Aleksander Byrski (ID)
Krzysztof Węgrzyński
Wojciech Radwański
Grażyna Starzec (ID)
Mateusz Starzec (ID)
Monika Bargiel
Aleksandra Urbańczyk (ID)
Marek Kisiel-Dorohinicki (ID)

POPULATION DIVERSITY IN ANT-INSPIRED OPTIMIZATION ALGORITHMS

Abstract　*Measuring the diversity in evolutionary algorithms that work in real-value search spaces is often computationally complex, but it is feasible; however, measuring the diversity in combinatorial domains is practically impossible. Nevertheless, in this paper we propose several practical and feasible diversity-measurement techniques that are dedicated to ant colony optimization algorithms, leveraging the fact that we can focus on a pheromone table even though an analysis of the search space is at least an NP problem where the direct outcomes of the search are expressed and can be analyzed. Besides sketching out the algorithms, we apply them to several benchmark problems and discuss their efficacy.*

1. Introduction

Optimization metaheuristics (particularly biologically-inspired techniques) have been gaining attention for more than three decades (c.f. [6, 32], for example). Such an approach is supposed to be universal, although critics point to the higher computation times and larger complexities of the algorithms. However, when facing difficult problems, it is usually effective to switch from a deterministic approach to stochastic search and optimization methods [29], which may justify the additional costs. An excellent overview of ACO methods can be found in [13].

Thus, the metaheuristic methods are often called "methods of last resort" and should not be used for solving simple tasks for which deterministic methods can be applied. One must also remember that these methods should not be used because "they look fancy," as they consist of managing groups of agents, ants, bees, or other similar beings [24].

Ant systems are a popular tool for solving many discrete optimization problems; e.g., the traveling salesman problem (TSP), vehicle routing problem (VRP), graph coloring problem (GCP) quadratic assignment problem (QAD), and others [10]. In this article, we present an ant system as a way of expressing the socio-cognitive behaviors of a population of ants by introducing various ant species that embody different behaviors from the point of view of their stigmergic interactions.

It is well-known that ant colony optimization algorithms often suffer from premature convergence and stagnation, which hinders the exploration of the solution space [14]; therefore, several techniques have been proposed to properly manage the balance between exploration and exploitation (see, e.g., [5,11]). This sought-after balance will surely remain one of the most important aspects of metaheuristics research (population-based research in particular).

Considering population-based metaheuristics like evolutionary algorithms (which usually work in a continuous space domain), there are many diversity enhancement techniques like niching [16] or the multi-deme model [3] that, when properly configured, may lead to obtaining a sufficiently diverse population of individuals, balancing between exploration and exploitation according to a user's needs and preferences.

In the genetic algorithm (GA), fitness evaluation and selection are directly related to intensification, and mutation is directly related to diversification. The trade-off between them can be explicitly controlled by adjusting the mutation rate. In ACO, pheromone depositing is equivalent to fitness evaluation and edge selection by ants and is comparable to selection in GA [22].

The proper configuration of certain algorithms that are aimed at enhancing diversity and reaching a sought-after balance requires either randomly choosing the parameters of an algorithm (such as the probabilities and ranges of mutations in evolutionary algorithms) in the trial-and-error process or employing certain measures of diversity that can be used for having a greater insight into the exploration and exploitation phenomena. In the continuous domain, the problem is not too complex, as

any analysis of the center of the weight of the population or the variance of the genes in both genotypic space and phenotypic space can be used (cf. [17, 21], for example).

In addition to the original selection defined by Equation (1), Nakamichi and Arita [22] introduced a mechanism of random selection in which an ant selects a target city with equal probability (equivalent to the mutation adopted in evolutionary computation). The rate of random selection continuously adjusts the balance between exploitation and exploration.

In applying standalone metaheuristic algorithms, it is possible to lose the diversity of the population through premature convergence; thus, the algorithm becomes stuck in the local optima. Mohsen [19] proposed maintaining a proper balance between diversification and intensification by combining an ACO algorithm with a simulated annealing approach and choosing one or another depending on the population diversity. His measure was based on the fitness of the ants.

Considering a discrete search space (combinatorial in particular), it is not so easy to compute the diversity or use such a measure in seeking the above-mentioned balance. This is imposed by the basic difficulty or even impossibility of analyzing such spaces, which is expressed and well-known as the closest string problem [18]. Thus, we are unable to compute the center of the weight in such spaces (e.g., when solving the traveling salesman problem) or cluster the results without solving an NP-hard problem. Therefore, the direct measuring of the diversity in such spaces is computationally impossible in a reasonable amount of time.

In this paper, we follow the method introduced in [28] by refining the diversity measurement methods that are devoted to seeking a balance between exploitation and exploration in ant colony optimization-type algorithms. Of course, we cannot directly analyze a search space [18]; instead, we focus on analyzing the pheromone table. We recall and refine three measures based on the pheromone table and apply them to analyzing the classic ant colony optimization algorithm and several of its variants – checking its efficacy for different parameters of the algorithm. Thus, the relationship between the introduced measurement methods and the effectiveness of the algorithm is checked. This prepares us for analyses of other ACO-like algorithms and devising auto-adaptive ACO algorithms in the future.

In the next section, the ACO algorithm and its most important variants are summarized along with its modifications that are aimed at enhancing the diversity of a search. Next, the diversity measurement techniques are evoked after [28] and refined. Finally, the impact of different parameters of the classic ACO on the results shown by the defined diversity measurement techniques is shown and discussed. Finally, the paper is concluded.

2. Enhancing diversity in ACO

The behavior of natural ant colonies was the inspiration for the ant colony optimization algorithm, which was first described by Dorigo [8] for solving TSP and further developed into a meta-heuristic [9].

In ACO algorithms, the optimization problem is considered to be a graph that is based on a finite set of *components* and edges with assigned costs. This problem also defines the restrictions and requirements that must be met by a path for it to be considered to be a feasible solution. Among the feasible solutions, we are looking for the optimal one; i.e., the one with the minimum cost, where the cost is a function of the costs of the edges that form the path.

2.1. ACO variants

Since the first description of the ACO algorithm, a lot of variants have been developed. Below, we provide descriptions of the basic algorithm and their most important modifications.

2.1.1. Ant System

In every iteration of an ACO algorithm, each agent (representing an ant) creates a complete solution. Each ant starts in some initial vertex and travels through a graph according to a probabilistic-decision rule. This rule assigns a probability (or *attractiveness*) of choosing a path from component i to j based on the edges' costs and pheromone trails. In the basic ACO algorithm – ant system (AS, [12]) – the attractiveness of path (i, j) is defined for ant k in iteration t as follows:

$$p_{ij}^k(t) = \begin{cases} \frac{\tau_{ij}^\alpha(t)\,\eta_{ij}^\beta(t)}{\sum_{l \in A_k} \tau_{il}^\alpha(t)\,\eta_{il}^\beta(t)} & \text{if } j \in A_k \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $\tau_{ij}(t)$ is the level of a pheromone (also called a *trail*) on edge (i, j), and $\eta_{ij}(t)$ is the visibility of the edge (which can be defined as an inversed distance between the cities in the case of TSP). The greater the product of τ_{ij} and η_{ij}, the more likely it is that j will be chosen as the next city. The trail and visibility are weighted by parameters α and β, respectively, and A_k is a set of allowed transitions for ant k in its current state.

After the solution is constructed, the pheromone levels need to be updated. In the classic AS version, the values of τ_{ij} are modified at the end of each iteration according to the following formula:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \sum_{k=1}^{m} \Delta\tau_{ij}^k \tag{2}$$

where $\rho \in [0, 1)$ is the pheromone trail *evaporation rate*, and m is the number of ants.

The pheromone update level for each ant is defined as follows:

$$\Delta\tau_{ij}^k = \begin{cases} \frac{Q}{L_k} & \text{if } k\text{-th ant uses edge } (i, j) \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where L_k is the cost of the k-th ant's tour, and Q is a constant.

2.1.2. Elitist Ant System

A potential drawback of the AS algorithm that was noticed in [12] is that, during the search process, all of the information about the best solutions found thus far is lost. An improvement that was proposed in the same paper and described further in [23] was to introduce an additional number of e "elitist" ants that report during each iteration as if they have traversed the best solution found thus far by the algorithm. Such a modification results in a minor change to the pheromone update formula:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \sum_{k=1}^{m} \Delta\tau_{ij}^{k} + e\,\Delta\tau_{ij}^{*} \tag{4}$$

$$\Delta\tau_{ij}^{*} = \begin{cases} \frac{Q}{L^{*}} & \text{if the best solution uses edge } (i,j) \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where L^{*} is the cost of the best solution found thus far by the algorithm. In this way, the paths that belong to the best solution will have higher pheromone levels. This modification of the AS algorithm is called the elitist ant system (EAS).

2.1.3. Rank-based Ant System

The rank-based ant system (ASRank) [2] is an extension of EAS. It uses elitist ants while also modifying the way the regular ants update their pheromone trails. The solutions found from each iteration are ordered by their cost; only the m-best solutions are taken into account in the pheromone-update process. The update value is also weighted according to the solution's rank μ. Usually, $m = e - 1$; then, the pheromone update formula is as follows:

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \sum_{\mu=1}^{e-1} (e-\mu)\Delta\tau_{ij}^{\mu} + \Delta\tau_{ij}^{*} \tag{6}$$

$$\Delta\tau_{ij}^{\mu} = \begin{cases} \frac{Q}{L_{\mu}} & \text{if the } \mu\text{th-best solution uses edge } (i,j) \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where L_{μ} is the tour length of the μth-best ant.

2.1.4. Max-Min Ant System

Another variant of the ACO algorithm, the max-min ant system (MMAS) [27], introduces a few changes to the AS algorithm:
- values of pheromones limited to interval $[\tau_{\min}, \tau_{\max}]$;
- initial values of pheromones set to $\tau_{\max}$;
- after each iteration, only one ant updates pheromone levels – either best from iteration or best found thus far by algorithm.

The performance of the MMAS algorithm was further improved by the *pheromone trail-smoothing* mechanism, which increases the pheromone trails proportionally to their differences to $\tau_{\max}$ when the algorithm is very close to convergence. This mechanism can also be effectively applied to other elitist ant systems.

2.1.5. Ant Colony System

The ant colony system (ACS) [11, 20] differs from the previous variations in three aspects:

- extended state transition rule;
- global updating rule applied only to edges from global-best solution;
- additional local updating rule.

Ant k in iteration t chooses to move from component i to j according to Equation (8):

$$
j = \begin{cases} \max_{l \in A_k} \{ \tau_{il}(t)\, \eta_{il}^{\beta}(t) \} & \text{if } q \leq q_0 \\ S & \text{otherwise} \end{cases} \tag{8}
$$

where q is a random number that is uniformly distributed in $[0, 1]$, q_0 is a parameter, and S is a random component that is selected by using Formula (1). The state transition rule favors short edges and edges with large levels of pheromones.

To make the search more directed, only the global-best solution is reinforced by applying the standard formula after each iteration in the *Global Updating* stage. After each state transition, each ant applies the *Local Update*:

$$
\tau_{ij}(t+1) = (1 - \rho)\tau_{ij}(t) + \rho\tau_0 \tag{9}
$$

where τ_0 is the initial pheromone level.

2.2. Exploration and exploitation balance

Just like with other meta-heuristics, ACO algorithms require us to keep an appropriate balance between exploration and exploitation. Failing to take sufficient care in this regard may lead to a premature convergence that hinders an in-depth exploration of the solution space [14]. To prevent this from happening, ACO algorithms are extended with various techniques. In the standard AS algorithm, this balance can be controlled with the careful choice of the α and β parameters, which affect the probability function that determines the next move of each ant. MMAS uses the *pheromone trail-smoothing* mechanism, and the updates of the pheromones after each iteration are either based on the total-best solution or the best solution in the iteration. It is, however, recommended to first favor exploration by choosing the iteration's best solution and only then focus on the exploitation of the total-best solution. In ACS, the ants sometimes select the next step according to the maximized value of $\tau_{ij}\eta_{ij}^{\beta}$ instead of the standard formula (which exploits the information that is contained in the pheromone levels).

2.2.1. Solution analysis

The effectiveness of any technique that is introduced to avoid a premature convergence should not be assessed based solely on the results that are achieved by the modified algorithm. Also, the automatic detection of the close-to-convergence state might be advisable. Thus, it is necessary to be able to determine the state of the ants' colony by defining a measure of its diversity. One of the simplest approaches to this problem is to evaluate the differences among a single iteration's solutions, where a difference can be defined as the quotient between the standard deviation and the average of the tours' lengths. This is called the variation coefficient [14]. Another way of measuring the distance between two solutions is by counting the number of arcs that differentiate one tour from the others. When the average distance between the solutions drops, this means that preferred paths start to appear in the colony. However, there is a major disadvantage of this measure – high computational complexity [14].

2.2.2. λ-branching

The author of [15] introduced a measure called λ-*branching*, which is based on the distribution of the pheromone levels over the edges. For each component i, the λ-branching factor is the number of arcs that are incident to it for which the pheromone trail level satisfies the following condition:

$$\tau_{ij} \geq \tau_{\min}^i + \lambda(\tau_{\max}^i - \tau_{\min}^i) \tag{10}$$

where $\tau_{\min}$ and $\tau_{\max}$ are the minimal and maximal values, respectively, of the pheromones on the arcs that are incident to i, and $\lambda \in [0, 1]$ is a parameter. The average value of the λ-branching factor conveys the current level of the exploration of the solution space. However, dependence on the value of λ is considered a disadvantage of this approach.

2.2.3. Selection entropy

Few diversity measures exist that are used to increase the self-adaptability of the algorithms. Some of the known strategies are based on entropy; e.g., [4, 5, 7, 30, 31]. The entropy for component i is defined as follows:

$$\epsilon_i = - \sum_{j \in A_k} p_{ij} \log p_{ij} \tag{11}$$

where p_{ij} is the probability of choosing arc (i, j) as given by Equation (1).

2.3. Summary

As the dimension of a combinatorial search space usually precludes the possibility of using it for investigating diversity in a reasonable amount of time (cf. [18]), several diversity measures that are based on the values of pheromone trails are proposed in this paper to be the missing component in an ACO algorithm analysis.

The pheromone trails represent the state of a search conducted by a colony. Together with the visibility values, they fully determine the ants' behaviors – see (1).

3. ACO diversity measuring

The direct measuring of diversity in a discrete space is practically impossible. However, a "derivative" of the information that is contained in the search space (especially that which is connected with the search abilities of the ants, resulting in their behavior [exploration – when they actively search for new solutions, and exploitation – when they fine-tune those already found) may be read from the pheromone table according to the results presented in [28]. Thus the main goal of this paper is to define and test some diversity measures based directly on an analysis of the pheromone table.

3.1. Pheromone Ratio (PR)

The number of pheromone-marked edges of the graph (those with pheromone levels that are above a threshold value) should directly show the diversity of a search: when a small number of edges are marked, the ants will only travel by using these edges and possibly getting stuck in a local extremum. Otherwise, the ants will roam through the entire graph when a large number of edges are marked. Therefore, we propose treating pheromone dispersion as the first measure of diversity for ACO. In other words, this measure is based on the ratio of the number of pheromone-marked edges to the total number of edges:

$$PR = 100\% \, \frac{\#\{e_{ij} : \tau_{ij} > \gamma\}}{\#E}, \; e_{ij} \in E \tag{12}$$

where E is a set of all graph edges, and γ is a threshold value.

The original measure introduced in [28] assumed a constant value of γ. This caused the unreliable behavior of the measure for other variants of ACO because the pheromones were initialized with positive values[1].

Manually specifying the threshold values to get meaningful results is not a trivial task because of its dependence on algorithm implementation. Therefore, we used a modified PR measure where threshold γ is calculated at each iteration as the average pheromone level over each edge in the entire graph:

$$\gamma = \frac{\sum_{(i,j) \in E} \tau_{ij}}{\#E} \tag{13}$$

Summing up, the greater the value of PR, the better the diversity (more graph edges have pheromone levels that are higher than average, which encourages exploration). PR is a percentage-based measure.

---

[1]the JACOF library was used for the implementation and running of the systems described in this paper; cf. https://github.com/thiagodnf/jacof

3.2. Attractiveness Dispersion (AD)

The second measure is based on the attractiveness given by Eq. (1), as the ants are directly driven by this value during their quests. In the extreme case where the ants are randomly roaming everywhere (choosing directions with equal probabilities), all of the edges would have equal attractiveness. Therefore, one can compute the attractiveness of each edge and measure its dispersion throughout the whole graph. If the dispersion (measured, for example, by the means of standard deviation) is low, the attractiveness values tend to be close to the mean. If it is high, the values are spread out over a wider range, and only small part of the graph is marked with high attractiveness. Therefore, a second measure of diversity that is based on the attractiveness of the edges is defined as follows:

$$AD = \sigma(\{\tau_{ij}\}), \ \forall e_{ij} \in E \tag{14}$$

where σ stands for the standard deviation.

We use τ_{ij} here instead of p_{ij} since the remaining parameters from Eq. (1) are constant throughout the simulation.

Summing up, the lower the value of AD, the better the diversity (a high value of AD would mean that there are only few edges marked with a sufficiently high pheromone levels). Of course, the aim of using this measure is not that all of the pheromones should have the same uniform level. This is a real-value-based measure, which means a standard deviation that is computed for attractiveness for all of the edges in the graph.

3.3. Attractiveness Ratio (AR)

The third measure is also based on attractiveness; however, the rationale for it is different. If the ants have chosen only one solution (they have lost diversity), they will travel only along one Hamiltonian in the graph. Therefore, only the edges that belong to this Hamiltonian will have a positive attractiveness. Consequently, one can compute the sum of the attractivenesses of the edges that belong to the current-best solution and divide it by the sum of the attractivenesses of the other edges (as given by the following formula):

$$AR = \frac{\sum_{i,j} \tau_{ij}}{\sum_{k,l} \tau_{kl}}, \ (i,j) \in E_{\text{opt}}, \ (k,l) \in E \setminus E_{\text{opt}} \tag{15}$$

where E_{opt} is the set of edges of the current-best solution.

Summing up, the lower the value of AR, the better the diversity (similar to AD, a high value of AR would mean that there are few edges marked with sufficiently high levels of pheromones). Yet again, the aim of using this measure is not that all of the pheromones should have the same uniform level.

3.4. Graphical representation

In this work, we propose a graphical representation of the rationale behind the presented diversity measures (PR, AD, AR) that aims to facilitate their understanding and develop the intuition as to why these measures are meaningful. We have chosen to represent this information on an arbitrary graph by using the opacities, widths, and colors of the edges.
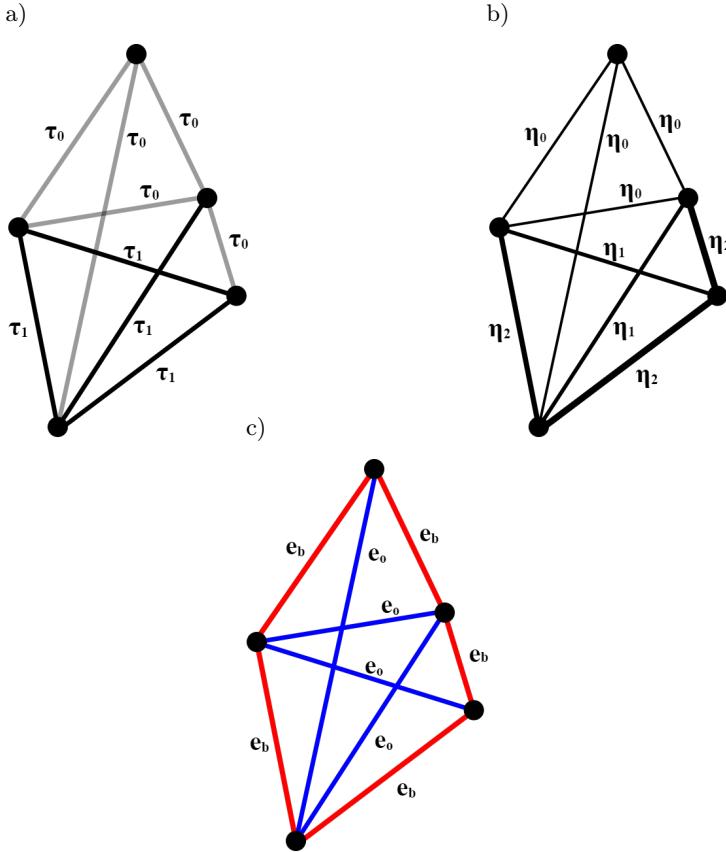
Figure 1. Graphical representation of diversity metrics and final results for considered ACO variants: a) pheromone levels as edge opacities; b) attractiveness values as edge widths; c) red edges contained within current-best solution, and blue edges represent all other edges

Figure 1a represents the state of the pheromone table upon which PR is directly based; the edges labeled τ_0 are below-average pheromone levels, and those labeled τ_1 are those that are above-average. In this case, PR is at 44% (as 4 out of 9 edges have gathered more than the average levels of pheromones). Figure 1b representing AD shows attractiveness as the width of an edge; three different levels of attractiveness are presented as η_0, η_1, and η_2 (in ascending order). It can be easily seen that a higher

ratio of thick-to-thin edges will lead to a loss of search diversity by the ants. Finally, Figure 1c represents those edges that are labeled e_b (b for best) that are contained within an example Hamiltonian cycle (which can be a solution to the TSP problem) as red, and all of the other edges that are labeled e_o (o for other) as blue. The attractivenesses gathered on the red edges divided by the sum of the attractivenesses gathered on the blue edges forms the AR measure. The higher the AR measure is, the more likely the ants will travel using the red e_b edges (thus, losing the diversity of their searches).

4. Experimental results

The experiments were conducted on the Prometheus supercomputer, which is a peta-scale (2.4 PFlop) machine located in the AGH Cyfronet computing center in Krakow, Poland. As of November 2017, Prometheus was ranked as the 77th-fastest supercomputer according to the TOP500 list. Prometheus is a cluster that is based on HP Apollo 8000 nodes with Xeon E5-2680v3 CPUs working at 2.5GHz (each node having 24 physical cores, with 55,728 computing cores in total). The nodes are connected via an InfiniBand FDR network. For the purpose of implementation, an open-source implementation of different ACO-type algorithms (JACOF) was used. The diversity was calculated at the *daemon actions* step of each iteration. Each experiment for the classical ant system was repeated ten times, the values of the proposed measures were shown as-is, and the quality of the obtained solutions were presented at the end of the simulation using a box-and-whiskers plot. Experiments for other ACO versions (ACS, EAS, ASRank, MMAS) were repeated 30 times.

The most important parameters of the classical ACO algorithm were as follows:

- Number of ants: $\{5, 20, 35, 50, 65, 80, 95, 100\}$, default 100;
- Number of iterations: 100;
- Pheromone influence on attractiveness: $\alpha \in \{0, 1, 2, 3, 4, 5\}$, default $\alpha = 2.0$;
- Distance influence on attractiveness: $\beta \in \{0, 1, 2, 3, 4, 5\}$, default $\beta = 3.0$;
- Pheromone evaporation rate: $\rho \in \{0.01, 0.05, 0.09, 0.13, 0.17\}$, default $\rho = 0.01$.

Similar ranges of parameters have been suggested to be advantageous by other authors; see [22], for example.

The parameters for the other considered ACO variants were the same as above (default values) except for the method-specific parameters, which are as follows:

- Local update rate (ACS): $r = 0.1$;
- Probability of unbiased exploitation (ACS): $q_0 = 0.9$;
- Number of "elitist" ants (EAS): $e = 30$;
- Number of "elitist" ants (ASRank): $e = 6$.

We were solving the "Berlin52" problem, which is available in the TSPLIB library[2].

---

[2]http://comopt.ifi.uni-heidelberg.de/software/TSPLIB95/

4.1. Experiments with different values of α

The graphs in Figure 2 show the dynamics of the diversity for the measurement methods described in Section 3. Considering all of the tested variants of the algorithm, it is easy to see that, as in the case of most metaheuristics (and according to common sense), the diversity values decrease as the simulation progresses. As stated in Section 3, the PR measure is designed to be proportional, while AD and AR are inversely proportional to the diversity of the ant population. These observations are consistent with those presented in [14].

The case when $\alpha = 0$ should be considered separately. In this situation, the ant algorithm becomes greedy, considering only distance but not taking care of the levels of the pheromones. Therefore, the search is the most diverse (high values observed on two diversity measurement graphs – Figures 2a, 2c). However, this is not a desirable configuration, as the algorithm ceases to "intelligently" seek the optimum; it becomes similar to purely statistical Monte Carlo methods. In other words, it may be treated as a heuristic algorithm but not a metaheuristic one anymore.

An apparent disturbance in Figure 2b will be seen later throughout the other experiments. The proposed metrics are fine and statistically repeatable for most of the observed configurations of the algorithm; however, when they become unstable, one should consider building an ensemble metric instead of relying on an individual one.

Comparing the final solutions that are obtained for different values of α (see Fig. 9a), it can be seen that the best solution (and, in fact, one that is quite close to the global optimum for Berlin52 (7542[3])) was observed for $\alpha = 2.0$. The diversity measure does not point out this value unanimously; however, the diversity that was observed for the best configuration in the considered case lies somewhere in the middle of the other diversity curves in all cases.

An interesting pattern can be seen in Figure 2c. For $\alpha = 5$, the AR value quickly increases at first and then starts to fall. It is understandable that larger values of α make the algorithm more exploratory. Apparently, the algorithm very quickly locates a suboptimal solution for the case of $\alpha = 5.0$ while losing its primary diversity. It then slowly turns to explore the other possibilities, thus causing a slow diversity increase.

4.2. Experiments with different values of β

Similar tendencies (high diversity in the beginning and lower in the end) are shown in Figure 3, which present the diversity measures for different values of β. When this parameter is equal to zero, the ACO becomes a random-walk algorithm; therefore, this is not a desirable solution (even though the PR diversity is high).

It seems that the β parameter does not affect the final result to a large extent (see Fig. 9b), as the observed differences are small; however, the best final quality was still obtained for $\beta = 2$. Again, there is not a very direct connection between the observed diversity dynamics and the final result (even though the AD was the highest for $\beta = 1$ in the considered case – see Figure 3b).
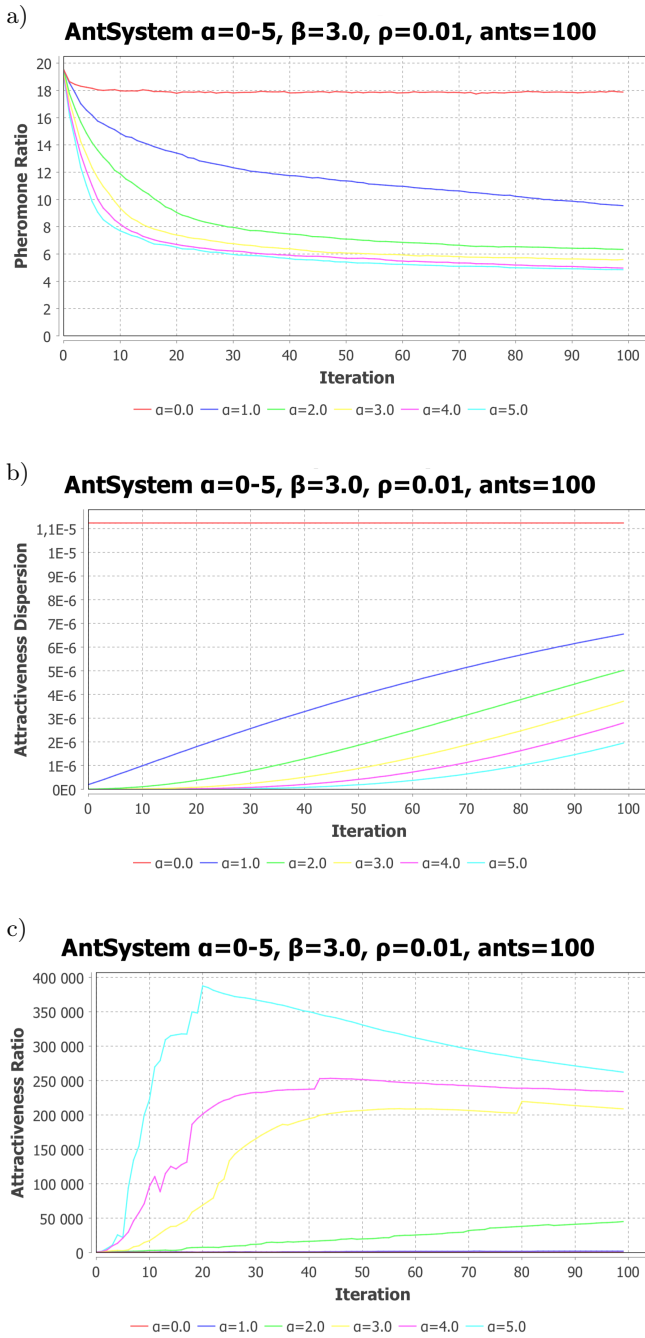
---

[3]https://www.iwr.uni-heidelberg.de/groups/comopt/software/TSPLIB95/STSP.html

a)

b)

c)

Figure 2. Diversity measures and final results found for different values of α:
a) PR; b) AD; c) AR

a)

AntSystem α=2.0, β=0-5, ρ=0.01, ants=100

b)

AntSystem α=2.0, β=0-5, ρ=0.01, ants=100

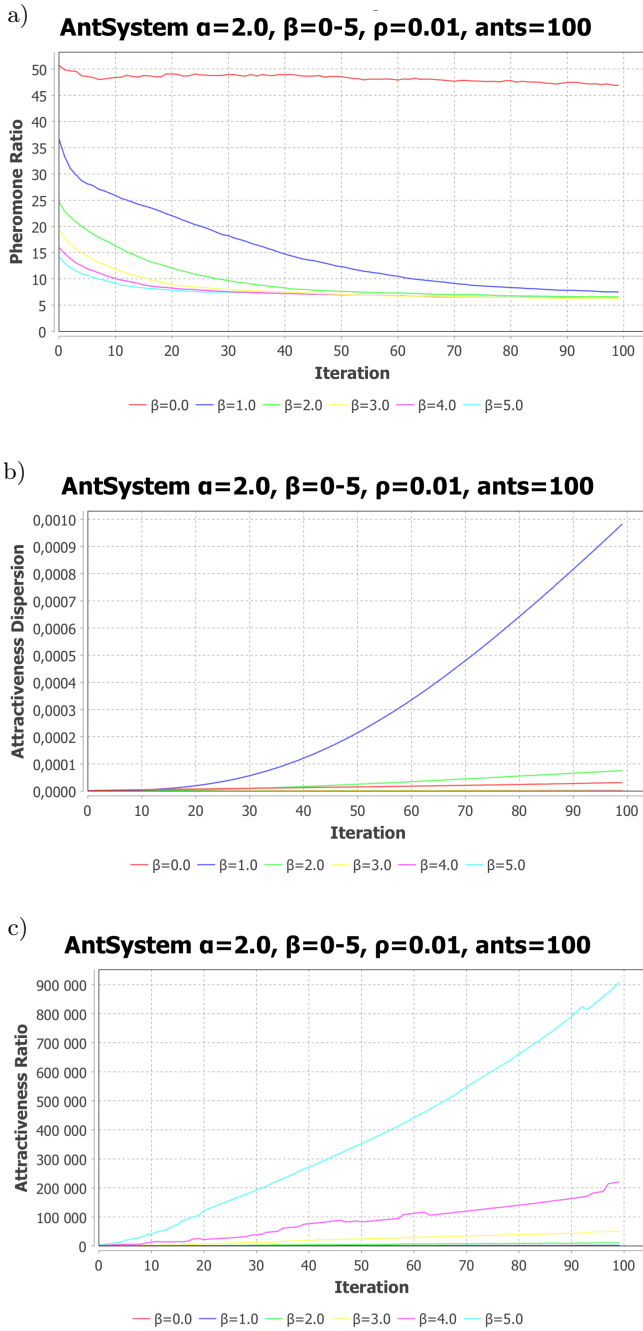c)

AntSystem α=2.0, β=0-5, ρ=0.01, ants=100

Figure 3. Diversity measures and final results found for different values of β:
a) PR; b) AD; c) AR

4.3. Experiments with different values of ρ

The diversity curves for different values of ρ exhibit similar dynamics as were described in the previous subsections for the α and β parameters.

The tested values of ρ do not affect the final result of the simulation to a large extent; however, the highest dispersion can be observed in the case of $\rho = 0.09$ (see Fig. 9c). Moreover, if the pheromone persists longer, the PR diversity is higher (cf. Fig. 4a).

The ρ value has a significant influence on the AR diversity; thus, it is an important factor when modifying the other parameters of the ACO algorithm. It is easy to see that the quick evaporation of the pheromone leads to a very early loss of diversity (cf. Fig. 4c).

The AD measure (cf. Fig. 4b) was observed to be inversely proportional to the value of ρ. This was caused by the overall lower levels of pheromones that were contained in the graph due to the quicker evaporation.

4.4. Experiments with different numbers of ants

When observing the results that were obtained for different numbers of ants in the simulation, the same trend as before is easy to see: the diversity decreases with increased simulation time (see Fig. 5).

A quite predictable result is observed in the case of the AD measure (Fig. 5b); namely, decreasing the number of ants increases the diversity. However, as in the case of zero values for α and β, a minimal number of ants will not lead to the achievement of good results (which is confirmed in Fig. 9d).

The AR diversity measure shown in Fig. 5c is quite similar for all of the tested values of the numbers of ants. It is also quite predictable since the behavior of the ants should be similar for the same search parameters, as they follow the same algorithm regardless of their numbers.

The above observation is also true for the PR measure (see Fig. 5a); this is most likely due to the same explanation.

4.5. Experiments with different ACO variants

Figure 6 shows the changes in the PR measure for each tested modification to the classic algorithm. It can be seen that these variants brought significant changes in the observed trends. In the case of ACS (see Fig. 6a), diversity increases with each iteration; this suggests a good exploration of the search space by the ACS variation, which is specifically aimed at by the introduction of its *pseudo-random-proportional rule*. The graphs for EAS and ASRank show similar behaviors (see Figs. 6b and 6c); this is to be expected, as they operate on very similar premises. These results indicate that PR can be successfully used to measure diversity across different ACO flavors.
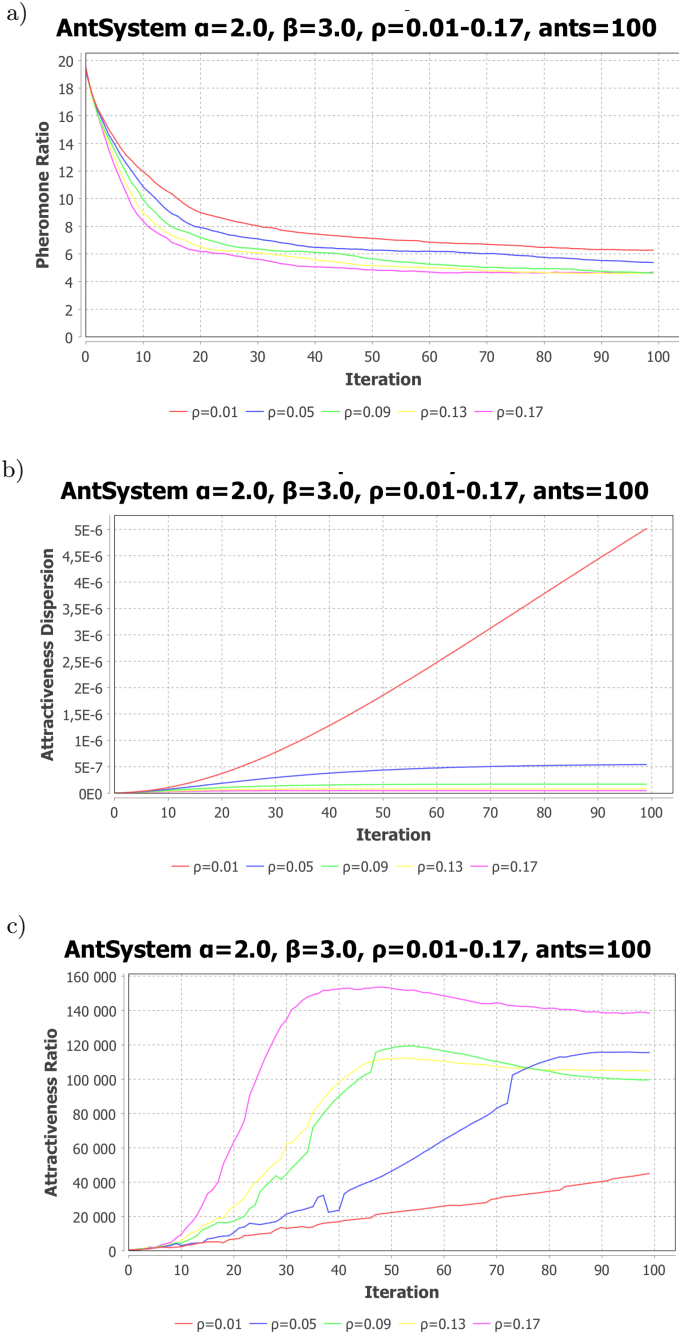
a)

AntSystem α=2.0, β=3.0, ρ=0.01-0.17, ants=100

b)

AntSystem α=2.0, β=3.0, ρ=0.01-0.17, ants=100

c)

AntSystem α=2.0, β=3.0, ρ=0.01-0.17, ants=100

Figure 4. Diversity measures and final results found for different values of ρ:
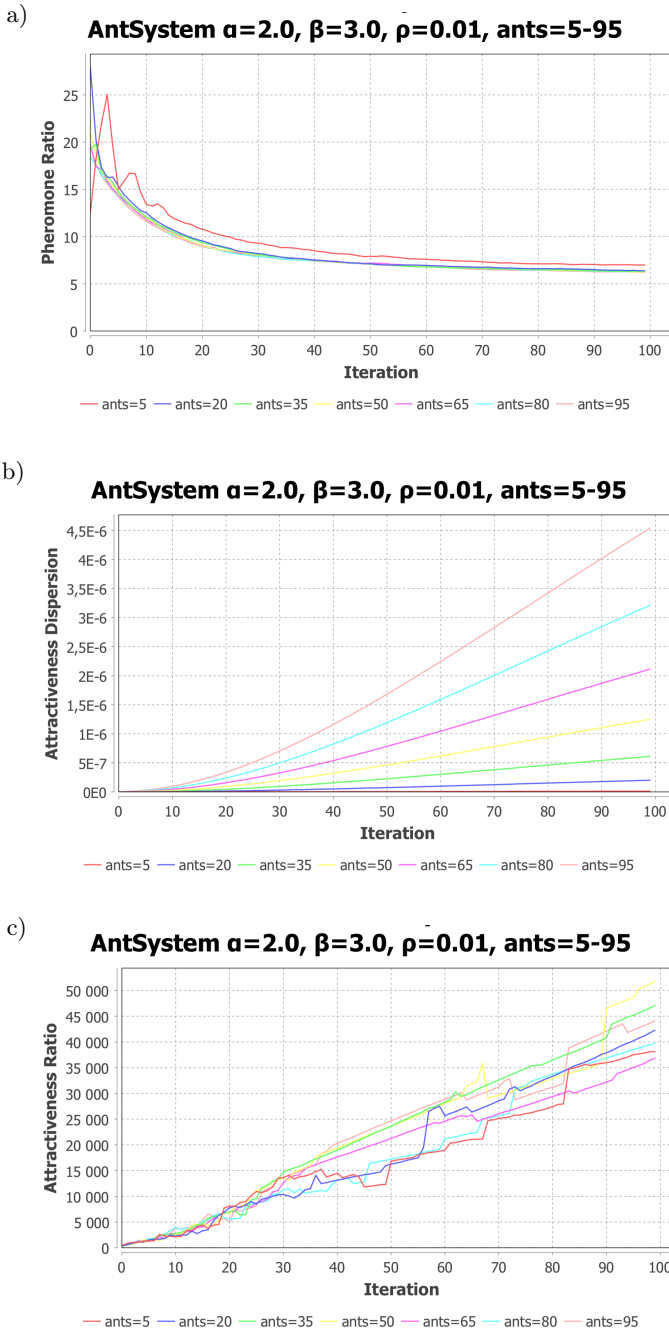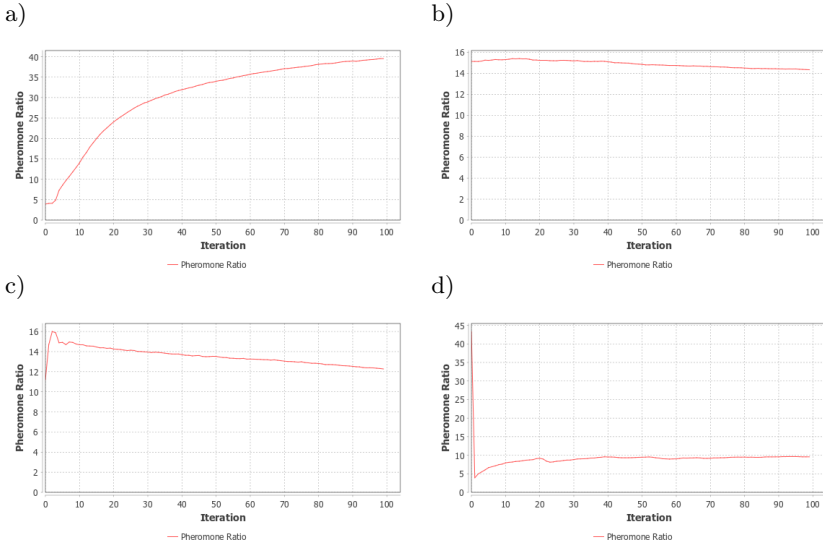a) PR; b) AD; c) AR

a)

b)

c)

Figure 5. Diversity measures and final results found for different numbers of ants:
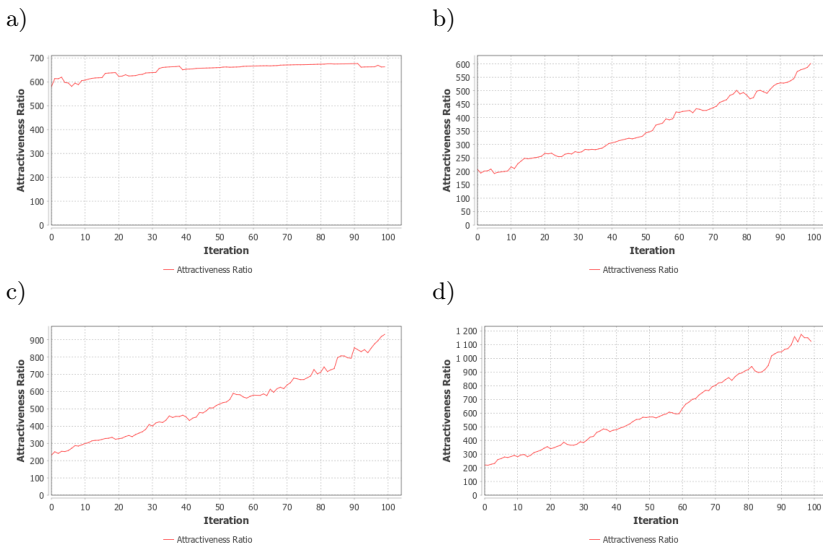a) PR; b) AD; c) AR

Figure 6. PR measures for considered ACO modifications: a) Ant Colony System;
b) Elitist Ant System; c) Rank-based Ant System; d) Max-Min Ant System

The AD measures for the different ACO modifications shown in Fig. 7 can be difficult to compare, as its values may vary by orders of magnitude. However, we observed that ACS exhibits the best performance in respect to this measure (see Fig. 7a). Also, each tested ACO variation (with the exception of EAS) performed better than the classical approach.
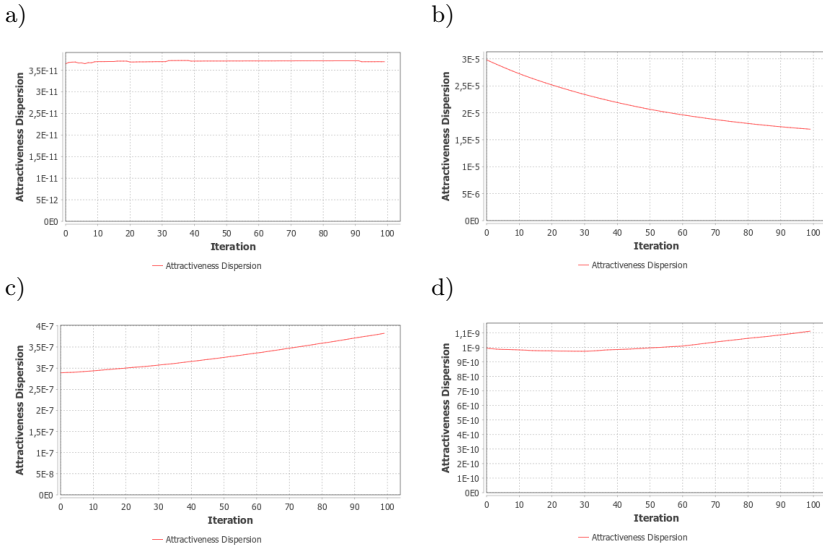
Figure 7. AD measures for considered ACO modifications: a) Ant Colony System;
b) Elitist Ant System; c) Rank-based Ant System; d) Max-Min Ant System

Considering the AR measure, we found that each tested ACO variant converges at a much slower rate than the classical algorithm (see Figs. 8, 2c, 3c, 4c, 5c). Again, we observed the strong performance of ACS (see Fig. 8a); moreover, its AR value stayed at a nearly constant level throughout the simulation. For the other ACO modifications, the diversity measure grew with a very similar trend to the classical approach with $\rho = 0.01$ (cf. Fig. 4c), albeit at a much slower pace.

Figure 8. AR measures for considered ACO modifications: a) Ant Colony System; b) Elitist Ant System; c) Rank-based Ant System; d) Max-Min Ant System

Except for ACS, we can see that each tested ACO modification performed significantly worse than the classical approach (see Fig. 9e). This was likely caused by the specific character of the Berlin52 problem, in which good results are achieved with high exploitation and low exploration of the solution space.

4.6. Remarks on parameter values

Summarizing, the following tendencies can be observed concerning the ACO parameter values:

1. If α is too high as compared to β, algorithm tends to show stagnation behavior without finding good solutions;
2. If α is too low, algorithm operates like stochastic multi-greedy algorithm;
3. If ρ value is high, pheromone evaporates too fast and algorithm cannot exploit positive feedback. If ρ is too small, there is danger of early convergence of algorithm.

Other studies [2] have shown similar dependencies.
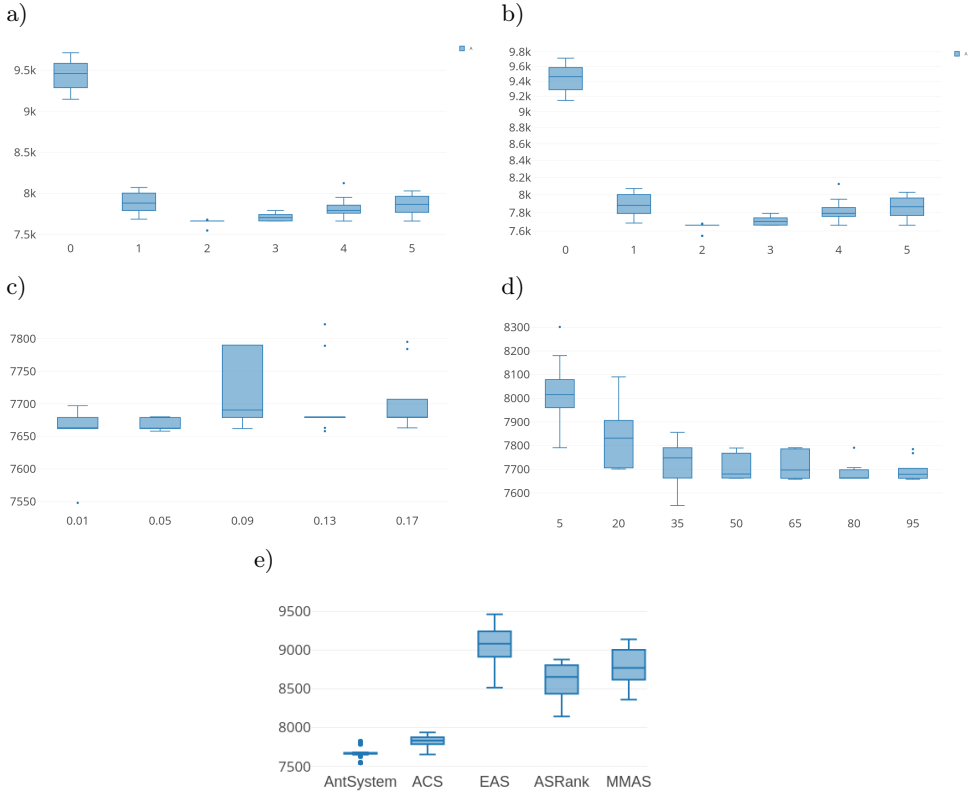
Figure 9. Final results of *Berlin52* problem obtained for ACS for different algorithm parameter values – final results for all tested algorithms: a) different values of α; b) different values of β; c) different values of ρ; d) different numbers of ants; e) different ACO variations

5. Conclusion

In this work, we studied the population diversity of the ACO algorithm and its variants using three diversity measures – all based on attractiveness values. We compared the behavior of these measures for the different parameter settings and known algorithm modifications.

Specifically, we have:

- refined and stabilized the method that was first sketched out in [28] regarding the evaluation of the diversity of a population of computing ants;
- extended and detailed the obtained results for classic ACO and applied the defined diversity measures to other popular and efficient ACO algorithms;
- carefully analyzed the results and concluded that the defined diversity measures were useful when their trends are observed;

- come to the conclusion that, although the findings are sound and repeatable, an ensemble classification that is based on many proposed diversity measures may be needed to properly evaluate the diversity – this should be included in any future work plans;
- established that, for each of the used alternative ACO algorithms, the pheromone levels were processed differently so that the ranges of the values of the diversity measures were different – however, comparing the trends can supply us with meaningful information.

Based on an analysis of the pheromone table, we can conclude that our diversity measures might be very helpful to support the maintenance of a balance between exploitation and exploration in ant colony optimization-type algorithms.

Diversity curves show the dynamics of a population; thus, the diversity descends with simulation time in most cases. Moreover, populations with medium dynamics of the diversity measures produce better final results. This follows the expected behavior of stochastic search methods – when the moves in a search space are sudden and their amplitudes are high, the optimal solution may be passed by. On the other hand, too-small movements might lead to a premature convergence. Ergo, a compromise of these of these extreme cases should be achieved.

There are obvious non-trivial relationships between the different parameters of ACO and the diversity measures. In few cases, a direct connection between the diversity measures and the final quality was observed, so further work is required in the future that involves data farming or variance-based sensitivity analysis, for example [1]. However, the trends presented by the diversity curves confirmed many reasonable behaviors of the ants (e.g., the exact value of the evaporation coefficient predictably affected the AR measure).

6. Future work

In the future, we plan to deal with auto-adaptive algorithms in the context of ACO to leverage our diversity measures. Also, we would like to test other flavors of ACO (e.g., distributed ACO with a higher number of ants [25] or desynchronized ACO in an HPC environment [26] for large-scale TSP problems) using our diversity-measurement techniques.

The proposed measures have been tested on a single instance of TSP (*berlin52*) that included a small number of nodes. In the future, we plan to extend our tests to larger instances of TSP and/or other problems.

Acknowledgements

References

[1] Bachniak D., Rauch L., Krol D., Liput J., Slota R., Kitowski J., Pietrzyk M.: Sensitivity analysis on HPC systems with Scalarm platform, *Concurrency and Computation: Practice and Experience*, vol. 29(9), 2017.

[2] Bullnheimer B., Hartl R., Strauss C.: A New Rank Based Version of the Ant System – A Computational Study, *Central European Journal of Operations Research*, vol. 7, pp. 25–38, 1999.

[3] Cantu-Paz E.: *Efficient and Accurate Parallel Genetic Algorithms*, Springer, 2001.

[4] Chen J., You X., Liu S., Li J.: Entropy-Based Dynamic Heterogeneous Ant Colony Optimization, *IEEE Access*, vol. 7, pp. 56317–56328, 2019. doi: 10.1109/ACCESS.2019.2900029.

[5] Colas S., Monmarché N., Gaucher P., Slimane M.: Artificial ants for the optimization of virtual keyboard arrangement for disabled people. In: *International Conference on Artificial Evolution (Evolution Artificielle)*, pp. 87–99, Springer, 2007.

[6] Cui Z., Li F., Zhang W.: Bat algorithm with principal component analysis, *International Journal of Machine Learning & Cybernetics*, vol. 10(3), pp. 603–622, 2019.

[7] Deng W., Xu J., Zhao H.: An Improved Ant Colony Optimization Algorithm Based on Hybrid Strategies for Scheduling Problem, *IEEE Access*, vol. 7, pp. 20281–20292, 2019. doi: 10.1109/ACCESS.2019.2897580.

[8] Dorigo M.: Optimization, learning and natural algorithms, *PhD Thesis, Politecnico di Milano*, 1992.

[9] Dorigo M., Di Caro G.: Ant colony optimization: a new meta-heuristic. In: *Evolutionary Computation, 1999. CEC 99. Proceedings of the 1999 Congress on*, vol. 2, pp. 1470–1477, IEEE, 1999.

[10] Dorigo M., Di Caro G., Gambardella L.M.: Ant Algorithms for Discrete Optimization. Technical Report, IRIDIA/98-10, Université Libre de Bruxelles, Belgium, 2009.

[11] Dorigo M., Gambardella L.M.: Ant colony system: a cooperative learning approach to the traveling salesman problem, *IEEE Transactions on Evolutionary Computation*, vol. 1(1), pp. 53–66, 1997.

[12] Dorigo M., Maniezzo V., Colorni A.: Ant system: optimization by a colony of cooperating agents, *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 26(1), pp. 29–41, 1996.

[13] Dorigo M., Stützle T.: Ant Colony Optimization: Overview and Recent Advances. IRIDIA – Technical Report Series, Université Libre de Bruxelles, 2009.

[14] Dorigo M., Stützle T.: *Ant colony optimization*, chap. 3.6.1, MIT, 2004.

[15] Gambardella L.M., Dorigo M.: Ant-Q: A reinforcement learning approach to the traveling salesman problem. In: *Machine Learning Proceedings 1995*, pp. 252–260, Elsevier, 1995.

[16] Glibovets N.N., Gulayeva N.M.: A Review of Niching Genetic Algorithms for Multimodal Function Optimization, *Cybernetics and Systems Analysis*, vol. 49(6), pp. 815–820, 2013. doi: 10.1007/s10559-013-9570-8.

[17] Herrera F., Lozano M.: Adaptation of Genetic Algorithm Parameters Based on Fuzzy Logic Controllers, *Genetic Algorithms and Soft Computing*, vol. 8, pp. 95–125, 1996.

[18] Li M., Ma B., Wang L.: On the Closest String and Substring Problems, *Journal of the ACM*, vol. 49(2), pp. 157–171, 2002. doi: 10.1145/506147.506150.

[19] Mohsen A.M.: Annealing Ant Colony Optimization with Mutation Operator for Solving TSP, *Computational Intelligence and Neuroscience*, vol. 2016, p. 8932896, 2016. doi: 10.1155/2016/8932896.

[20] Montemanni R., Gambardella L.M., Rizzoli A.E., Donati A.V.: Ant Colony System for a Dynamic Vehicle Routing Problem, *Journal of Combinatorial Optimization*, vol. 10, 2005.

[21] Morrison R.W., Jong K.A.D.: Measurement of Population Diversity. In: P. Collet, C. Fonlupt, J.K. Hao, E. Lutton, M. Schoenauer (eds.), *Proc. of EA 2001, LNCS 2310*, pp. 31–41, Springer, 2002.

[22] Nakamichi Y., Arita T.: Diversity control in ant colony optimization, *Artificial Life and Robotics*, vol. 7(4), pp. 198–204, 2004. doi: 10.1007/BF02471207.

[23] Negulescu S.C., Oprean C., Kifor C.V., Carabulea I.: Elitist Ant System for Route Allocation Problem. In: *Proceedings of the 8th WSEAS International Conference on Applied Informatics and Communications (AIC08)*, pp. 62–67, Rhodes, Greece, 2008.

[24] Sörensen K.: Metaheuristics the metaphor exposed, *International Transactions in Operational Research*, vol. 22(1), pp. 3–18, 2015. doi: 10.1111/itor.12001.

[25] Starzec M., Starzec G., Byrski A., Turek W.: Distributed ant colony optimization based on actor model, *Parallel Computing*, vol. 90, p. 102573, 2019.

[26] Starzec M., Starzec G., Byrski A., Turek W., Pietak K.: Desynchronization in distributed Ant Colony Optimization in HPC environment, *Future Generation Computer Systems*, vol. 109, pp. 125–133, 2020.

[27] Stützle T., Hoos H.H.: MAX–MIN ant system, *Future Generation Computer Systems*, vol. 16(8), pp. 889–914, 2000.

[28] Świderska E., Łasisz J., Byrski A., Lenaerts T., Samson D., Indurkhya B., Nowé, A., Kisiel-Dorohinicki M.: Measuring Diversity of Socio-Cognitively Inspired ACO Search. In: G. Squillero, P. Burelli (eds.), *Applications of Evolutionary Computation*, pp. 393–408, Springer International Publishing, Cham, 2016.

[29] Wolpert D.H., Macready W.G.: No free lunch theorems for optimization, *IEEE Transactions on Evolutionary Computation*, vol. 1(1), pp. 67–82, 1997.

[30] Yang K., You X., Liu S., Pan H.: A novel ant colony optimization based on game for traveling salesman problem, *Applied Intelligence*, vol. 50, pp. 4529–4542, 2020. doi: 10.1007/s10489-020-01799-w.

[31] Zhang D., You X., Liu S., Yang K.: Multi-Colony Ant Colony Optimization Based on Generalized Jaccard Similarity Recommendation Strategy, *IEEE Access*, vol. 7, pp. 157303–157317, 2019. doi: 10.1109/ACCESS.2019.2949860.

[32] Zhang M., Wang H., Cui Z., Chen J.: Hybrid multi-objective cuckoo search with dynamical local search, *Memetic Computing*, vol. 10(2), pp. 199–208, 2018.

Affiliations

Aleksander Byrski
AGH University of Science and Technology, al. Mickiewicza 30, 30-059 Krakow, Poland, olekb@agh.edu.pl, ORCID ID: https://orcid.org/0000-0001-6317-7012

Krzysztof Węgrzyński
AGH University of Science and Technology, al. Mickiewicza 30, 30-059 Krakow, Poland, krzysiu06123@gmail.com

Wojciech Radwański
AGH University of Science and Technology, al. Mickiewicza 30, 30-059 Krakow, Poland, wojciech.radwanski@gmail.com

Grażyna Starzec
AGH University of Science and Technology, al. Mickiewicza 30, 30-059 Krakow, Poland, gstarzec@agh.edu.pl, ORCID ID: https://orcid.org/0000-0002-0813-798X

Mateusz Starzec
AGH University of Science and Technology, al. Mickiewicza 30, 30-059 Krakow, Poland, mstarzec@agh.edu.pl, ORCID ID: https://orcid.org/0000-0001-8258-2443

Monika Bargiel
AGH University of Science and Technology, al. Mickiewicza 30, 30-059 Krakow, Poland, mstarzec@agh.edu.pl

Aleksandra Urbańczyk
AGH University of Science and Technology, al. Mickiewicza 30, 30-059 Krakow, Poland, aurbanczyk@agh.edu.pl, ORCID ID: https://orcid.org/0000-0002-6040-554X

Marek Kisiel-Dorohinicki
AGH University of Science and Technology, al. Mickiewicza 30, 30-059 Krakow, Poland, doroh@agh.edu.pl, ORCID ID: https://orcid.org/0000-0002-8459-1877