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Fractional order PI controllers for TCP packet flow ensuring given modulus margins*†

by

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Abstract: An Active Queue Management (AQM) robust control strategy for Traffic Control Protocol (TCP) data transfer is proposed. To this purpose, the TCP behaviour is first approximated by a second–order model with delayed input obtained from the linearization of an efficient and commonly used nonlinear fluid–based model. The adopted feedback control structure uses a fractional–order PI controller. To ensure the desired robustness, the parameter regions where such a controller guarantees a given modulus margin (inverse of the H_∞ norm of the sensitivity function) are derived. An example commonly used in the literature is worked out to show that the suggested graphically–based design technique is simple to apply while it limits the effects of disturbances and of the unmodelled dynamics.

Keywords: fractional-order PI controllers, robust control, TCP congestion control, delayed systems

1. Introduction

Transmission Control Protocol (TCP) is one of the core protocols of the internet protocol suite (Forouzan, 2010). It is used by about 90% of the Internet Protocol (IP) traffic in the Internet. The IP task is to exchange pieces of information called datagrams. Due to network congestion, traffic load balancing, or other unpredictable causes, IP packets can be lost or delivered out of order. TCP

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detects these situations, requests retransmission of lost packets, rearranges out–of–order packets, and then passes the restored data to the application program, thus separating the application's communication from networking details.

Network congestion depends on the limitation of all network resources, such as router processing time and link throughput. To avoid, at least in part, this problem, core routers mark, or even drop, TCP packets with the objective of managing network utilization and queuing delay. This task is called Active Queue Management (AQM) (Low, Paganini and Doyle, 2002; Chatranon, Labrador and Banerjee, 2004). Various AQM algorithms have been proposed and applied, from the first Random Early Detection (RED) (Floyd and Jacobson, 1993; Hollot et al., 2001) method to the more robust and reliable Proportional plus Integral (PI) control that can be run with its default parameters in most circumstances.

To enable the application to AQM of the control engineering principles, such as the PI scheme, a suitable dynamic model of the TCP behaviour is required. An effective TCP model has been developed in Misra, Gong and Towsley (2000) and adopted in Hollot et al. (2002) to tune the parameters of P and PI controllers. A similar model for wireless networks has also been used in Yanping (2011). Simulations using the network simulator ns-2 have confirmed the usefulness of such a control engineering approach.

The two main sources of difficulties in the analysis of TCP dynamics are the input and state delay, on the one hand, and the parameter uncertainties and variations, on the other. To deal with the control of uncertain time–delayed systems in a proper way, different approaches have been proposed recently. Particularly interesting in this regard is the geometrical method described in Klamka and Tańcula (2010, 2012a,b), where the set of uncertain parameter values ensuring either asymptotic stability or D-stability has been determined with reference to the popular Random Early Detection (RED) algorithm for queue management (modelled as a first–order transfer function in the feedback control channel).

Noninteger–order systems have been considered with increasing interest in the recent control literature, because many plants can be described more satisfactorily by models of this kind (Chen, Petráš and Xue, 2009; Petráš, 2009) or because noninteger–order controllers provide a better performance than the classic integer–order ones (Podlubny, 1999). In fact, it has been shown that in many instances the fractional–order PID controllers outperform the best integer–order PID controllers (Luo and Chen, 2009; Chao et al., 2009). In the following, we consider the situation in which queue management is carried out using a fractional–order PI controller and the controlled system is described by a second–order model plus a time delay obtained from the nonlinear model derived in Misra, Gong and Towsley (2000). A similar approach has recently been followed with reference to wireless networks in Yanping (2011) using classic frequency–domain specifications.

Robustness is a fundamental issue in AQM and in noninteger–order control, too (see. e.g., Quet and Ozbay, 2004). In particular, it is very important to determine the set of PI^λ controllers that satisfy certain stability margins. Among these margins, the modulus margin (also called H_∞ margin because it is the inverse of the H_∞ norm of the sensitivity function) seems to be the most meaningful (Garcia, Karimi and Longchamp, 2004; Krajewski, Lepschy and Viaro, 2004). Determining the controllers that ensure a given modulus margin, however, is not an easy task even for integer–order systems. Such problem has been tackled, e.g., in Krajewski and Viaro (2012) for integer–order time-delay plants and PID controllers using different approaches. Here, we extend the essentially graphic method from Krajewski and Viaro (2012) to the aforementioned case of PI^λ controllers. The entire stability region in the space of controller parameters has already been determined in Hamamci (2007), Ruszewski (2008), and, for particular classes of fractional–order controllers and time–delay plants, in Hamamci and Koksal (2010), Rahimian and Tavazoei (2010), where, however, no indication is given regarding the loci of the constant modulus margin.

The remainder of this paper is organized as follows. In Section 2, the aforementioned TCP fluid–based model is briefly recalled. The considered control problem is stated in Section 3. The equation of the stability boundary in the controller parameter plane is derived in Section 4 along the lines followed in Krajewski and Viaro (2012) for the integer–order case. The loci of constant modulus margin are determined in Section 5 and plotted using a dedicated Matlab problem. An example frequently considered in the literature shows the effectiveness of the adopted approach in Section 6. A few concluding remarks are made in Section 7.

2. TCP flow model

By ignoring the TCP timeout mechanism, the nonlinear dynamic model of TCP behaviour developed in Misra, Gong and Towsley (2000) is described by the following pair of nonlinear differential equations:

$$\dot{W}(t) = \frac{1}{R(t)} - \frac{W(t)W(t-R(t))}{2R(t-R(t))} p\,(t-R(t)) \tag{1}$$

$$\dot{q}(t) = \frac{W(t)}{R(t)} N(t) - C \tag{2}$$

where:

W = expected TCP window size in packets,
q = expected queue length in packets,
$R = \dfrac{q}{C} + T_p$ = round–trip time in seconds,
C = link capacity in packets per second,
T_p = propagation delay in seconds,
N = number of TCP sessions (load factor),
p = probability of packet marking or dropping.

The window size W and the queue length q are non–negative and bounded, that is, $W \in [0, W_{max}]$, $q \in [0, q_{max}]$, where W_{max} and q_{max} denote the maximum window size and buffer capacity, respectively. Obviously, $p \in [0,1]$. The number N of sessions and the link capacity C are assumed constant.

The above model has a unique equilibrium point (W_0, q_0, p_0), characterized by

$$W_0 = \frac{R_0 C}{N}\,, \quad W_0^2 p_0 = 2\,, \;\; R_0 = \frac{q_0}{C} + T_p\,. \tag{3}$$

By linearizing equations (1)–(2) about this point and taking into account the considerations from Hollot et al. (2001), the model becomes

$$\delta\dot{W}(t) \;=\; -\frac{2N}{R_0^2 C}\delta W(t) - \frac{R_0 C^2}{2N^2}\delta p(t-R_0)\,, \tag{4}$$

$$\dot{\delta q}(t) \;=\; \frac{N}{R_0}\delta W(t) - \frac{1}{R_0}\delta q(t)\,, \tag{5}$$

where $\delta W(t) \doteq W(t) - W_0$, $\delta q(t) \doteq q(t) - q_0$, and $\delta p(t) \doteq p(t) - p_0$. The deviations $\delta W(t)$ and $\delta q(t)$ represent the state variables, whereas $\delta p(t)$ is the control variable.

The main task of the AQM algorithm is to relate the queue length q at the bottleneck router to the marking probability p, and then inform the TCP sender on the state of congestion. For the adopted model the transfer function between $\delta p(t)$ and $\delta q(t)$ turns out to be:

$$G(s) = \frac{-\dfrac{C^2}{2N}\; e^{-sR_0}}{(s + \dfrac{2N}{R_0^2 C})(s + \dfrac{1}{R_0})}, \tag{6}$$

so that the TCP behaviour is locally described by a linear second–order model with delayed input and poles at $-2N/(R_0^2 C)$ and $-1/R_0$.

Clearly, the control signal $\delta p(t)$ should drive the state $[\delta W(t),\ \delta q(t)]^T$ of the linearized model to the origin. The main goals of the control system design are: (i) to ensure suitable stability margins, (ii) to allow for efficient queueing utilization, and (iii) to guarantee robustness against uncertainties and disturbances.

3. Problem statement

Consider a unity–feedback control system and assume that the controlled plant is described by the transfer function:

$$G(s) = \frac{n(s)}{d(s)}\, e^{-Ts}\,, \tag{7}$$

where: $n(s) = \dfrac{C^2}{2\,N}$, $d(s) = s^2 + (\dfrac{2N}{R_0^2 C} + \dfrac{1}{R_0})s + \dfrac{2N}{R^3 C}$, and $T = R_0$ is a time delay.

Assume also that plant is controlled by means of a standard PI^{λ} controller described by the transfer function:

$$C(s) = k_p + \frac{k_i}{s^{\lambda}}, \tag{8}$$

where $\lambda > 0$, and k_p, k_i are the proportional and integral gain, respectively. Clearly, this setting encompasses any combination of integer–order time–delay plant and integer–order or noninteger–order controller.

The problem we refer to is that of finding a controller as in equation (8) in such a way that the overall unity–feedback control system is stable with a modulus margin greater than a prescribed value. In the usual case of integer–order controllers, there are two design parameters, i.e., k_p and k_i, whereas the aforementioned control problem allows for one more design parameter, i.e., λ, and this greater flexibility can be exploited in order to achieve a better performance.

4. Stability regions

The Nyquist diagram of the loop function:

$$L(s) = C(s)G(s) \tag{9}$$

crosses the unit circle centred at the origin with a phase equal to $m_\varphi - \pi$, where m_φ is the *phase margin*, if:

$$L(\jmath\omega_a) = e^{\jmath(m_\varphi - \pi)}, \tag{10}$$

where ω_a denotes *the gain crossover frequency* and $\jmath$ denotes the imaginary unit. Taking into account equations (7) and (8), the interpolation condition, expressed by equation (10), can be written as

$$\left[k_p(\jmath\omega_a)^{\lambda} + k_i\right] n(\jmath\omega_a) = d(\jmath\omega_a)(\jmath\omega_a)^{\lambda}\, e^{\jmath(T\omega_a + m_\varphi - \pi)}. \tag{11}$$

By decomposing $n(\jmath\omega_a)$ and $d(\jmath\omega_a)$ into their real and imaginary parts according to

$$n(\jmath\omega_a) = n_r(\omega_a) + \jmath\, n_i(\omega_a), \quad d(\jmath\omega_a) = d_r(\omega_a) + \jmath\, d_i(\omega_a), \tag{12}$$

equation (11) can be split into two equations relating the real and imaginary parts on both of its sides, leading, after some algebra, to

$$k_p \sin\lambda\frac{\pi}{2} = -A(\omega_a)\,\sin\left(\omega_a T + m_\varphi + \lambda\frac{\pi}{2}\right) + B(\omega_a)\,\cos\left(\omega_a T + m_\varphi + \lambda\frac{\pi}{2}\right), \tag{13}$$

$$k_i \sin\lambda\frac{\pi}{2} = \omega_a^{\lambda}\left[A(\omega_a)\,\sin\left(\omega_a T + m_\varphi\right) - B(\omega_a)\,\cos\left(\omega_a T + m_\varphi\right)\right] \tag{14}$$

with

$$A(\omega_a) = \frac{d_r(\omega_a)n_r(\omega_a) + d_i(\omega_a)n_i(\omega_a)}{n_r^2(\omega_a) + n_r^2(\omega_a)}, \tag{15}$$

$$B(\omega_a) = \frac{d_r(\omega_a)n_i(\jmath\omega_a) - d_i(\jmath\omega_a)n_r(\omega_a)}{n_r^2(\omega_a) + n_i^2(\omega_a)}, \tag{16}$$

which provide the parametric equations (with parameter ω_a) of a curve in the (k_p, k_i)–plane.

When $m_\varphi = 0$, at each point of these curves $1 + L(\jmath\omega_a) = 0$, i.e., the characteristic equation $1 + L(s) = 0$ of the feedback control system exhibits the *purely imaginary roots* $\pm\jmath\omega_a$. Therefore, on the parameter plane (k_p, k_i), these curves separate regions characterized by different numbers of right half–plane (RHP) and left half–plane (LHP) roots of the system characteristic equation, and some of these regions may correspond to a stable behaviour. This property was proved in Krajewski and Viaro (2012) for integer–order systems and controllers.

For $\lambda = 1$ (integer–order PI controller) equations (13) and (14) simplify to

$$k_p = -A(\omega_a)\,\cos\Big(\omega_a T + m_\varphi\Big) - B(\omega_a)\,\sin\Big(\omega_a T + m_\varphi\Big), \tag{17}$$

$$k_i = \omega_a\Big[A(\omega_a)\,\sin\big(\omega_a T + m_\varphi\big) - B(\omega_a)\,\cos\big(\omega_a T + m_\varphi\big)\Big], \tag{18}$$

which of course coincide with equations (10) and (11) in Krajewski and Viaro (2012).

The stability boundaries have been determined for the sample network considered in Hollot et al. (2002), where $C = 3750$ packets/s (15Mbps), $N = 60$ and $R_0 = 0.246$ s. In particular, Fig. 1 shows the stability regions for $\lambda = 0.8, 1.0, 1.1, 1.25$.

By way of example, the loci described by equations (13) and (14) inside the stability region for different values of the phase margin m_φ when $\lambda = 1.25$, are shown in Fig. 2.

5. Loci of constant modulus margin

An indicator of system robustness that is more adequate than the phase and gain margins is the *modulus margin* defined as:

$$\delta := \min_\omega |1 + L(\jmath\omega)| \tag{19}$$

which represents the minimal distance of the Nyquist diagram of the loop function from the critical point $-1 + \jmath 0$ and corresponds to the reciprocal of the infinity norm of the sensitivity function.

Now, the locus of the parameter points where $\delta =$ const is the envelope of the loci:

$$|1 + L(\jmath\omega)| = \delta,\ \forall\omega, \tag{20}$$

which is equivalent to

$$L(\jmath\omega) + \overline{L(\jmath\omega)} + |L(\jmath\omega)|^2 = \delta^2 - 1\,, \tag{21}$$

where the overbar denotes complex conjugate.

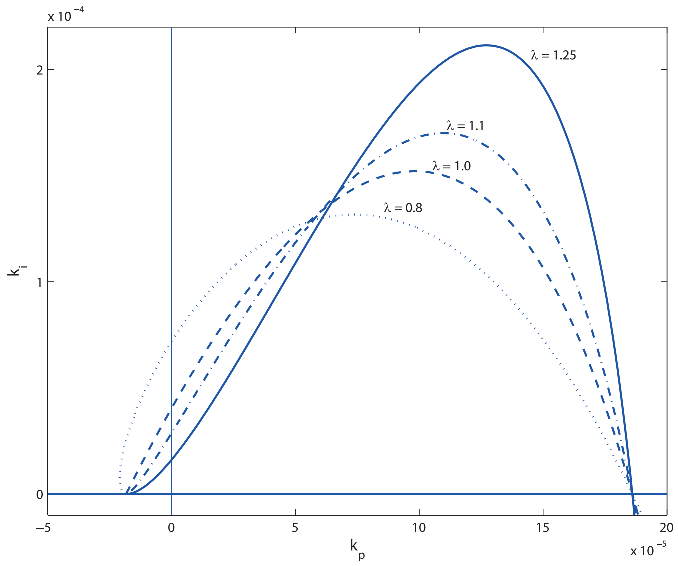

Figure 1. Stability regions for different values of the exponent λ in equation (8) (regions below the curves, corresponding to each value of λ, and above the horizontal axis) when the plant transfer function is given by equation (6), with $C = 3750$ packets/s (15Mbps), $N = 60$ and $R_0 = 0.246$ s

Simple algebra leads from equation (21) to

$$\begin{aligned}&\big(k_p\omega^\lambda\big)^2 + k_i^2 + 2k_i\big(k_p\omega^\lambda\big)\cos\lambda\frac{\pi}{2} + 2k_p\omega^{2\lambda}\Big[A(\omega)\cos\omega T + B(\omega)\sin\omega T\Big]\\ &+2k_i\omega^\lambda\Big[A(\omega)\cos(\omega T + \lambda\frac{\pi}{2}) + B(\omega)\sin(\omega T + \lambda\frac{\pi}{2})\Big]\\ &= \omega^{2\lambda}\,\frac{d_r^2(\omega) + d_i^2(\omega)}{n_r^2(\omega) + n_i^2(\omega)}(\delta^2 - 1),\end{aligned} \tag{22}$$

which is the equation of an ellipse in the (k_p, k_i)–plane, whose centre can be found using standard procedures of analytic geometry.

The region, where $\delta > 0.2$ inside the stability area for the plant model in equation (6) with $C = 3750$ packets/s, $N = 60$, $R_0 = 0.246$, and $\lambda = 1.25$ in equation (8), is shown in Fig. 3: it is given by the area under the lower envelope of the family of ellipses.

6. Example

In Hollot et al. (2002), to control the considered plant, the integer–order PI controller $C(s) = k_p + \frac{k_i}{s}$ with $k_p = 1.822 \cdot 10^{-5}$ and $k_i = 9.64 \cdot 10^{-6}$ has been adopted. It locally stabilizes the equilibrium point corresponding to the

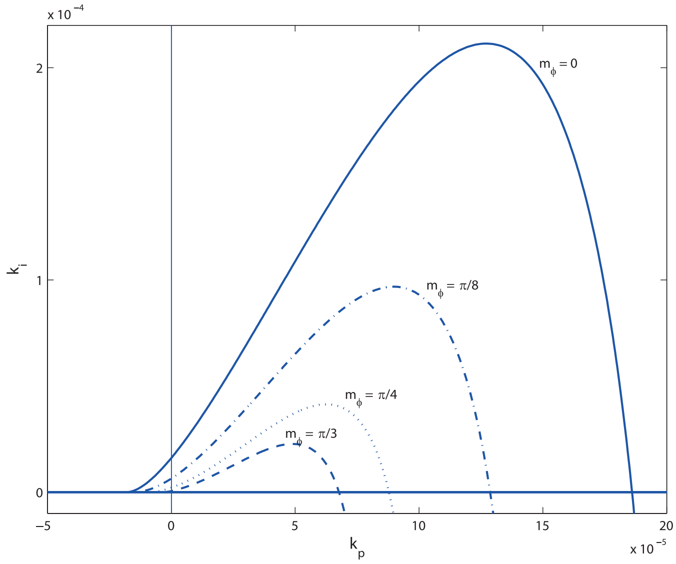

Figure 2. Loci described by equations (13) and (14) for $\lambda = 1.25$ and phase margin $m_\varphi = 0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{\pi}{3}$, when the plant transfer function is given by equation (6) with $C = 3750$ packets/s (15Mbps), $N = 60$ and $R_0 = 0.246$ s

parameter values that characterize the *nominal* plant, i.e., $C = 3750$ packets/s, $N = 60$ and $R_0 = 0.246$. However, this PI controller is rather fragile since the point $(k_p, k_i) = (1.822 \cdot 10^{-5}, 9.64 \cdot 10^{-6})$ is close to the stability boundary (see the point P_1 in Fig. 4).

A less fragile controller is obtained for $k_p = 3.7 \cdot 10^{-5}$ and $k_i = 2.2 \cdot 10^{-5}$ (corresponding to the point P_2 in Fig. 4) which ensure almost the same behaviour of the feedback control system as the controller proposed in Hollot et al. (2002), when the plant parameter take the nominal values.

Fig. 5 shows the unit step responses of the control system with this less fragile controller for the *nominal* plant with $C = 3750$ packets/s, $N = 60$, $R_0 = 0.246$, as well as for the two partly perturbed plant models with $C = 3750$ packets/s, $N = 80$, $R_0 = 0.15$ and, respectively, $C = 3750$ packets/s, $N = 45$, $R_0 = 0.4$. In the last case the considered PI controller (and, to a greater extent, the less robust controller derived in Hollot et al., 2002) does not even guarantee stability.

Fig. 6 depicts the step responses for the same plant models when a fractional–order PI^λ controller, characterized by the same values of k_p and k_i, and by $\lambda = 1.25$, is adopted. The control system performance is better than that afforded by the integer–order controller. In particular, the fractional–order controller locally stabilizes the system in all of the three cases, thus ensuring greater robustness.

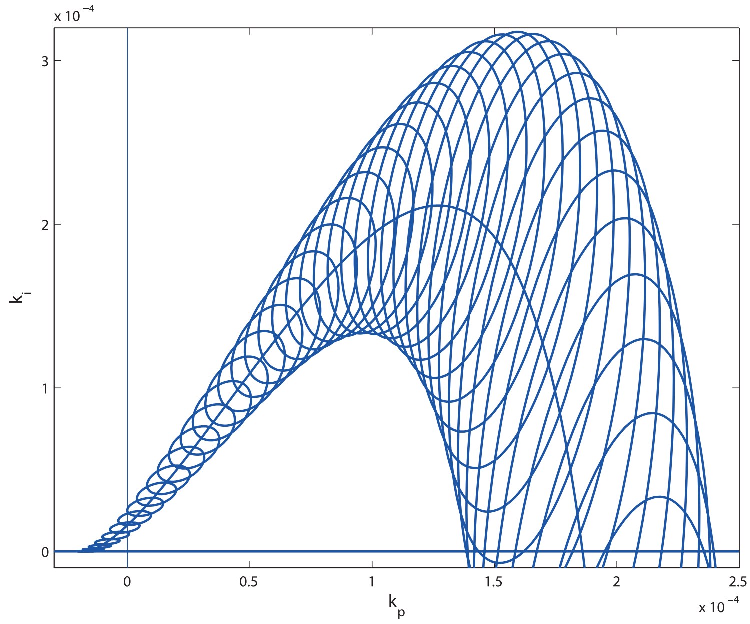

Figure 3. Family of ellipses corresponding to $\omega \in (0.2, 5.0)$ rad/s when $\delta = 0.2$ and $\lambda = 1.25$ in equation (22). The lower envelope of this family delimits the area of the first two quadrants, where stability is ensured, and the modulus margin satisfies the inequality $\delta > 0.2$

7. Conclusions

A fractional–order PI^λ controller has been applied to the control of TCP packet flows. The regions of the controller parameter space, where given stability margins are ensured, have been determined. Particular attention has been given to the so–called modulus margin that accounts well for system robustness. The loci of constant stability margins in the parameter space have been plotted using a simple Matlab program. As shown in Section 6, these curves can profitably be used to design robust fractional–order controllers in a simple and intuitive way.

It is believed that considerable improvements over conventional AQM techniques can be achieved in terms of both performance and robustness using fractional–order controllers in conjunction with modulus margin specifications.

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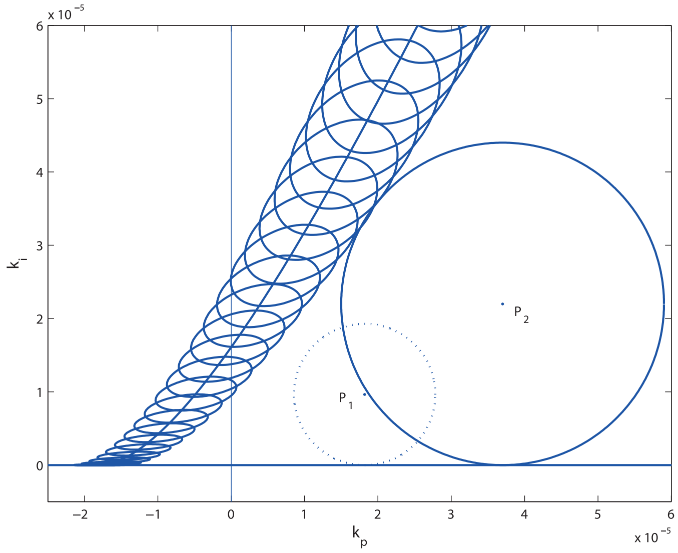

Figure 4. A part of the area where stability is ensured and the modulus margin satisfies the inequality $\delta > 0.2$. The point $P_1 = (k_p, k_i) = (1.822 \cdot 10^{-5}, 9.64 \cdot 10^{-6})$ corresponds to the PI controller derived in Hollot et al. (2002), whereas the point $P_2 = (k_p, k_i) = (3.7 \cdot 10^{-5}, 2.2 \cdot 10^{-5})$ corresponds to a less fragile controller ensuring almost the same dynamic behaviour for the nominal plant

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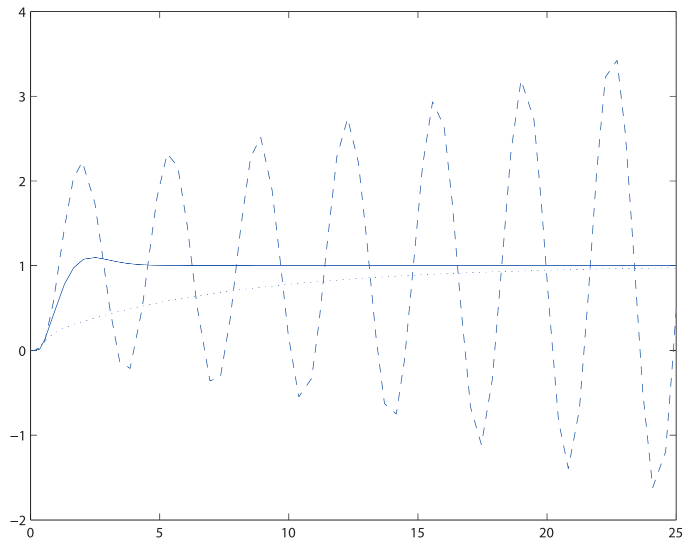

Figure 5. Step responses for the integer–order PI controller with $k_p = 3.7 \cdot 10^{-5}$ and $k_i = 2.2 \cdot 10^{-5}$ (point P_2 in Fig. 4) when the plant parameters are: (i) $C = 3750$ packets/s, $N = 60$, $R_0 = 0.246$ (solid line), (ii) $C = 3750$ packets/s, $N = 80$, $R_0 = 0.15$ (dotted line), and (iii) $C = 3750$ packets/s, $N = 45$, $R_0 = 0.4$ (dashed line)

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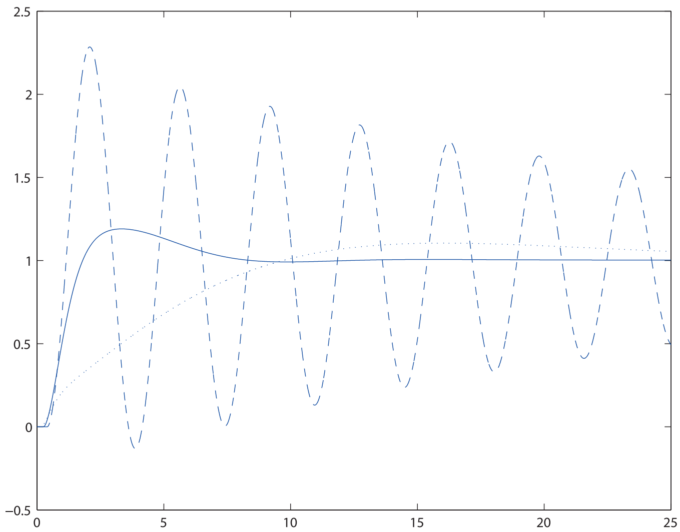

Figure 6. Step responses afforded by the fractional–order PI controller with $k_p = 3.7 \cdot 10^{-5}$, $k_i = 2.2 \cdot 10^{-5}$ and $\lambda = 1.25$ when the plant parameters are: (i) $C = 3750$ packets/s, $N = 60$, $R_0 = 0.246$ (nominal plant, solid line), (ii) $C = 3750$ packets/s, $N = 80$, $R_0 = 0.15$ (dotted line), and (iii) $C = 3750$ packets/s, $N = 45$, $R_0 = 0.4$ (dashed line)

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