $\bar{\bar{X}}$ is the grand mean.

The classical EWMA control chart limits are established as follows.

$$\begin{aligned} UCL &= \mu_0 + L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]} \\ CL &= \mu_0 \\ LCL &= \mu_0 - L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]} \end{aligned} \quad (1)$$

where UCL, CL and LCL are upper control limit, centre line and lower control limit respectively, ***L*** is the width of the control limits, λ is the smoothing constant and σ is the standard deviation.

If σ^2/n is the variance of $\bar{X}_i$ independent random variables drawn from a population with known standard deviation σ , with sample size n , then
for a small sample number *i*

$$\sigma_{zi}^2 = \sigma^2/n\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2i}\right],$$

for a moderate and large sample number *i*

$$\sigma_{zi}^2 = \sigma^2/n\left(\frac{\lambda}{2-\lambda}\right).$$

For a moderate and large sample size n, the statistic below can be used to obtaine the estimates of the upper control limit, center line and the lower control limit of the classical EWMA control chart.

$$UCL_{ewma} = \overline{\overline{X}} + {}^{3\sigma}\!/\!_{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}$$
$$CL_{ewma} = \overline{\overline{X}}$$
$$LCL_{ewma} = \overline{\overline{X}} - {}^{3\sigma}\!/\!_{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}$$

(2)

4.2. Interval Type-2 fuzzy Exponentially Weighted Moving Average Control Chart (IT2FEWMA)

For a large sample number when the σ is known, the IT2FEWMA control limits of the control chart is obtained by fuzzification and expressed as

$$UCL_{IT2FEWMA} = \left(\overline{X}_a^U, \overline{X}_b^U, \overline{X}_c^U, \overline{X}_d^U\right) + \frac{3}{\sqrt{n}}\left(\sigma_a^U, \sigma_b^U, \sigma_c^U, \sigma_d^U\right)\sqrt{\left(\frac{\lambda}{2-\lambda}\right)},$$
$$\left(\overline{X}_a^L, \overline{X}_b^L, \overline{X}_c^L, \overline{X}_d^L\right) + \frac{3}{\sqrt{n}}\left(\sigma_a^L, \sigma_b^L, \sigma_c^L, \sigma_d^L\right)\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}$$
$$= \begin{bmatrix} \overline{X}_a^U + \frac{3\sigma_a^U}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}, \overline{X}_b^U + \frac{3\sigma_b^U}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}, \overline{X}_c^U + \frac{3\sigma_c^U}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}, \\ \overline{X}_d^U + \frac{3\sigma_d^U}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}; \min\left(H_1\left(A_i^U\right), H_2\left(A_i^U\right)\right), \\ \overline{X}_a^L + \frac{3\sigma_a^L}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}, \overline{X}_b^L + \frac{3\sigma_b^L}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}, \overline{X}_c^L + \frac{3\sigma_c^L}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}, \\ \overline{X}_d^L + \frac{3\sigma_d^L}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}; \min\left(H_1\left(A_i^L\right), H_2\left(A_i^L\right)\right) \end{bmatrix}$$

$$CL_{IT2FEWMA} = \begin{pmatrix} \left(\overline{X}_a^U, \overline{X}_b^U, \overline{X}_c^U, \overline{X}_d^U\right); \min\left(H_1\left(A_i^U\right), H_2\left(A_i^U\right)\right), \\ \left(\overline{X}_a^L, \overline{X}_b^L, \overline{X}_c^L, \overline{X}_d^L\right); \min\left(H_1\left(A_i^L\right), H_2\left(A_i^L\right)\right) \end{pmatrix}$$

(3)

$$UCL_{IT2FEWMA} = \left(\bar{X}_a^U, \bar{X}_b^U, \bar{X}_c^U, \bar{X}_d^U\right) - \frac{3}{\sqrt{n}}\left(\sigma_a^U, \sigma_b^U, \sigma_c^U, \sigma_d^U\right)\sqrt{\left(\frac{\lambda}{2-\lambda}\right)},$$

$$\left(\bar{X}_a^L, \bar{X}_b^L, \bar{X}_c^L, \bar{X}_d^L\right) - \frac{3}{\sqrt{n}}\left(\sigma_a^L, \sigma_b^L, \sigma_c^L, \sigma_d^L\right)\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}$$

$$= \begin{bmatrix} \bar{X}_a^U - \frac{3\sigma_a^U}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}, \bar{X}_b^U - \frac{3\sigma_b^U}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}, \bar{X}_c^U - \frac{3\sigma_c^U}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}, \\ \bar{X}_d^U - \frac{3\sigma_d^U}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}; \min\left(H_1\left(A_i^U\right), H_2\left(A_i^U\right)\right), \\ \bar{X}_a^L - \frac{3\sigma_a^L}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}, \bar{X}_b^L - \frac{3\sigma_b^L}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}, \bar{X}_c^L - \frac{3\sigma_c^L}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}, \\ \bar{X}_d^L - \frac{3\sigma_d^L}{\sqrt{n}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}; \min\left(H_1\left(A_i^L\right), H_2\left(A_i^L\right)\right) \end{bmatrix}$$

4.3. Defuzzication of Interval Type-2 fuzzy Exponentially Weighted Moving Average Control Chart (IT2FEWMA)

$$DIT2_{trap(i)}^U = \frac{\left(a_{i4}^U - a_{i1}^U\right) + \left(H_2\left(\tilde{A}_1^U\right)a_{i2}^U - a_{i1}^U\right) + \left(H_1\left(\tilde{A}_1^U\right)a_{i3}^U - a_{i1}^U\right)}{4} + a_{i1}^U$$

$$DIT2_{trap(i)}^L = \frac{\left(a_{i4}^L - a_{i1}^L\right) + \left(H_2\left(\tilde{A}_1^L\right)a_{i2}^L - a_{i1}^L\right) + \left(H_1\left(\tilde{A}_1^L\right)a_{i3}^L - a_{i1}^L\right)}{4} + a_{i1}^L \quad (4)$$

$$DIT2_{trap(i)} = \frac{DIT2_{trap(i)}^U + DIT2_{trap(i)}^L}{2}$$

$$i = 1, 2, 3, \ldots, n.$$

Similarly, from the equation (4) the modified BNP technique is being transformed to the interval type-2 fuzzy exponentially moving average (IT2FEWMA) control chart limits as expressed below.

$$DIT2FEWMA_{trap(i)}^U = \frac{\left(\bar{X}a_{.4}^U - \bar{X}a_{.1}^U\right) + \left(H_2\left(\tilde{A}_1^U\right)\bar{X}a_{.2}^U - \bar{X}a_{.1}^U\right) + \left(H_1\left(\tilde{A}_1^U\right)\bar{X}a_{.3}^U - \bar{X}\,a_{.1}^U\right)}{4} + \bar{X}a_{.1}^U$$

$$DIT2FEWMA^{L}_{trap(i)} = \frac{\left(\bar{X}a^{L}_{.4} - \bar{X}a^{L}_{.1}\right) + \left(H_2\left(\tilde{A}^{L}_{1}\right)\bar{X}a^{L}_{.2} - \bar{X}a^{L}_{.1}\right) + \left(H_1\left(\tilde{A}^{L}_{1}\right)\bar{X}a^{L}_{.3} - \bar{X}a^{L}_{.1}\right)}{4} + \bar{X}a^{L}_{.1} \tag{5}$$

$$DIT2FEWMA_{trap(i)} = \frac{DIT2^{U}_{trap(i)} + DIT2^{L}_{trap(i)}}{2}. \tag{6}$$

$i = 1, 2, 3, \ldots, n$

For the upper control limit:

$$UCL(DIT2FEWMA^{U}_{trap(i)}) = \frac{\left(\bar{X}a^{U}_{.4} - \bar{X}a^{U}_{.1}\right) + \left(H_2\left(\tilde{A}^{U}_{1}\right)\bar{X}a^{U}_{.2} - \bar{X}a^{U}_{.1}\right) + \left(H_1\left(\tilde{A}^{U}_{1}\right)\bar{X}a^{U}_{.3} - \bar{X}\,a^{U}_{.1}\right)}{4} + \bar{X}a^{U}_{.1}$$

$$UCL(DIT2FEWMA^{L}_{trap(i)}) = \frac{\left(\bar{X}a^{L}_{.4} - \bar{X}a^{L}_{.1}\right) + \left(H_2\left(\tilde{A}^{L}_{1}\right)\bar{X}a^{L}_{.2} - \bar{X}a^{L}_{.1}\right) + \left(H_1\left(\tilde{A}^{L}_{1}\right)\bar{X}a^{L}_{.3} - \bar{X}a^{L}_{.1}\right)}{4} + \bar{X}a^{L}_{.1}$$

$$UCL(DIT2FEWMA_{trap(i)}) = \frac{UCL(DIT2FEWMA^{U}_{trap(i)}) + UCL(uDIT2^{L}_{trap(i)})}{2}. \tag{7}$$

For the lower control limit:

$$LCL(DIT2FEWMA^{U}_{trap(i)}) = \frac{\left(\bar{X}a^{U}_{.4} - \bar{X}a^{U}_{.1}\right) + \left(H_2\left(\tilde{A}^{U}_{1}\right)\bar{X}a^{U}_{.2} - \bar{X}a^{U}_{.1}\right) + \left(H_1\left(\tilde{A}^{U}_{1}\right)\bar{X}a^{U}_{.3} - \bar{X}\,a^{U}_{.1}\right)}{4} + \bar{X}a^{U}_{.1}$$

$$LCL(DIT2FEWMA^{L}_{trap(i)}) = \frac{\left(\bar{X}a^{L}_{.4} - \bar{X}a^{L}_{.1}\right) + \left(H_2\left(\tilde{A}^{L}_{1}\right)\bar{X}a^{L}_{.2} - \bar{X}a^{L}_{.1}\right) + \left(H_1\left(\tilde{A}^{L}_{1}\right)\bar{X}a^{L}_{.3} - \bar{X}a^{L}_{.1}\right)}{4} + \bar{X}a^{L}_{.1}$$

$$LCL(DIT2FEWMA_{trap(i)}) = \frac{LCL(DIT2FEWMA^{U}_{trap(i)}) + LCL(DIT2FEWMA^{L}_{trap(i)})}{2} \tag{8}$$

For the centre line:

$$CL(DIT2FEWMA^{U}_{trap(i)}) = \frac{\left(\bar{X}a^{U}_{.4} - \bar{X}a^{U}_{.1}\right) + \left(H_2\left(\tilde{A}^{U}_{1}\right)\bar{X}a^{U}_{.2} - \bar{X}a^{U}_{.1}\right) + \left(H_1\left(\tilde{A}^{U}_{1}\right)\bar{X}a^{U}_{.3} - \bar{X}\,a^{U}_{.1}\right)}{4} + \bar{X}a^{U}_{.1}$$

$$CL(DIT2FEWMA^{L}_{trap(i)}) = \frac{\left(\bar{X}a^{L}_{.4} - \bar{X}a^{L}_{.1}\right) + \left(H_2\left(\tilde{A}^{L}_{1}\right)\bar{X}a^{L}_{.2} - \bar{X}a^{L}_{.1}\right) + \left(H_1\left(\tilde{A}^{L}_{1}\right)\bar{X}a^{L}_{.3} - \bar{X}a^{L}_{.1}\right)}{4} + \bar{X}a^{L}_{.1}$$

$$CL(DIT2FEWMA_{trap(i)}) = \frac{CL(DIT2FEWMA^{U}_{trap(i)}) + CL(DIT2FEWMA^{L}_{trap(i)})}{2} \tag{9}$$

This transformation technique above can be used to defuzzify each sample, such that every defuzzified sample point is monitored with the defuzzified control limits. And the criterion for the in-control criteria expressed as

$$LCL(DIT2FEWMA) < DIT2FEWMA_{(i)} < UCL(DIT2FEWMA) \cdot \tag{10}$$

The equation (10) above indicates the in-control process, otherwise the process is out of control.

5. Application of the Interval Type-2 fuzzy Exponentially Weighted Moving Average Control Chart

The application of the IT2FEWMA was conducted by simulating the data. Below is the data with 60 sample points with trapezium fuzzy number of measurement and the upper and lower membership values.

Table 1. Interval type-2 trapezoidal fuzzy number of measurement simulated from the parameters of the existing process data

Sn	X_a^u	X_b^u	X_c^u	X_d^u	$H_1(A_i^u)$	$H_2(A_i^u)$	X_a^l	X_b^l	X_c^l	X_d^l	$H_1(A_i^l)$	$H_2(A_i^l)$
1	3.62	3.81	3.99	4.21	1	1	3.50	3.69	3.90	4.12	0.7	0.5
2	3.61	3.77	3.99	4.18	1	1	3.51	3.69	3.87	4.11	0.8	0.6
3	3.62	3.81	4.00	4.20	1	1	3.50	3.69	3.88	4.11	0.7	0.6
4	3.59	3.81	3.98	4.20	1	1	3.52	3.71	3.89	4.08	0.8	0.6
5	3.60	3.81	4.00	4.20	1	1	3.52	3.70	3.91	4.09	0.6	0.5
6	3.62	3.80	3.99	4.18	1	1	3.49	3.69	3.91	4.08	0.8	0.6
7	3.60	3.78	4.00	4.21	1	1	3.51	3.72	3.90	4.11	0.9	0.8
8	3.59	3.80	3.98	4.19	1	1	3.50	3.71	3.90	4.11	0.7	0.5
9	3.59	3.80	4.00	4.21	1	1	3.49	3.70	3.90	4.09	0.8	0.6
10	3.60	3.80	4.00	4.20	1	1	3.49	3.73	3.92	4.10	0.8	0.7
11	3.61	3.81	3.97	4.21	1	1	3.52	3.69	3.91	4.10	0.6	0.5
12	3.59	3.80	4.01	4.19	1	1	3.49	3.70	3.90	4.09	0.7	0.6
13	3.61	3.79	4.00	4.18	1	1	3.50	3.70	3.91	4.10	0.7	0.5
14	3.60	3.80	3.99	4.22	1	1	3.48	3.69	3.90	4.10	0.8	0.7
15	3.58	3.81	3.99	4.20	1	1	3.50	3.71	3.92	4.11	0.9	0.8
16	3.61	3.80	4.01	4.21	1	1	3.50	3.70	3.89	4.09	0.8	0.7
17	3.62	3.80	4.01	4.20	1	1	3.51	3.72	3.91	4.10	0.9	0.8
18	3.59	3.82	3.99	4.19	1	1	3.50	3.72	3.91	4.10	0.7	0.5
19	3.60	3.80	4.00	4.21	1	1	3.50	3.70	3.91	4.08	0.8	0.6
20	3.60	3.79	4.00	4.21	1	1	3.51	3.70	3.90	4.11	0.7	0.6
21	3.61	3.80	3.98	4.21	1	1	3.50	3.71	3.91	4.12	0.8	0.6
22	3.60	3.80	3.99	4.19	1	1	3.50	3.71	3.90	4.09	0.6	0.5
23	3.60	3.80	4.00	4.21	1	1	3.51	3.71	3.91	4.09	0.8	0.6
24	3.60	3.80	4.00	4.21	1	1	3.51	3.71	3.90	4.11	0.9	0.8
25	3.61	3.79	4.00	4.19	1	1	3.49	3.71	3.93	4.11	0.7	0.5

Table 1. Interval type-2 trapezoidal fuzzy number of measurement simulated from the parameters of the existing process data (cont.)

Sn	X_a^u	X_b^u	X_c^u	X_d^u	$H_1(A_i^u)$	$H_2(A_i^u)$	X_a^l	X_b^l	X_c^l	X_d^l	$H_1(A_i^l)$	$H_2(A_i^l)$
26	3.61	3.81	4.00	4.22	1	1	3.51	3.69	3.90	4.10	0.8	0.6
27	3.59	3.81	4.01	4.19	1	1	3.52	3.70	3.90	4.10	0.8	0.7
28	3.60	3.80	4.01	4.21	1	1	3.51	3.71	3.89	4.09	0.6	0.5
29	3.61	3.81	3.98	4.19	1	1	3.49	3.71	3.90	4.09	0.7	0.6
30	3.61	3.81	4.00	4.20	1	1	3.50	3.69	3.91	4.08	0.7	0.5
31	3.59	3.79	4.00	4.20	1	1	3.50	3.69	3.91	4.11	0.8	0.7
32	3.59	3.80	3.97	4.20	1	1	3.50	3.70	3.91	4.10	0.9	0.8
33	3.58	3.79	4.00	4.20	1	1	3.51	3.70	3.88	4.09	0.7	0.6
34	3.61	3.81	4.00	4.19	1	1	3.50	3.71	3.92	4.09	0.7	0.5
35	3.60	3.81	3.99	4.21	1	1	3.50	3.72	3.90	4.09	0.7	0.5
36	3.61	3.79	4.01	4.20	1	1	3.49	3.70	3.91	4.09	0.8	0.6
37	3.61	3.79	4.01	4.21	1	1	3.49	3.69	3.89	4.09	0.7	0.6
38	3.61	3.80	4.01	4.17	1	1	3.51	3.72	3.90	4.10	0.8	0.6
39	3.60	3.81	3.98	4.21	1	1	3.51	3.70	3.89	4.12	0.6	0.5
40	3.61	3.81	4.00	4.20	1	1	3.50	3.70	3.89	4.10	0.8	0.6
41	3.61	3.81	4.00	4.19	1	1	3.51	3.70	3.91	4.11	0.9	0.8
42	3.60	3.79	4.00	4.20	1	1	3.48	3.70	3.90	4.10	0.7	0.5
43	3.62	3.80	4.00	4.19	1	1	3.51	3.69	3.89	4.10	0.8	0.6
44	3.60	3.80	4.01	4.19	1	1	3.50	3.71	3.91	4.09	0.8	0.7
45	3.61	3.80	3.99	4.20	1	1	3.50	3.71	3.91	4.09	0.6	0.5
46	3.58	3.79	4.00	4.20	1	1	3.49	3.71	3.89	4.08	0.7	0.6
47	3.59	3.78	4.01	4.19	1	1	3.51	3.70	3.88	4.11	0.7	0.5
48	3.59	3.81	3.99	4.19	1	1	3.49	3.69	3.88	4.10	0.8	0.7
49	3.59	3.78	4.00	4.21	1	1	3.51	3.71	3.89	4.11	0.9	0.8
50	3.59	3.79	4.01	4.19	1	1	3.50	3.70	3.90	4.11	0.7	0.5
51	3.60	3.80	4.00	4.20	1	1	3.48	3.70	3.90	4.09	0.8	0.6
52	3.61	3.80	4.00	4.18	1	1	3.50	3.71	3.89	4.10	0.8	0.7
53	3.61	3.81	3.98	4.21	1	1	3.51	3.72	3.91	4.08	0.6	0.5
54	3.61	3.80	4.00	4.20	1	1	3.49	3.68	3.90	4.11	0.7	0.5
55	3.60	3.81	4.00	4.20	1	1	3.49	3.72	3.87	4.11	0.8	0.6
56	3.61	3.81	4.00	4.20	1	1	3.50	3.72	3.89	4.10	0.7	0.6
57	3.60	3.80	3.99	4.20	1	1	3.50	3.72	3.90	4.11	0.8	0.6
58	3.59	3.80	4.00	4.20	1	1	3.50	3.70	3.89	4.08	0.6	0.5
59	3.61	3.79	4.00	4.20	1	1	3.50	3.68	3.89	4.11	0.8	0.6
60	3.59	3.79	3.99	4.20	1	1	3.51	3.71	3.89	4.09	0.9	0.8

5.1. Result and discussion

The fuzzy numbers are modelled with the interval type-2 FEWMA control chart using trapezoidal membership functions.

Table 2. The representation of the result of the interval type-2 FEWMA control chart

UCL_{FEWMA}	3.606533	3.806300	4.006047	4.206225	1	1	3.506508	3.706719	3.906252	4.106251	0.6	0.5
CL_{FEWMA}	3.600000	3.800000	4.000000	4.200000	1	1	3.500161	3.700372	3.899905	4.099904	0.6	0.5
LCL_{FEWMA}	3.593838	3.793660	3.993353	4.193530	1	1	3.493813	3.694024	3.893558	4.093557	0.6	0.5

Table 2 above shows the limits of the FEWMA Control Chart and this include the upper control limit, centre line and lower control limit. This is obtained from equation 3.

Table 3. The representation of the defuzzification values of the IT2FEWMA control chart

$DIT2FEWMA^{U}_{(i)}$	3.906291	$UCL-DIT2FEWMA_{(i)}$	3.823862	$DIT2FEWMA^{L}_{(i)}$	3.741432
$DIT2FEWMA^{U}_{(i)}$	3.899944	$CL-DIT2FEWMA_{(i)}$	3.817514	$DIT2FEWMA^{L}_{(i)}$	3.735085
$DIT2FEWMA^{U}_{(i)}$	3.893597	$LCL-DIT2FEWMA_{(i)}$	3.811167	$DIT2FEWMA^{L}_{(i)}$	3.728737

Table 3 above shows the limits of the diffuzified IT2FEWMA Control Chart and this include the upper control limit, centre line and the lower control limit. It is obtained from equation 7, 8 and 9.

Table 4. The values of the defuzzification of each and every sample and its corresponding decision criteria

$UCL(DIT2FEWMA)$	$DIT2FEWMA_{(i)}$	$LCL(DIT2FEWMA)$	$3.811167 < DIT2FEWMA_{(i)} < 3.823862$
3.900753	3.818206	3.735658	In-control
3.898874	3.816990	3.735105	In-control
3.900749	3.817966	3.735183	In-control
3.899352	3.817340	3.735328	In-control
3.900304	3.818219	3.736135	In-control
3.899481	3.816832	3.734183	In-control
3.900007	3.817989	3.735971	In-control
3.899131	3.817155	3.735178	In-control
3.900071	3.817340	3.734610	In-control
3.899979	3.817546	3.735113	In-control
3.899754	3.817950	3.736145	In-control
3.899773	3.817041	3.734309	In-control
3.899344	3.817188	3.735031	In-control
3.900413	3.817038	3.733664	In-control
3.899769	3.817680	3.735590	In-control
3.900462	3.817564	3.734666	In-control

Table 4. The values of the defuzzification of each and every sample and its corresponding decision criteria (cont.)

$UCL(DIT2FEWMA)$	$DIT2FEWMA_{(i)}$	$LCL(DIT2FEWMA)$	$3.811167 < DIT2FEWMA_{(i)} < 3.823862$
3.900647	3.818358	3.736069	In-control
3.900034	3.817611	3.735188	In-control
3.900183	3.817433	3.734683	In-control
3.899973	3.817875	3.735776	In-control
3.899969	3.817745	3.735522	In-control
3.899457	3.817105	3.734754	In-control
3.900235	3.817901	3.735567	In-control
3.900266	3.818138	3.736009	In-control
3.899793	3.817553	3.735314	In-control
3.900891	3.818070	3.735249	In-control
3.899879	3.817843	3.735806	In-control
3.900303	3.817711	3.735119	In-control
3.899988	3.817285	3.734583	In-control
3.900272	3.817237	3.734202	In-control
3.899423	3.817221	3.735018	In-control
3.899181	3.817042	3.734902	In-control
3.899181	3.817022	3.734862	In-control
3.900298	3.817679	3.735061	In-control
3.900225	3.817659	3.735093	In-control
3.900152	3.817387	3.734623	In-control
3.900463	3.817348	3.734233	In-control
3.899939	3.817813	3.735687	In-control
3.899884	3.817897	3.735911	In-control
3.900545	3.817807	3.735069	In-control
3.90005	3.818053	3.736056	In-control
3.899764	3.817109	3.734455	In-control
3.900386	3.817791	3.735196	In-control
3.899959	3.817511	3.735062	In-control
3.899905	3.817429	3.734952	In-control
3.899195	3.816681	3.734167	In-control
3.899326	3.817301	3.735277	In-control
3.899648	3.816942	3.734237	In-control
3.899549	3.817576	3.735603	In-control
3.899507	3.81727	3.735033	In-control
3.900055	3.816914	3.733773	In-control
3.899708	3.817343	3.734979	In-control
3.900194	3.817774	3.735353	In-control
3.900445	3.817423	3.734401	In-control
3.900095	3.817479	3.734863	In-control
3.900643	3.818006	3.735369	In-control
3.899956	3.817829	3.735702	In-control
3.899821	3.817094	3.734366	In-control
3.900089	3.817403	3.734717	In-control
3.898954	3.817156	3.735358	In-control

Table 4 above show the results obtained from equation (10), which is the difuzzified values set against the difuzzified control limits. The results indicate that the process is in control, since all the sample points are within the control limits.

6. Conclusion

Several publications with great application surfaces in the literature recently on statistical process control incorporated with fuzzy system. Amalgamation of different SPC tools with type-1 fuzzy, intuitionistic fuzzy, hesitant fuzzy, type-2 fuzzy, and recently the interval type-2 fuzzy control chart has come into existence in recent publications. However, no publication exists in Interval Type-2 Exponentially Weighted Moving Average (IT2FEWMA) control chart.

This paper extends the control limits of the classical control chart of the exponentially weighted moving average (EWMA). The IT2FEWMA is advantageous over the classical EWMA due to its flexibility over the control limits, but it is not capable of detecting a big shift in the process due to the fact that classical EWMA does not have such capacity too. This paper is a new addition to the existing Statistical Process Control Tools. It is useful when the process engineer needs to monitor a process whose measurement is obtained in fuzzy environment and a small shift needs to be detected.

Future research: Multivariate IT2FEWMA control chart, interval type-2 intuitionistic FEWMA control chart and interval type-2 Hesitant FEWMA control chart can be developed, also comparative studies can be established between the IT2FEWMA and FEWMA control chart.

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