Eddy Viscosity Turbulence Models employed by Computational Fluid Dynamic

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ABSTRACT

This paper is discussing the advantages and disadvantages of the two-equation eddy-viscosity turbulence models employed to carry out computational fluid dynamic analyses. Simulating turbulence by means of numerical methods is one of the most critical problems in modeling fluid flow. The most popular, two-equation eddy-viscosity turbulence models are reviewed in this paper. These models rely on the eddy-viscosity, which according to Boussinesq approach relates Reynolds stress with mean velocity gradients. The consequences of such approach for two-equation models' capabilities in predicting fluid motion are analyzed. Moreover, the current work discusses: (i) Approaches taken to obviate the shortcomings of the Boussinesq assumption; (ii) Performance of the k-ε model and some of its modifications in predicting fluid structure; (iii) Advantages offered by the k-ω model in simulating some class of the flows; (iv) The Shear Stress Transport (SST) model benefiting from the features of the k-ε and k-ω models.

This paper discusses the vital problems related to simulating turbulence flows by means of CFD codes.

1. INTRODUCTION

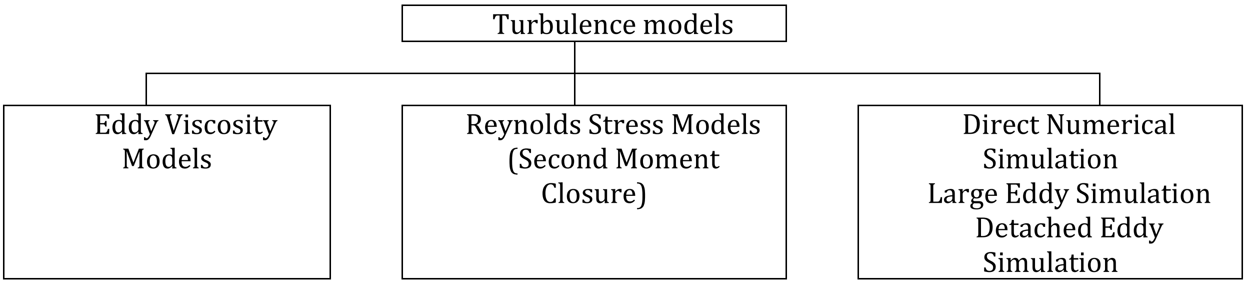
International industry develops very fast, imposing high demands on the technical devices to meet customer's expectations in terms of quality and reliability. Progress of the computer technology enables numerical modeling of the most complicated phenomenon described by differential equations. Simulating fluid motions described by Navier-Stokes equation is one of the most challenging problems in aerodynamics and has therefore received a lot of attention from the researchers and engineers. Industry demands more accurate computer codes, which would enable numerical testing of new applications prior to the experimental investigation in order to reduce time required for developing new products and improving their performance.

In order to simulate turbulent fluid motions, Reynolds decomposed the flow quantities into its average and fluctuating components. The new quantities were substituted to the Navier-Stokes equations and as a result Reynolds Averaged Equations (RANS) were obtained. The fluctuating components in the transport equations are unknown and cause system of the equation not to be closed. Additional equations, which enable to solve the new terms, are defined as turbulence models.

The turbulence models may be divided into three groups (see table 1). The first one relies on the Boussinesq assumption according to which, Reynolds' stress is proportional to mean strain rate. These models are often called eddy-viscosity models.

Second group involves modeling Reynolds stress transport equations – known as a second moment closure (SMC) method. The capabilities of the Reynolds stress transport equations in modeling turbulent flows are reviewed in Ref. [19, 13, 4, 5, 32]. The last group concerns the Direct Numerical Simulation (DNS), the Large Eddy Simulation (LES), and the Detached Eddy Simulation (DES). These models rely on a direct resolution of the Navier-Stokes equation for some class of the flows (DNS) or direct resolution of the Navier-Stokes equation for “large scales” and modeling “sub-grid scales” (LES, DES). The evaluation of the efficiency of these models in simulating turbulent flows may be found in the following Ref. [30, 8, 31, 10, 1].

Table 1. Classification of turbulence models.



This paper deals with the two-equation eddy-viscosity models, which in spite of their shortcomings are capable of delivering solutions of reasonable accuracy and low computational cost. The objective of this paper is to discuss some of the deficiencies and advantages of eddy-viscosity models available in the published research results. The discussion starts from analyzing the consequences of the Boussinesq approaches for prediction capabilities of turbulence models. Subsequently, the k-ε turbulence model will be discussed. It is probably one of the most popular models applied in the industrial calculations. Then the k-ω turbulence model considered as the first two-equation model will be presented. Relying on the advantages of the k-ε and k-ω models, Menter proposed two new turbulence models. The first one is the Base Line (BSL) model and second is the Shear Stress Transport model (SST).

2. CONSEQUENCES OF THE BOUSSINESQ ASSUMPTION

Boussinesq relates the Reynolds stress with mean strain-rate tensor by the dynamic eddy-viscosity μ_t according to the following equation [40]:

$$\tau_{ij} = 2\mu_t \left[S_{ij} - \frac{1}{3}\frac{\partial U_k}{\partial x_k}\delta_{ij} \right] - \frac{2}{3}\rho k \delta_{ij} \tag{2.1}$$

where the mean strain rate tensor is [40]:

$$S_{ij} = \frac{1}{2}\left[\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right] \tag{2.2}$$

and

k - turbulent kinetic energy,

u, U – velocity,

ε - turbulent dissipation rate,

μ_t - dynamic eddy-viscosity,

ρ - density,

τ - shear stress,

δ_{ij} -Kronecker delta

i, j - indicial notation with Einstein's summation convention, i,j=1,2,3 – refers to all equations presented in this paper.

Eddy-viscosity is calculated by means of different turbulence models. The Boussinesq eddy-viscosity assumption is analogous to the Stokes approximation for laminar flow. However, in turbulent flows eddy-viscosity is not a property of the fluid, but depends upon the characteristics of the flow under investigation. It is affected by the shape and nature of any solid boundaries, freestream turbulence intensity, and perhaps most significantly, flow history effects [38]. Two-equation eddy-viscosity turbulence models take advantage of the Boussinesq assumption of proportionality of the Reynolds stress to strain rates. This assumption allows the investigation of different flow phenomena.

As the understanding of the different closure models increases, it appears that this assumption needs to be revised. Application of the Boussinesq approximation leads to failure:

- to predict anisotropy of the normal stresses,
- to account for streamline curvature effects,
- to model secondary and swirling flows.

Additionally, it is not clear if the model's predictions have any relation to physical reality for flows that are unsteady and/or include boundary layers separation [40]. Wilcox, Ref. [39] pointed out another shortcoming of existing two-equation models, which is related to the turbulence that asymptotically, approaches isotropy. However, for Reynolds stress assumed to be proportional to mean strain rate, the two-equation model will predict an instantaneous return to isotropy [39].

Employing algebraic Reynolds stress models or non-linear eddy-viscosity formula may alleviate some of the deficiencies of the Boussinesq assumption.

2.1. ALGEBRAIC REYNOLDS STRESS MODELS

The Reynolds Stress Models, contrary to the eddy-viscosity models, represent the Reynolds stress tensor using partial differential equations. By the introduction of an approximation for the transport terms, Reynolds stress model can be reduced to a set of algebraic equations. These equations form an algebraic stress model (ASM), which implicitly determines the Reynolds stresses (locally) as a function of k, ε, and mean velocity gradients. Due to the approximation, algebraic stress model are inherently less general and less accurate then Reynolds' stress models. However, because of their relative simplicity, they have been widely used as turbulence models [30].

The advantages and shortcomings of the algebraic stress models were discussed by Gatski et al. in Ref. [9]. In this reference, algebraic stress models were obtained from second-order closure by means of an equilibrium hypothesis, in which the Reynolds stress convection and transport terms were neglected. In these models Reynolds stresses were related implicitly to the mean velocity gradients. This provided the first formal means, based on a higher-order closure, to justify the extension of the Boussinesq hypothesis to incorporate nonlinearties into the mean velocity gradients. However, this type of algebraic stress model is cumbersome to implement in complex flows since the stress-strain relation is not explicit; numerical stiffness problems can result from the need for successive matrix inversions at each iteration. Pope [29] presented a methodology for obtaining an explicit relation for the Reynolds stress tensor from implicit algebraic stress models. Gatski et al. [9] extended the results of Pope [29] to three-dimensional turbulent flows in non-internal frames starting from a more general hierarchy of second-order closure models. The presented explicit models lead to considerable savings in computational expense by avoiding the need for successive matrix inversions to obtain the Reynolds stresses for a given set of mean velocity gradients. Gatski et al. stressed that when algebraic stress formulations are applied to non-equilibrium turbulent flows with localized strain rates that are large, there is a need for regularization. Model presented in [9] was regularized by a Padé approximation and verified in applications to non-trivial, two-dimensional turbulent flows involving shear and rotation. It was shown that the new explicit algebraic stress models represent the equilibrium Reynolds stress anisotropies predicted by second-order closure in homogeneous turbulent flows [9].

Example of the algebraic Reynolds stress approach, taken to alleviate the Boussinesq assumption drawbacks is presented in Ref. [39]. The objective of this reference was to build an algebraic stress model delivering an accurate method for describing effects of streamline curvature and secondary motion. The improved representation of the Reynolds-stress tensor was accomplished through the introduction of a multiscale description of the turbulence, i.e. two energy scales are used corresponding to upper and lower partitions of the turbulence energy spectrum. In Ref. [39] the general observation is made that eddies in the lower partition are expected to contain most of the vorticity, to be isentropic, and to dissipate rapidly into heat. A key feature of eddies in the upper partition is that they are more or less inviscid. In terms of the model equation these approaches yield i) the two-equation k-ω model devised by Wilcox [40] and ii) an inviscid tensor equation for the upper partition contribution to the Reynolds-stress tensor. A novel feature of the formulation is that the differential equation for the Reynolds stress tensor is of first order, which in effect corresponds to what can be termed as an “algebraic stress model” with convective terms. Author of Ref. [39] summarized the complete set of equations that constitute the model, which can be applied to a wide range of turbulent flows including homogeneous turbulence, compressible and incompressible two-dimensional boundary layer, and unsteady boundary layers including periodic separation and reattachment. Comparisons with corresponding experimental data show that this formulation reproduces all salient features of the flows considered [39]. The evaluation of the Multi-Scale model capabilities regarding the wall-jet flows was discussed in Ref. [27].

2.2. NON-LINEAR EDDY-VISCOSITY MODIFICATIONS

Nonlinear eddy-viscosity models are considered another method of improving the Boussinesq assumption. According to discussion in Ref. [16] some attempts aimed at developing and using nonlinear eddy-viscosity models rely on introducing quadratic terms to the Boussinesq relations. These quadratic nonlinear eddy-viscosity models successfully reproduced turbulence driven secondary flows and streamline curvature (including swirl). Kazuhiko [16] pointed out an inherent defect in the stress-strain relation and tried to remove it. In shear-free turbulence appearing for example near the free surface of an open channel flow, all strain and vorticity tensor components vanish. Thus, the linear and nonlinear stress-strain relations always return isotropic turbulence there, while the actual turbulence is anisotropic. Therefore, additional cubic terms were introduced to the stress-strain relation. Kazuhiko [16] investigated the near-wall performance of the typical linear and nonlinear eddy-viscosity models. The use of nonlinear terms in the stress-strain relations is essential to predict complex strain fields. Moreover the cubic terms are necessary to mimic streamline curvature and swirl effects. The combined effects of the new local near-wall parameters and nonlinear stress-strain relations have significantly enhanced performance of the eddy-viscosity schemes [16].

According to the results provided in Ref. [37], the modification to the eddy-viscosity formulation relaying on introducing quadratic terms leads to the improvement in prediction capabilities of the k-τ model in recirculating flows. The nonlinear eddy-viscosity turbulence models (NLEVM) were also validated in Ref. [3]. The study shows that NLEVM improve the numerical predictions in shock/boundary-layer interaction, compared with the linear models, but they require longer computing times. The nonlinear models improve considerably the prediction of the maximum value of shear stress, but the results are still far from the experimental [3].

Some of the shortcomings of the Boussinesq approach can be partially overcome by using two-equation models with a nonlinear algebraic correction of the eddy-viscosity or by application of algebraic Reynolds stress models. Major improvement can only be achieved by means of higher order closure [13].

3. CAPABILITIES OF TWO-EQUATION EDDY-VISCOSITY TURBULENCE MODELS

Two-equation turbulence models that employed transport equation for turbulent kinetic energy capture interest of many researchers. Turbulence kinetic energy equation has been developed to incorporate nonlocal and flow history effects in the eddy-viscosity. Prandtl postulated computing a characteristic velocity scale for the turbulence. He chose the kinetic energy (per unit mass) of the turbulent fluctuations, *k*, as the basis of the velocity scale

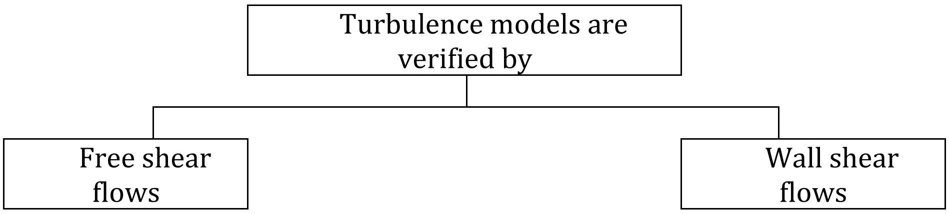
$$k = \frac{1}{2}\overline{u_i u_i} \tag{3.1}$$

One of the key conclusions, of the 1980-81 AFOSR-HTTM-Stanford Conference on Complex Turbulent Flows, was that the greatest amount of uncertainty about two-equation models lies in the transport equation complementing the equation for *k*. However, two-equation models are complete, i.e., can be used to predict properties of a given turbulent flow with no prior knowledge of turbulent structure. They are, in fact, the simplest complete models of turbulence [38]. The two-equation models rely on Boussinesq assumption so as a result suffer from all the deficiencies of this assumption discussed in point 2.

Two-equation turbulence models closing the RANS contain constants, which appeared during the process of derivation. These constants determine the capabilities of two-equation models in simulating fluid flow. Menter [22] recognizes, based on his experience, that small changes [5-10%] in simulating constants could lead to significant improvement or deterioration of model predictions. The validity of the proposed model ought to be tested against a number of challenging and well-documented research flows. It is often unclear whether the improvements presented for one type of flows will not lead to deterioration for another class of equally important flows [22].

Advantages and limitations of turbulence models are verified in terms of providing predictions of turbulent free shear flows and predictions of turbulent wall shear flows, (see table 2). Hwang et al. [13] concluded that each turbulence model can be applied successfully to some turbulent flows but obtains unsatisfactory prediction results for other flows, especially for flows, which are very different from those to which the models were calibrated [13].

Table 2. Fluid flow conditions applied to verify turbulence models capabilities.



Free shear flow is considered as a flow, which is not bounded by any solid surface, for example far wake flow downstream of any object or jets ejected from a nozzle. Computation of free shear flows enables to verify the accuracy of modeling the Reynolds stress, turbulent kinetic energy, and its dissipation rate. The pressure gradient in a free shear flows is negligible, therefore results are most sensitive to the mentioned above flow quantities. Also the complexity of the near-wall turbulence is absent in free shear flows, so that the accuracy of a turbulence model in predicting the general flow field can be carefully visualized without interference from wall turbulence [13].

Wall shear flow is considered as a flow bounded by solid surface, characterized by no-slip boundary conditions prescribed. Backward facing step flow is one of the wall bounded test case flows. This configuration is very popular because the geometry is simple and it covers many important flow phenomena, like separation, recirculation, reattachment, shear-layer mixing and the development of a boundary layer downstream of the reattachment point [11]. In the literature there are many examples of test cases of different geometrical and physical configuration. Reviewing these is beyond the scope of this paper. Nevertheless it is vital to stress the importance of the test cases in terms of their flow character verification capabilities.

4. EXAMINATION OF THE k-ε MODEL PERFORMANCE IN SIMULATING TURBULENT FLOWS

The k-ε model is an example of one of the most popular turbulence models. Due to its capabilities, it is used both by scientists and engineers. It is also available in most commercial CFD codes. Jones and Launder [14] were the first to derive the equations that constitute the model. Further analyses leading to modifications of the constants were carried out by Launder and Sharma [18]. Their approach is considered as a parent (standard) k-ε model. Transport equation for turbulent kinetic energy and turbulent dissipation rate can be written [38]:

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial(\rho U_j k)}{\partial x_j}=\tau_{ij}\frac{\partial U_i}{\partial x_j}-\rho\varepsilon+\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] \tag{4.1}$$

$$\frac{\partial(\rho \varepsilon)}{\partial t}+\frac{\partial(\rho U_j \varepsilon)}{\partial x_j}=c_{\varepsilon 1}\frac{\varepsilon}{k}\tau_{ij}\frac{\partial U_i}{\partial x_j}-c_{\varepsilon 2}\rho\frac{\varepsilon^2}{k}+\frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] \tag{4.2}$$

where turbulent viscosity is specified as follows:

$$\mu_t = c_\mu\frac{k^2}{\varepsilon} \tag{4.3}$$

and

$c_{\varepsilon 1}, c_{\varepsilon 2}$ - constants,

t - time,

σ - closure coefficient.

The ε-equation is the transport equation complimenting the model and represents the dissipation rate of energy transfer from the large eddies.

4.1. THE k-ε MODEL APPLIED TO MODEL WALL SHEAR FLOWS

In the derivation of the k-ε model, it was assumed that the flow is fully turbulent, and the effects of molecular viscosity are neglected. The standard k-ε model is valid only for fully turbulent flows [7]. Therefore it is not capable of handling calculations of the low-Reynolds number flows or wall bounded flows. Hence, the standard k-ε model was modified and their altered formulations are called low-Reynolds number models. The advantages and disadvantages of these models are discussed in Ref. [28, 38]. The reviewing methods aimed at modeling wall bounded flows are beyond the scope of this paper. Nevertheless it is important to stress that according to Vieser et al. [43] the ε-equation has severe limitations in the near-wall region. Turbulence models based on the ε-equation lead to overprediction of the turbulent length scale in flows with adverse pressure gradients, resulting in high wall shear stress and high heat transfer rates. In combination with low-Re number extensions, the ε-equation proved to be numerically stiff, leading to a significant reduction in numerical robustness. The experience with low-Reynolds number formulations for heat transfer predictions using the ε-equation was that it produces significant overprediction of the local Nusselt number at reattachment point [43].

Another deficiency of the k-ε model can be the source of substantial inaccuracies and numerical stiffness, and is related to the fact the balance of terms at the wall in transport equation modeling dissipation rate depends on higher-order correlations [33]. According to the results of the analysis presented in Ref. [33] the k-ε model suffers from the lack of the natural boundary conditions for the dissipation rate. The available boundary conditions are either asymptotically inconsistent or numerically stiff. Commonly used boundary conditions are presented in Ref. [33]. The example of the derived boundary conditions for the dissipation rate is

$$2\frac{\mu}{\rho}\left(\frac{\partial\sqrt{k}}{\partial y}\right)^2=\varepsilon, \tag{4.4}$$

where

μ - dynamic molecular viscosity,

which requires information at the wall on the second-order derivative of the turbulent kinetic energy – feature that can lead to considerable numerical stiffness. Another approach is to utilize the Neumann boundary condition

$$\frac{\partial\varepsilon}{\partial y}=0, \tag{4.5}$$

which has been applied in a variety of applications of the k-ε model with no solid theoretical or experimental justification [33].

4.2. THE k-ε MODEL APPLIED TO MODEL FREE SHEAR FLOWS

Authors of Ref. [13] verified the capabilities of the k-ε turbulence models in predicting free shear flows. Approximations of plane jet, round jet, plane mixing layer, and plane wake turbulent free shear flows were investigated. The predicting spreading rate of free shear flow is a proper criterion for examining the effectiveness of the considered models. Based on results of the analyses it was pointed out that only results of the analyses of the rates of spread for the round jet flow were not satisfying. All formulations considered in Ref. [13] (k-ε and its modifications, SMC) have difficulties in approximating intermittence of turbulence at the edge of this flow. A similar difficulty is also found in other free shear flows. Authors of Ref. [13] pointed out that multi-time-scale model of Hanjalic et al. [12] performs better for a round jet flow. The deficiencies of the k-ε model computations of the rate of spreading for the round jet were also investigated in Ref. [30]. Another paper where authors have pointed out the spreading rate modeling problems by the k-ε turbulence model is Ref. [25]. The k-ε model fails to capture the experimentally observed difference between the spreading rate of a plane jet and the spreading rate of a round jet. In experiments, the spreading rate of a round jet is 15% lower than that of a plane jet, but in simulations with standard k-ε model, the round-jet spreading rate is 15% higher. Attempts to overcome the round jet/plane jet anomaly involved, almost invariably, modifying the closure coefficients in the dissipation-rate equations of the k-ε model. Discussed in Ref. [25] modifications of the closure coefficients in the standard k-ε model reduced either the generality or the numerical stability of the CFD code.

4.3. EXAMPLES OF MODIFICATIONS OF THE STANDARD k-ε MODEL

(i) The realizable k-ε model

The realizable k-ε model is an example of approach taken to improve capabilities of the standard k-ε turbulence model. The term "realizable" means that the model satisfies certain mathematical constraints on the Reynolds stresses, consistent with the physics of turbulent flows. Neither the standard k-ε model nor the RNG k-ε model is realizable. The realizable k-ε model differs from the standard k-ε model in two important ways. The first one is that the former contains a new formulation for the turbulent viscosity. The second differences concerns a new transport equation for the dissipation rate, ε, that was derived from an exact equation for the transport of the mean-square vorticity fluctuation. An immediate benefit of the realizable k-ε model is that it more accurately predicts the spreading rate of both planar and round jets. It is also likely to offer superior performance for flows involving rotation, boundary layers under strong adverse pressure gradients, separation and recirculation [7].

(ii)The RNG k-ε model

Example of broadening the applicability of the standard k-ε model to simulate different fluid motion is also the RNG k-ε model. It was derived using a rigorous statistical technique (renormalization group theory). The analytical derivation results in constants different from those in the standard k-ε model, and additional terms and functions in the transport equations for k and ε [7]. The RNG formulation has additional term in its ε-equation, due to which it is more responsive to the effects of rapid strain and streamline curvature, which explains its superior performance for certain class of flows [7]. The RNG theory provides also an analytical formula for turbulent Prandtl numbers, while the standard k-ε model uses user-specified, constant values. The scale elimination procedure in RNG theory results in a differential equation for turbulent viscosity. This equation is integrated to obtain an accurate description of variation of the effective turbulent transport with the effective Reynolds number (or eddy scale), allowing better description of low-Reynolds number and near-wall flows. The RNG model presented in Ref. [7] provides an option to account for the effects of swirl or rotation by modifying the turbulent viscosity [7].

Application of the RNG k-ε models to determine an excessive solid particle erosion damage of the fourth stage rotor disc of an 110MW double flow geothermal turbine is presented in Ref. [20]. Combustion of two heavy-duty DI diesel engines was simulated by means of the RNG k-ε models in Ref. [36].

(iii) The $(k\text{-}\varepsilon)_{1E}$ one equation model

In Ref. [24] a transformation of the high Reynolds number version of the k-ε model to a one-equation model $(k\text{-}\varepsilon)_{1E}$ is presented. The derivation is based on the Bradshaw's assumption that the turbulent shear stress is proportional to the turbulent kinetic energy. Menter, in order to arrive at one-equation model expressed the time derivative of the eddy-viscosity by the time derivative of k and ε. One-equation models are beyond the scope of this paper, however it is worth to discuss Menter's formulation capabilities in describing backward facing step flow. The results of this test case depict that the k-ε model underpredicts the reattachment location by about 5 percent revise, and is generally not in good agreement with data in the separated region and near reattachment. Different low Reynolds number forms of the k-ε model give different skin friction distribution, so this behavior is certainly a result of the low-Reynolds number terms. However, the $(k\text{-}\varepsilon)_{1E}$ model is in very good agreement with the experimental data. The reattachment location is predicted with sufficient accuracy. Both of the presented in Ref [24] models fail to correctly approximate the recovery of the velocity profiles downstream of reattachment. This is a general problem with existing models [24].

Research resulting in alleviating the shortcoming of the standard k-ε model is highly demanded by the industry. Thus, in the literature many modifications of the basic model are available. Some of them involve additional transport equations. Ref. [3] discussed the three-equation model ($k\text{-}\varepsilon\text{-}A_2$) and in Ref. [43, 26] the four-equation model (k-ε-v2f) is presented. These models will not be considered here.

Most of the altered models provide improved prediction capabilities for some class of the flows. However, general formula that might be applicable to most of the flows under investigation is yet to be specified.

5. Assessment of the k-ω model predictability of turbulent flows

Wilcox developed k-ω turbulence model that is popular in performing CFD computations. This model employs transport equations for turbulent kinetic energy k and specific dissipation rate ω. Transport equation for these quantities can be written [40]:

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial(\rho U_j k)}{\partial x_j}=\tau_{ij}\frac{\partial U_i}{\partial x_j}-\beta^*\rho k\omega+\frac{\partial}{\partial x_j}\left[\left(\mu+\sigma^*\mu_t\right)\frac{\partial k}{\partial x_j}\right] \tag{5.1}$$

$$\frac{\partial(\rho\omega)}{\partial t}+\frac{\partial(\rho U_j\omega)}{\partial x_j}=\alpha\frac{\omega}{k}\tau_{ij}\frac{\partial U_i}{\partial x_j}-\beta\rho\omega^2+\frac{\partial}{\partial x_j}\left[\left(\mu+\sigma\mu_t\right)\frac{\partial\omega}{\partial x_j}\right] \tag{5.2}$$

where dynamic eddy-viscosity is specified as follows:

$$\mu_t=\rho\frac{k}{\omega} \tag{5.3}$$

and

α,β - constants,

σ - closure coefficient,

ω - specific dissipation rate.

The specific dissipation rate ω - is the rates of dissipation of turbulence per unite energy. Wilcox in Ref. [40] presented a comprehensive and critical review of closure approximations for two-equation turbulence models. According to Wilcox the greatest amount of uncertainty and controversy over two-equation models lies in the scale-determining equation. Based on the performed analysis, Wilcox postulated a new k-ω turbulence model. Ref. [40] gives the study of the most appropriate values of closure coefficients.

5.1. THE k-ω MODEL APPLIED TO MODEL WALL SHEAR FLOWS

The new k-ω turbulence model ensures that with no viscous damping of the model's closure coefficients and without the aid of wall functions, the model equations can be integrated through the viscous sublayer. Wilcox verified the applicability of the new model to resolve turbulent flows in the boundary layer by means of the perturbation analysis, and compared with experimental data. Analysis concerns the incompressibility defect layer, including effects of pressure gradient. Obtained results were compared with these by k-ε model proposed by Jones-Launder [14] and k-ω^2 proposed by Wilcox-Rubesin [41] and with experimental data. Comparison analysis indicated that new k-ω model is superior to the other models. Thus, Wilcox confined the further analysis to the new k-ω model [40].

The viscous sublayer including effects of surface roughness and surface mass injection were also investigated by means of perturbation analysis. The ω-oriented equations offer solutions in which the value of ω may be arbitrarily specified at the surface. This provides a natural way to incorporate effects of surface roughness through surface boundary conditions. The obtained results fall well within experimental errors bounds [40].

Ref. [40] includes results of attached boundary layer computations for boundary layers subjected to adverse pressure gradient, surface mass injection and compressibility. Author concludes the presented analyses that the k-ω model is much more accurate that comparable two-equation models (k-ε and k-ω^2) for the considered test cases [40].

The deficiency of the k-ω concerns the fact that the model does not correctly approximate the asymptotic behavior of the turbulence as it approaches the wall. Close to the surface the eddy-viscosity is much smaller than the molecular viscosity and the asymptotic behavior of the mean flow profile is independent of the asymptotic form of the turbulence. Therefore, even if the turbulence model is not asymptotically consistent, the mean flow profile and the wall skin friction are still predicted correctly [22].

The problem of the appropriate modeling of the behavior of the turbulence as it approaches the wall was also discussed in Ref. [33]. According to the conclusions of the analysis the k-ω model yields results for the turbulent kinetic energy – as well as other turbulence quantities – that are asymptotically inconsistent. By referring to the baseline ω-equation, it was proved that exact cross diffusion term (CDT) was neglected in the modeled ω-transport equation. The cross diffusion term (CDT) is specified by the following equation:

$$CDT = \left(\frac{2\mu}{k\rho}\right)\left(\frac{\partial k}{\partial x_j}\right)\left(\frac{\partial \omega}{\partial x_j}\right), \tag{5.3}$$

Based on presented in Ref. [33] analyses, the k-ω model can be asymptotically consistent by the addition of the viscous cross diffusion term and by damping the coefficient of ω^2 to one at wall. However, Speziale et al., derived a model transport equation for $\tau \equiv \frac{1}{\omega}$, since τ is not singular near the wall [33]. Equations that constitute the k-τ model are discussed in Ref. [33]. The evaluation of the k-τ model capabilities for the incompressible flat-plate boundary layer with adverse pressure gradient, and incompressible flow past backward steps are presented in Ref. [37]. The k-τ model is not discussed because is not very popular in the industrial calculations.

5.2. THE k-ω MODEL APPLIED TO MODEL FREE SHEAR FLOWS

According to Ref. [23] the k-ω model has a very strong sensitivity to the freestream values of the ω_f specified for ω outside the boundary layer. To investigate this disturbing feature of the equations more closely the self-similar equations for incompressible equilibrium boundary layers, as well as for the wake, were solved numerically in Ref. [23]. Additionally in order to emphasize, that the solution dependency of ω_f is not simply a property of the self-similar equations, the Navier-Stokes computations were performed for a flat plate boundary layer. The analyses were based on the results of investigation performed by Wilcox [40]. The presented predictions confirm that eddy-viscosity is strongly affected by the freestream values of ω_f. The computations performed for the equilibrium boundary layer reveal that the eddy-viscosity increases as ω_f decreases and does not go to zero at the boundary layer edge. The influence on the mean velocity flow profile is moderate, and that is the reason why the freestream dependency is of little consequence for boundary-layer flows. The results of the wake flows depict also the same character of eddy-viscosity changes as for boundary layer. However, in this case the velocity profile is strongly affected and shows a much larger spreading rate as ω_f decreases. The computations performed for the k-ε model show almost no dependency on the freestream values. The results are the same in both cases for self-similar equations and for the flat plate boundary layer computations based on the incompressible Reynolds-averaged Navier-Stokes equations. According to Menter the k-ω freestream dependency is severe in free shear layer applications [23].

Performance of the k-ω model is also discussed in Ref. [25]. The k-ω model describes the behavior of attached boundary layer in adverse pressure gradients more accurately than k-ε model, but performs poorly in free shear flows. In the revised Wilcox [42] turbulence model, closure coefficients, which were previously constant [39] became functions of the flow variables. Wilcox showed that the revised model works well in simulations of self-similar free-shear flows [25]. Wilcox attributes the success of the k-ω model in modeling adverse pressure gradient layers to the lack of the cross-diffusion term (CDT) in the modeled boundary layer. On the other hand, changing the dependent variables from ε to ω in the equation defining eddy-viscosity shows that cross-diffusion term is an implicit feature of the k-ε model. Apart from the differences in the closure coefficients, this is the major difference between the two models. The lack of the cross-diffusion term in the ω-equation is also believed to be responsible for the poor performance of the 1988 k-ω [39] model in free shear flows. In his 1998 model [42], Wilcox uses the cross diffusion parameter, which is large in free shear flows and small in boundary layers, to selectively increase the dissipation term in the k equation and hence to reduce the eddy-viscosity in free shear flows [25].

According to the Ref. [43] the ω-equation has significant advantages near the surface and accurately predicts the turbulent length scale in adverse pressure gradient flows, leading to improved wall shear stress and heat transfer predictions. Furthermore, the model has a very simple low-Re formulation, which does not require additional non-linear wall damping terms. The correct sublayer behavior is achieved through Dirichlet boundary conditions for ω [43].

6. EFFECTIVENESS OF THE SHEAR-STRESS TRANSPORT (SST) MODEL IN SIMULATING TURBULENCE FLUID MOTIONS

In order to alleviate the shortcoming and take advantage of the k-ω and k-ε models the two new two-equation eddy-viscosity models were devised and presented in Ref. [22]. Menter developed the two new turbulence models using some of the elements of already existing models. The first model, referred to as baseline (BSL) model, utilizes the original k-ω model of Wilcox in the inner region of the boundary layer and switches to the standard k-ε model in the outer region and in free shear flows. The second model results from a modification to the definition of the eddy-viscosity in the BSL model, which accounts for the effect of the transport of the principal turbulent shear stress (SST) [22].

6.1. BASE LINE k-ω MODEL

Author of Ref. [22], in order to achieve the desired features in different flow regions, transformed the standard high-Reynolds-number version of the k-ε model to a k-ω formulation.

Transformed k-ε model was written in the following form [22]:

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial(\rho U_j k)}{\partial x_j}=\tau_{ij}\frac{\partial U_i}{\partial x_j}-\beta^*\rho\omega k+\frac{\partial}{\partial x_j}\left[(\mu+\sigma_{k2}\mu_t)\frac{\partial k}{\partial x_j}\right] \quad (6.1)$$

$$\frac{\partial(\rho\omega)}{\partial t}+\frac{\partial(\rho U_j\omega)}{\partial x_j}=\frac{\gamma_2}{\nu_t}\tau_{ij}\frac{\partial U_i}{\partial x_j}-\beta_2\rho\omega^2+\frac{\partial}{\partial x_j}\left[(\mu+\sigma_{\omega2}\mu_t)\frac{\partial\omega}{\partial x_j}\right]+2\rho\sigma_{\omega2}\frac{1}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial\omega}{\partial x_j} \quad (6.2)$$

where:

β, β^*, γ - constants,

σ - closure coefficient,

ν_t - kinematic eddy-viscosity.

The difference between this formulation and the original k-ω model is that an additional cross-diffusion term appears in the ω-equation and that the modeling constants are different. Furthermore, the standard k-ω model (equation [5.1] and [5.2]) was multiplied by a blending function F_1 and then equations (6.1) and (6.2) were multiplied by $(1-F_1)$ and the corresponding equations of each set were added together to give the new model [22]:

$$\frac{\partial(\rho k)}{\partial t}+\frac{\partial(\rho U_j k)}{\partial x_j}=\tau_{ij}\frac{\partial U_i}{\partial x_j}-\beta\rho\omega k+\frac{\partial}{\partial x_j}\left[(\mu+\sigma_k\mu_t)\frac{\partial k}{\partial x_j}\right] \quad (6.3)$$

$$\begin{aligned}\frac{\partial(\rho\omega)}{\partial t}+\frac{\partial(\rho U_j\omega)}{\partial x_j}&=\frac{\gamma}{\nu_t}\tau_{ij}\frac{\partial U_i}{\partial x_j}-\beta\rho\omega^2+\frac{\partial}{\partial x_j}\left[(\mu+\sigma_\omega\mu_t)\frac{\partial\omega}{\partial x_j}\right]\\&+2\rho(1-F_1)\sigma_{\omega2}\frac{1}{\omega}\frac{\partial k}{\partial x_j}\frac{\partial\omega}{\partial x_j}\end{aligned} \quad (6.4)$$

where

F_1 - blending function,

β, γ - constants,

σ - closure coefficient.

The blending function F_1 was designed to be one in sublayer and logarithmic region of the boundary layer and to gradually approach zero in the wake region. This means that the new model is based on the k-ω formulation, with the original Wilcox model activated in the near wall region and the standard k-ε model activated in the outer wake region and in free shear layers. This first step leads to a new model that will be termed the baseline (BSL) model. The BSL model has a performance very similar to that of the original k-ω model, but without the undesirable freestream dependency [22].

6.2. THE SHEAR STRESS TRANSPORT MODEL (SST)

The mathematical analysis of the behavior of two-equation models in adverse pressure gradient flows was largely restricted to the logarithmic region. Although, the behavior of the model in the logarithmic region is important, especially in flows with moderate pressure gradients, it is the level of the eddy-viscosity in the wake region that ultimately determines the ability of an eddy-viscosity model to simulate strong adverse pressure gradient flows. This was demonstrated by the improvement that the Johnson-King model [15] introduced over standard algebraic models by reducing the wake region eddy-viscosity in adverse gradient flows. The limited influence of the logarithmic region on the results for strong adverse pressure gradient is also evident in the failure of the k-ω model to accurately determine pressure-induced separation despite its superior log-region characteristic. The basic idea behind the Johnson-King model [15] is to enforce Bradshaw's observation that the principal turbulent shear stress τ is proportional to the turbulent kinetic energy k in the wake region of the boundary layer [22]:

$$\tau = \rho a_1 k \tag{6.4}$$

with a_1 being a constant. On the other hand, in two-equation models, the shear stress is computed from [22]:

$$\tau = \mu_t \frac{\partial u}{\partial y} \tag{6.5}$$

For conventional two-equation models, Eq. (6.5) can be rewritten to give [22]:

$$\tau = \rho \sqrt{\frac{\mathrm{PRODUCTION}_k}{\mathrm{DISSIPATION}_k}}\, a_1 k \tag{6.6}$$

In adverse pressure gradient flows the ratio of production to dissipation can be significantly larger than one as found from the experimental data presented in Ref. [22] and therefore Eq. (6.6) leads to an overprediction of τ. To satisfy Eq. (6.4) within the framework of an eddy-viscosity model, the kinematic eddy-viscosity is redefined in the following way [22]:

$$\nu_t = \frac{a_1 k}{\max\left(a_1\omega; \frac{\partial u}{\partial x} F_2\right)} \tag{6.7}$$

where F_2 is a function that is one for the boundary layer flows and zero for free shear layers. In an adverse pressure gradient boundary layer, production of k is larger than its dissipation and Eq. (6.7) therefore guarantees that Eq. (6.4) is satisfied whereas the original formulation $\nu_t = k/\omega$ is used for the rest of the flow [22].

To recover the original formulation of the eddy-viscosity for free shear layers {where Bradshaw's assumption, expressed in Eg. (6.4) does not necessarily hold} the modification to the shear-stress transport (SST) model is limited to wall bounded flows.

This is achieved in the same way as it is for the BSL model by applying a blending function F_2. For general flows $\frac{\partial u}{\partial y}$ is taken to be the absolute value of the velocity [22].

Enforcing Bradshaw's observation introduces a lag effect into the equations that accounts for the transport of the principal turbulent shear stress. It was shown in [22] that the classical formulation of the eddy-viscosity in two-equation models violates Bradsaw's relation and thereby neglects this important effect. Thus, in the second step of the analyses presented in Ref. [22], the definition of the eddy-viscosity was modified relying on Bradshaw's observation to account for the transport of the principal turbulent shear stress. The resulting model was called the Shear-Stress Transport (SST) model. Modification of the eddy-viscosity definition leads to a major improvement in performance over both the original k-ε and k-ω models.

6.3. THE SST MODEL APPLIED TO MODEL WALL SHEAR FLOWS

The BSL and SST models were carefully tuned and tested for a large number of challenging research flows and results of the analyses were presented in Ref. [22]. The test cases included:

- flows involving adverse pressure gradients,
- flow over a backward-facing step,
- NACA 4412 airfoil flow,
- transonic bump flow.

The original k-ω, as well as the standard k-ε model is included in the comparison.

One of the most important aspects of turbulence model for aerodynamic application is its ability to accurately predict adverse pressure gradient boundary layer flows. It is especially important that a model is able to compute the location of flow separation and the displacement effect associated with it. For flow over circular cylinder with strong adverse pressure gradient, the SST model predicted the largest amount of separation, whereas the k-ε model stayed firmly attached. The velocity profiles predicted by SST model clearly were in the best agreement with the experiments.

The skin friction distribution and the reattachment length for flow over a backward-facing step were approximated by all the k-ω models better then the k-ε model.

For the NACA airfoil flow, the SST model predictions of the displacement effect were in very good agreement with experiments. The results of the analysis also exhibit poor performance of the original k-ω model caused by its freestream dependency. The change in ω_f had very little impact on the computation with the SST model, whereas the original k-ω model predicted significantly different results [22].

Approximations of the wall pressure distribution for the axisymetric transonic shockwave/turbulent boundary-layer experiment provided by the SST formulation are significantly better than other models [22]. As expected, the BSL model gives results very close to the original k-ω model of Wilcox but avoids its freestream dependency. The SST model leads to a significant improvement for all flows involving adverse pressure gradients. According to Ref. [22] the SST model demonstrated the ability to accurately calculate pressure-induced separation and the resulting viscous-inviscous interaction is also superior to other formulation with regard to numerical stability [22].

Vieser et al. in Ref. [43] tested the functioning of the eddy-viscosity two-equation models in heat transfer simulations. A series of test cases was carried out with the k-ε, k-ω and the SST model. According to the performed analyses it was shown that the SST model, in combination with optimal wall treatment, does provide highly accurate results for a wide variety of heat transfer test cases [43].

The capabilities of SST model in simulating the drag of engine-airframe interference are presented in Ref. [17]. The recent advances in CFD and computer power make it possible to simulate complete airplane configurations in a short enough period of time and to have a significant impact on the design cycle. The test case chosen was the DLR-F6 configuration with and without engines mounted underneath the wing. Satisfactory results were obtained for lift, drag and pitching moments over a wide range of flight angles of attack [17].

Validation of nonlinear two- and three-equation, eddy-viscosity turbulence models (NLEVM) in transonic flows featuring shock/boundary-layer interaction and separation is given in Ref. [3]. The accuracy of the models is assessed by comparison with the experimental results for two transonic flows over bump geometries. The bump geometries are among most broadly used configurations for validating turbulence models in simulating shock/boundary layer interaction. Presented in Ref. [3] results of the analyses show that the linear SST k-ω model and the nonlinear k-ε models deliver the best estimates of separation and reattachment positions. Yet, the value of the maximum Mach number is better assessed by the nonlinear models, whereas the results are inconclusive regarding the shock locations. Moreover, it was showed that the SST k-ω model provides in many cases results comparable to the ones obtained by nonlinear models [3].

One of the examples of employing the SST model to compute the turbulent viscosity and other turbulent quantities in the analyses of separated and transitional boundary layers under low-pressure turbine airfoil conditions is presented in Ref. [34]. The SST produces almost fully turbulent flow in the leading edge of the boundary layer and therefore it was used as a baseline model. In order to execute the analysis, author proposed a new transport equation for the intermittency factor. The intermittent behavior of the transitional flows is taken into account and incorporated into formulation by modifying the dynamic eddy-viscosity, μ_t, with the intermittency factor. The intermittency factor is obtained from transport equation, which can not only reproduce the experimentally observed streamwise variation of the intermittency in the transitional zone, but also provide a realistic cross-stream variation of the intermittency profile [34]. Functioning of the SST model with respect to simulating the turbulence quantities in modeling the transitional flows in low-pressure turbines using intermittency transport equation is also examined in Ref. [35].

Detached-Eddy Simulation (DES) models also use Menter's formulation. These hybrid models combine the best features of the Reynolds-averaged Navier-Stokes and large-eddy simulation approaches. The intended application of detached-eddy simulation is the treatment of massively separated, high-Reynolds number flows over complex configurations [8]. The application of the SST in the hybrid formulation Detached Eddy Simulation (DES) is also presented in Ref. [21]. In this approach the SST model is utilized in the boundary layer and LES like formulation for free shear flows. The main reason, why these were selected as the underlying RANS models, lies in their improved separation prediction capabilities [21]. The RANS turbulence models do not determine correctly the level of the turbulent stresses in the detached shear layer emanating from the separation line. This in turn seems to be one of the main reasons for the incorrect flow recovery obtained by means of the models downstream of reattachment [21]. Improved recovery could be computed with the DES formulation, the justification was presented in the Ref. [21].

6.4. THE SST MODEL APPLIED TO MODEL FREE SHEAR FLOWS

According to the Ref. [22], for free shear layers the SST and the BSL model reduces to the same model *(F_1=0; F_2=0)*, and are virtually identical to the standard k-ε model. Since the behavior of the k-ε model for free shear layer has been already discussed, no further examples will be included.

The SST model takes advantage of the baseline formulation and modification of eddy-viscosity formulation. Thus, it offers improved capabilities in simulation of flows with adverse pressure gradient, drag and separation of the boundary layer. The SST is also chosen to simulate the transition between laminar and turbulent flow and in hybrid applications.

7. CONCLUSIONS

Table 3. Evaluation of turbulence models capabilities.

Eddy-viscosity Models	Free shear flows	Wall shear flows
k-ε	Only some problems with predicting of the rates of spread for the round jet flow.	The ε-equation has severe limitations in the near-wall region. Turbulence models based on the ε-equation lead to overprediction of the turbulent length scale in flows with adverse pressure gradients, resulting in high wall shear stress and high heat transfer rates.
Realizable k-ε	More accurately predicts the spreading rate of both planar and round jets, provides superior performance for flows involving rotation.	Provides superior performance for boundary layers flows under strong adverse pressure gradients, separation and recirculation
RNG k-ε	Provides an option to account for the effects of swirl or rotation by modifying the turbulent viscosity.	Model better handles low-Reynolds number and near-wall flows.
k-ω	Exhibits poor performance in free shear flows due to very strong sensitivity to the freestream values of the ω_f specified for ω outside the boundary. layer	The ω-equation has significant advantages near the surface and accurately predicts the turbulent length scale in adverse pressure gradient flows, leading to improved wall shear stress and heat transfer predictions. Furthermore, the model has a very simple low-Re formulation, which does not require additional non-linear wall damping terms. The model equations can be integrated through the viscous sublayer
SST	Shear Stress Transport (SST) model benefits from the advantages of the k-ω and standard k-ε models. Modification of the eddy-viscosity leads to improved simulation capabilities of the model in adverse pressure boundary layer flows, also provides better approximation of the wall pressure distribution for the axisymetric transonic shockwave/turbulent boundary-layer.	

There are many two-equation turbulence models available in the literature. These are the result of modification of the k-ε and k-ω models or devised based on different scale of motion. The aim of this paper was to discuss the most popular models usually available in the CFD commercial codes. Based on the overview it might be concluded that there is a lack of general formulation, which might be used with high level of reliability for different flow tests. Nevertheless, for some class of the flows there are turbulence models, which provides the effective simulation capabilities. According to the presented above discussion, some recommendations, concerning the application of turbulence models to certain test flow cases, may be suggested. These are summarized in the table 3. However, each case flow is different and is subject to specific boundary conditions. Engineers dealing with CFD must bear in mind that provided recommendations (table 3), can only be taken into consideration at the first step of the CFD iterative computation procedure.

Based on the examined results the SST model seems to be the most promising. It groups best features of the k-ε and k-ω models and also accounts for the transport effect of the turbulent shear stress. Improved prediction capabilities for wall-bounded flows provided by the k-ω models are emphasized. This model, according to the literature, seems to be especially suitable for modeling flows in adverse pressure gradient. Modification of the standard k-ε model, leading to RNG k-ε, offers improvement in accuracy for some classes of the flows, in particular for flows with recirculation or flow curvature effects. Analyses executed by means of non-linear eddy-viscosity formulation offer improvements in assessing streamlines curvature and swirl effects.

The tests of fluid flows run by means of the CFD tools must be associated with the carefully chosen approach. In spite of the deficiencies of the two-equation models, they represent reasonable compromise between accuracy and time required in order to perform turbulent flow simulations. Two-equation eddy-viscosity models enable to analyze fluid motion in the complicated configurations delivering results that enhance the understanding of the considered phenomena. These models are continuously explored in order to improve their performance in different industrial applications.

This paper broadens the basic understanding of tools available in the most popular commercial CFD codes.

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SYMBOLS USED IN THE TEXT:

Latin symbols

$a_1, c_{\varepsilon 1}, c_{\varepsilon 2}$ - constants

F_1 - blending function

F_2 - blending function

k - turbulent kinetic energy

u, U - velocity

t – time

Greek symbols

$\alpha, \beta, \beta^*, \gamma$ - constants

δ_{ij} - Kronecker delta

ε - turbulent dissipation rate

μ_t - dynamic eddy-viscosity

μ - dynamic molecular viscosity

ρ - density

σ - closure coefficient

τ_{ij} - shear stress

ν_t - kinematic eddy-viscosity

ω - specific dissipation rate $\omega = \varepsilon / k$

ACRONYMS:

CFD – Computational Fluid Dynamic

SST – Shear Stress Transport

RANS – Reynolds Averaged Navier Stokes Equations

SMC – Second Moment Closure

DNS – Direct Numerical Simulation

LES – Large Eddy Simulation

DES – Detached Eddy Simulation

ASM – Algebraic Stress Model

NLEVM – Nonlinear Eddy Viscosity Turbulence Models

BSL – Base Line

RNG – Renormalization Group Theory

CDT – Cross Diffusion Term

REFERENCES

[1] Azzi A., Lakehal D.; Perspectives in modeling film cooling of turbine blades by transcending conventional two-equation turbulence models, *Transaction of ASME*, Vol. 124, July 2002

[2] Baldwin B., Lomax H.; - This-Layer Approximation and Algebraic Model for Separated Turbulent flows, *AIAA Paper* 78-257, 1978

[3] Barakos G., Drikakis D.; Investigation of nonlinear-eddy viscosity turbulence models in shock/boundary-layer interaction, *AIAA Journal*, Vol. 38, No. 3, March 2000

[4] Basara B., Jakirlic S, Hanjalic K.; Numerical simulation of turbulent flows in a car compartment with a second-moment turbulence closure, *ASME Fluids Engineering Division Summer Meeting ASME, FEDSM97*-so23, June 22-26, 1997

[5] CHANG Keh-Chin; Turbulent flow computations in combustors, *Proc. Natl. Sci. Counc. ROC(A)*, Vol. 24, No. 6, 2000, pp.413-421

[6] Dash S. M., Beddini R. A., Wolf D. E., Sinha N.; Viscous/Inviscid Analysis of Curved Sub- or Supersonic Wall Jets, *AIAA Paper* 83-1679, 1983

[7] Fluent V5.0, User's guide, vol. 2, *Fluent Inc. Canterra Resource Park, 10 Cavendish Court, Lebanon NH 03766, EUA, 1999*

[8] Forsythe J.R., Hoffmann K. A., Cummings R. M., Squires K. D.; Detached-Eddy simulation with compressibility corrections applied to a supersonic axisymmetric base flow, *Journal of Fluids Engineering*, pp.911-923, Vol. 124, December 2002

[9] Gatski T. B., Speziale C. G.; On explicit algebraic stress models for complex turbulent flows, *Journal of Fluid Mechanic* (1993), Vol. 254, pp.59-78

[10] Georgiadis N. J., Alexander J. I. D., Reshotko E.; Hybrid Reynolds-Averaged Navier-Stokes/Large-Eddy simulations of supersonic turbulent mixing, *AIAA Journal*, Vol. 41, No. 2, February 2003

[11] Grotjans H, Menter F. R.; Wall function for General Application CFD Codes, *ECCOMAS 98 Published in 1998 by John Wiley & Sons, Ltd.*

[12] Hanjalic K., Lanunder B. E., Schiestel R.; - Multiple-time-scale concept in turbulent transport modeling, Turbulent Shear Flows, Vol. 2 1980, pp.36-49, L.J.S. Bradbury, F. Durst, B.E. Launder, F.W. Schmidt, and J.H. Whitelaw *Eds. Springer-Verlag, Berlin, Germany*

[13] Hwang R. R., Shenq-Yuh Jaw; Second-Order Closure Turbulence Models: Their Achievement and Limitations, *Proc. Natl. Sci. Counc. ROC(A),* Vol. 22, No. 6, pp. 703-722, 1998

[14] Jones W. P., Launder B. E.; The predictions of Laminarization with a two-equation model of turbulence, *Int. J. Heat Mass Transfer* 15 1972, 301-314

[15] Johnson D. A., King L. S.; Mathematically Simple Turbulence Closure Model for Attached and Separated Turbulent Boundary Layer, *AIAA Journal*, Vol. 23, No. 11, 1985, pp. 1684-1692

[16] Kazuhiko Suga; Recent development in Eddy Viscosity Modeling of Turbulence, *R&D Review of Toyota CRDL* Vol. 33, No. 1 (1998.3)

[XX] Fluid Dynamics, Vol. 17, No. 1, pp. 87 – 93, February 2003

[17] Langtry R., Kuntz M., Menter F. R.; Drag prediction of engine-airframe interference effects with CFX 5; *CFX contribution to 2nd AIAA Drag Prediction workshop* Orlando, Florida, June 21-22, 2003, CFX Report

[18] Launder B. E., Sharma D. B.; Application of the energy-dissipation model of turbulence to the calculation of flow near a spinning disc., *Lett. Keat Mass Transfer* 1 1974, 131-138

[19] Launder B. E.; Shima N.; Second Moment Closure for the Near-Wall Sublayer: Development and Application, *AIAA Journal*, Vol. 27, No. 10, pp. 1319-1325, October 1989
[20] Mazur Z., Sierra-Espinosa F. Z., Urquiza-Beltrán G., Kubiak-S J.; Erosion of the rotor disc in a geothermal turbine of 110 MW, *Proceeding of 2000 Internal Joint Power Generation Conference* Miami Beach, Florida, July 23-23, 2000
[21] Menter F. R.; Kuntz M.; Langtry R. Ten Years of Industrial Experience with the SST Turbulence Model; *Turbulence, Heat and Mass Transfer* 4
[22] Menter F. R. Two Equation Eddy Viscosity Turbulence Models for Engineering Applications; *AIAA Journal*, Vol. 32, No. 8, August 1994
[23] Menter F. R. Influence of Freestream Values on k-ω Turbulence Model Predictions; *AIAA Journal,* Vol. 30, No. 6, June 1992
[24] Menter F. R.; Eddy viscosity transport equations and their relation to the k-ε model, *Transactions of the ASME*, Vol. 119, December 1997, p.876-884
[25] Morgans R. C., Dally B. B., Nathan G. J., Lanspeary PP. V., Fletcher D. F.; Application of the revised Wilcox (1998) k-ω turbulence model to a jet in co-flow, pp.479-484, *Second International Conference on CFD in the Minerals and Process Industries* CSIRO, Australia, 6-8 December 1999
[26] Ooi A., Petterson Reif B.A., Iaccarino G., Durbin P.A.; Evaluation of RANS models for rotating flows, *Center for Turbulence Research, Preceeding of Summer Program* 2000
[27] Pajayakrit P., Kind R. J.; Assessment and modification of two-equation turbulence models, *AIAA Journal*, Vol. 38, No. 6, June 2000
[28] Patel, V. C.; Rodi W.; Scenerer G.; Turbulence models for near wall and low Reynolds number flows. *AIAA Journal*, Vol. 23, No. 9, ppp. 1308-1319, September 1985
[29] Pope S. B.; A more general effective viscosity hypothesis, *J. Fluid Mech.* 1975-72, pp.331-340
[30] Pope S. B.; Turbulent flows; *Cambbridge University Press* 2000
[31] Schlüter J. U., Wu X., Kim S., Alonso J. J., Pitsch H.; Coupled RANS-LES computation of a compressor and combustor in a gas turbine engine, *40th AIAA/ASME/SAE/ASEE Joint Propulsion Conference and Exibit* July 11-13/ Ft. Lauderdale, Fl
[32] Sijerčić M., Stevanović Ž., Bološević S.; Modeling of non-confined turbulent flow of two coaxial streams under combustion conditions, *Facta Universitatis, Series: Mechanical Engineering Vol.1*, No.. 8, 2001, pp.981-988
[33] Speziale C. G., Abid R., Anderson E. C.; Critical Evaluation of Two-Equation Models for Near-Wall Turbulence, *AIAA Journal*, Vol. 30, No. 2, pp. 324-331, February 1992
[34] Suzen Y.B., Huang P. G., Hultgren L. S., Ashpis D. E.; Predictions of separated and transitional Boundary layers under low-pressure turbine airfoil conditions using an intermittency transport equation, *Journal of Turbomachinery*, Vol. 125, July 2003
[35] Suzen Y.B., Xiong G., Huang P. G.; Predictions of Transitional Flows in Low-Pressure Turbine Using Intermittency Transport Equation, *AIAA Journal*, Vol. 40, No. 2, February 2002
[36] Tanner F. X., Guang-Sheng Zu, Reitz R. D.; Emission predictions in disel engines using a non-equilibrium turbulence dissipation correction for the k-ε model, *Twelfth International Multidimensional Engine Modeling Meeting* at the SAE Congress, March 3, 202, Detroit, Michigan
[37] Thangam S., Abid R., Speziale C. G.; Application of a New k-τ Model to Near Wall Turbulent Flows, *AIAA Journal*, Vol. 30, No. 2, pp. 552-554, February 1992

[38] Wilcox D. C. Turbulence modeling for CFD, *La Cañada, CA:DCW Industries* (1993)
[39] Wilcox D. C. Multiscale Model for Turbulent Flows; *AIAA Journal*, Vol. 26, No. 11, November 1988
[40] Wilcox D. C. Reassessment of the Scale Determining Equation for Advanced Turbulence Models; *AIAA Journal*, Vol. 26, No. 11, November 1988
[41] Wilcox D. C., Rubesin M. W.; Progres in Turbulence Modeling for Complex Flow Field Including Effects of Compressibility, *NASA TP-1517*, April 1980
[42] Wilcox D. C.; Turbulence modeling for CFD, *La Cañada, CA:DCW Industries* (1998)
[43] Vieser W., Esch T., Menter F.; Heat Transfer Predictions using Advanced Two Equation Turbulence Models; *CFX Technical Memoradum* CFX-VAL10/0602

Michał Woelke

MODELOWANIE PRZEPŁYWÓW TURBULENTNYCH PRZY POMOCY NUMERYCZNYCH KODÓW MECHANIKI PŁYNÓW

W pracy zostały przedstawione wady i zalety dwu-równianiowych modeli turbulencji wykorzystujących zagadnienie lepkości burzliwej, które są stosowane w numerycznych analizach mechaniki płynów. Modelowanie przepływów turbulentnych stanowi jeden z najbardziej skomplikowanych zagadnień w mechanice płynów. Najczęściej stosowane dwu-równaniowe modele turbulencji zostały przedstawione w niniejszej pracy. Modele te wykorzystują lepkość burzliwą, która zgodnie z założeniem Boussinesq ustala związek pomiędzy naprężeniami Reynoldsa oraz gradientem prędkości średniej w przepływie turbulentnym. Konsekwencje tego założenia pod względem możliwości analizowania przepływów burzliwych przy pomocy modeli dwu-równaniowych zostały przeanalizowane w pracy.

Ponadto w pracy przedyskutowano:

(i) modyfikacje założenia Boussinesq w celu poprawy jego skuteczności,

(ii) możliwości modelu k-e oraz jego modyfikacji do modelowania przepływów turbulentnych,

(iii) zalety oferowane przez model k-w w zakresie analiz numerycznej mechaniki płynów,

(iv) cechy modelu transportu naprężeń stycznych (Shear Stress Transport - SST) będącego modyfikacją modeli k-e oraz k-w.

W pracy przedstawiono istotne problemy związane z modelowaniem przepływów turbulentnych przy pomocy numerycznych kodów mechaniki płynów.

Михал Воэлькэ

МОДЕЛИРОВАНИЕ ТУРБУЛЕНТНЫХ ТЕЧЕНИЙ ПРИ ПОМОЩИ ЧИСЛЕННЫХ КОДОВ МЕХАНИКИ ЖИДКОСТИ.

Резюме

В работе представлены недостатки и преимущества двухмерных моделей турбуленции использующих задачу турбулентной вязкости, которая применяется в численных анализах механики жидкости. Моделирование турбулентных течений является одной из наиболее сложных задач в механике жидкости, В работе представлены наиболее часто применяемые модели описывающее турбулентность двумя уравнениями. Эти модели используют турбулентную вязкость, которая в соответствии с предпосылкой Буссинеска определяет связь между напряжениями Рейнольдса и градиентом средней скорости в турбулентном течении. В работе анализируются последствия этой предпосылки в отношении возможности анализа турбулентных течений при помощи моделей двух уравнений. Кроме этого в работе обсуждаются :

(I) модификации предпосылки Буссинеска с целью улучшения ее эффективности,

(II) возможности модели к-е и ее модификации для моделирования турбулентных течений,

(III) преимущества модели к-в в области численных анализов механики жидкости,

(IV) особенности модели транспорта касательных напряжений (Shear Stress Transport - SST), являющейся модификацией к-е и к-в моделей.

В работе представлены важные проблемы связанные с моделированием турбулентных течений при помощи численных кодов механики жидкости.