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Analytical and numerical investigation of the free vibration of functionally graded materials sandwich beams

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ABSTRACT

Purpose: In this study, the free vibration analysis of functionally graded materials (FGMs) sandwich beams having different core metals and thicknesses is considered. The variation of material through the thickness of functionally graded beams follows the power-law distribution. The displacement field is based on the classical beam theory. The wide applications of functionally graded materials (FGMs) sandwich structures in automotive, marine construction, transportation, and aerospace industries have attracted much attention, because of its excellent bending rigidity, low specific weight, and distinguished vibration characteristics.

Design/methodology/approach: A mathematical formulation for a sandwich beam comprised of FG core with two layers of ceramic and metal, while the face sheets are made of homogenous material has been derived based on the Euler–Bernoulli beam theory.

Findings: The main objective of this work is to obtain the natural frequencies of the FG sandwich beam considering different parameters.

Research limitations/implications: The important parameters are the gradient index, slenderness ratio, core metal type, and end support conditions. The finite element analysis (FEA), combined with commercial Ansys software 2021 R1, is used to verify the accuracy of the obtained analytical solution results.

Practical implications: It was found that the natural frequency parameters, the mode shapes, and the dynamic response are considerably affected by the index of volume fraction, the ratio as well as face FGM core constituents. Finally, the beam thickness was dividing into frequent numbers of layers to examine the impact of many layers' effect on the obtained results.

Originality/value: It is concluded, that the increase in the number of layers prompts an increment within the frequency parameter results' accuracy for the selected models. Numerical results are compared to those obtained from the analytical solution. It is found that the dimensionless fundamental frequency decreases as the material gradient index increases, and there is a good agreement between two solutions with a maximum error percentage of no more than 5%.

Keywords: FGMs, Sandwich beam, Classical beam theory, Vibration, FEA, Frequency, ANSYS

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METHODOLOGY OF RESEARCH, ANALYSIS AND MODELLING

1. Introduction

Functionally graded materials (FGMs) being modern materials having compositions that change unceasingly via their thickness. They are generally produced from a combination of metal and ceramic and might consequently appear tougher when subjected to high temperatures [1]. The wide applications of functionally graded materials (FGMs) sandwich structures in automotive, marine construction, transportation, and aerospace industries have attracted much attention, because of its excellent bending rigidity, low specific weight, and distinguished vibration characteristics. Many researchers have investigated the stability and vibrational behavior of FGM structure structures using different theories such as 2D and 3D elasticity theory [2]. Le et al. [3] formulated an efficient third-order shear deformation beam element for free vibration and buckling analysis of bidirectional functionally graded sandwich beams. Mohammad Arefi and Farshid Najafitabar [4] studied buckling, and free vibration analyses of sandwich beam. The sandwich beam is composed of a soft core integrated with functionally graded graphene nanoplatelets reinforced composite face sheets using extended higher-order theory under various boundary conditions employing the Ritz method. Omidi Soroor et al. [5] used both the Euler-Bernoulli and Timoshenko beam theories to investigate the free vibration of sandwich beam consisting of magneto-rheological fluid core and an axially functionally graded constraining layer. The Rayleigh-Ritz method is used to derive the frequency-dependent eigenvalue problem through the kinetic and strain energy expressions of the sandwich beam.

Dynamic analysis of an inclined sandwich beam under a moving mass is presented based on a third-order shear deformation theory by Nguyen et al. [6]. The core of the sandwich beam is homogeneous while the two face sheets are made from FGMs. The numerical result reveals that the material gradation plays an important role on the dynamic response of the beam. Selmi and Mustafa [7] developed, an approximate method based on the continuous elements method to study the free vibration behavior of bi-dimensional FG Euler-Bernoulli beams with exponential law gradation. It is found that the material fraction index has a strong impact on the first mode shape and the influences become smaller for higher-order vibration modes. By employing both the beam Timoshenko theory and nonlocal strain gradient integral model, the free vibration response of functionally graded (FG) beam was examined by Tang and Qing [8]. The asymptotic development method (ADM) is used by Cao et al. [9] to investigate the free vibration analysis of uniform beams with different boundary conditions. The numerical results of the proposed method are confirmed by comparing the obtained results with those obtained via finite element analysis. Hadi Arvin et al. [10] provided numerical results of the free vibration treatment of pre-and post-buckled rotating functionally graded beams. The framework of the Euler-Bernoulli beam model alongside the von-Kármán strain-displacement relationship is employed using various beam theories. The free vibration analysis of rotating fully-bonded and delaminated sandwich beams; containing AL-foam flexible core and carbon nanotubes reinforced composite face sheets subjected to thermal and moisture field is investigated by using high-order generalized differential quadrature method [11]. Guo-dong Xu et al. [12] studied free vibration of composite sandwich beam with graded corrugated lattice core using Ritz Method. The natural frequencies and buckling loads are obtained in terms of weight fraction and distribution of graphene nanoplatelets, length to thickness ratio, core to surface ratio, and different boundary conditions.

S. Rajasekaran [13] studied the vibration and stability of the axially (FGM) non-uniform beams utilizing a differential transformation-based dynamic stiffness method. Karamanli and Vo [14] studied bending, buckling, and free vibration analyses of functionally graded (FG) sandwich microbeams using the third-order beam theory. The Mori-Tanaka homogenization technique is used to model the material distributions through the thickness. The finite element model is developed to solve the problems.

Sari and Butcher [15] used the Chebyshev collocation method and presented a new solution to identify the dynamic performance of rotating and non-rotating Timoshenko beams. Abbes Elmeiche et al. [16] investigated the analysis of the FG nanobeams free vibration depending upon the Ritz method of high-order beam theory. O. Rahmani and O. Pedram [17] studied depending upon the theory of nonlocal Timoshenko beam the size gradient effect upon the functionally graded nanobeam vibration. Mohamed Bouamama et al. [18] investigated the free vibration of the (FGMs) beams under different boundary conditions for examining the thickness effect of the sandwich beam panel upon the entire structure stability. Kanu et al. [19] made important comments on the fracture, vibration, buckling, and bending analysis of structures produced from FGMs. Li et al. [20] investigated the sandwich beams non-linear dynamic response with the (FGM) negative honeycomb core Poisson's ratio. The vibration response of simply supported FG and viscoelastic/fractionally damped beams located on the Pasternak foundation subjected to a point harmonic load is studied by Sepehri-Amin et al. [21]. Garg et al. [22] presented a comparative study that uses a finite element based on higher-order zigzag theory for observing bending analysis on sandwich functionally graded beams made up of different material property variation laws. Karamanli [23] employed a third-order shear deformation theory and the Lagrange equations to analyze free vibration of bi-directional FG straight beam subjected to different boundary conditions. Simsek and Kocatürk [24] used both the power-law and exponential forms to study the free vibration as well as the dynamic response of an (FGM) Euler–Bernoulli beam beneath a sinusoidal load.

Nguyen et al., [25] introduced an analytical based on Timoshenko beam theory to evaluate the free vibration of bi-dimensional functionally graded beams excited by a moving concentrated load. A finite element model is derived and used in combination with the Newmark method in computing the natural frequencies. Huang et al. [26] investigated the vibration characterizations of the axially (FGM) beams with various cross-sections. A New FEA approach has been employed to obtain the characteristics of the FG beam-free vibration [27]. Depending upon the Rayleigh-Ritz approach, the free vibration of the Timoshenko and Euler (FGM) beams exposed to various boundary conditions have been presented by Pradhan and Chakraverty [28]. Ngoc-Duong Nguyen et al. [29] analysed the dynamic response and free vibration of Timoshenko beams with internal hinges under various boundary conditions. Chang and Chen [30] proposed a Ritz-type solution for free vibration and buckling analysis thin-walled composite and functionally graded sandwich I-beams. Shahba and Rajasekaran [31] utilized the FEM for studying the problem of the stability and free vibration of the axial (FGM) non-uniform beam. Mashat et al. [32] investigated the problem of the free vibration for FGM layered beams depending upon different theories, and the results were verified employing FEM. Owing to the wide uses of the functionally graded materials FGMs, it's essential to investigate the dynamic and static analysis of the (FGM) sandwich structures, like plates, beams as well as shells.

The investigation aims to present an exact solution and an analytical mathematical model for the free vibration of a simply supported (FGM) sandwich beam to find the natural frequencies according to various FGM parameters. FGM parameters are power-law index, elastic parameters, core material, and length to thickness ratios. The beam material was assumed isotropic with a smooth variation in the thickness direction only.

There, the purpose of the presenting study was to investigate the effect of different FGM parameters, in addition to, various porous parameters on the vibration characterization of the sandwich beam structure.

2. Beam theory and formulations

Many beam theories use to signify the static, free vibration, and stability of beam structures. The Classical Beam Theory (CBT) or Euler–Bernoulli beam theory is the easiest theory used for evaluating the overall performance of thin beams. The alternative popular theory beam is the theory of Timoshenko beam or the (FSDT), in that the influence of the transverse shear stress regarding the coordinate of thickness is considered. Thus, a shear parameter is necessary to get accurate results. To avoid using the correction for shear and to obtain a better prediction of the transverse shear deformation and regular strains in beams, theories of higher-order shear deformation beam (HSDTs) have been suggested. Generally, these theories can be modified depending upon the higher-order variants of the in-plane displacements. The most common theories of HSDT beam are (i) the parabolic shear deformation theory, (ii) the trigonometric shear deformation theory, (iii) the hyperbolic shear deformation theory, (iv) the exponential shear deformation theory, and (v) the fresh shear deformation theory. A comprehensive study of static and FVA of the axially loaded (FGM) beams depending upon the (FSDT) was performed by Nguyen et al. [33]. The axial and transverse displacements of any point being, depending on the classical beam theory (CBT), given by,

$$u_x(x,y,z) = -z\frac{\partial w_o}{\partial x} \tag{1}$$

$$w(x,z) = w_o(x) \tag{2}$$

where, w_o is the transverse deflection upon the mid-plane (that means z is zero) of the beam. The displacement field in Equation (3) indicates that the straight lines normal to the mid-plane before deformation stay normal and straight to the mid-plane beyond deformation, as shown in Figure 1 [34]. By ignoring the transverse strain and the normal strain, the linear strain-displacement relationship can be expressed as,

$$e_{xx} = \frac{\partial u}{\partial x} = -z\frac{\partial w^2}{\partial x^2} \tag{3}$$

Regarding that the FG beam material follows the Hooke formula, the stress-strain relationship of the beam in the x-direction is,

$$S_{xx} = E_x\, e_{xx} \tag{4}$$

E_x is the modulus of elasticity; thus, an (FGM) beam's linear constitutive relations, such as the bending moments (M_{xx},) on a beam element case, can be written as follows,

$$M_{xx} = \int_{-\frac{h}{2}}^{\frac{h}{2}} S_{xx} z\, dz = E\int_{-\frac{h}{2}}^{\frac{h}{2}} (e_{xx})\, zdz = -D_{xx}\left(\frac{\partial^2 w}{\partial x^2}\right) \tag{5}$$

where $D_{xx}=E_xI_{yy}$ is the flexural rigidity of the beam, and $I_{yy} = \int_A z^2 dA$ is the 2nd moment of the area around the y-axis,

$$D = D_{xx} = \frac{1}{12}Eh^3 \tag{6}$$

Alternatively, the second-order equilibrium equation of the Kirchhoff beam theory may be written as,

$$\frac{\partial^2 M_{xx}}{\partial x^2} = I_0\frac{\partial^2 w}{\partial t^2} \tag{7}$$

By using bending and twisting moments in Equation (5), the beam governing equation can be obtained as detailed below,

$$D\left(\frac{\partial^4 w}{\partial x^4}\right) + I_0\frac{\partial^2 w}{\partial t^2} = 0 \tag{8}$$

where, I_0 is the inertial coefficient of the beam.

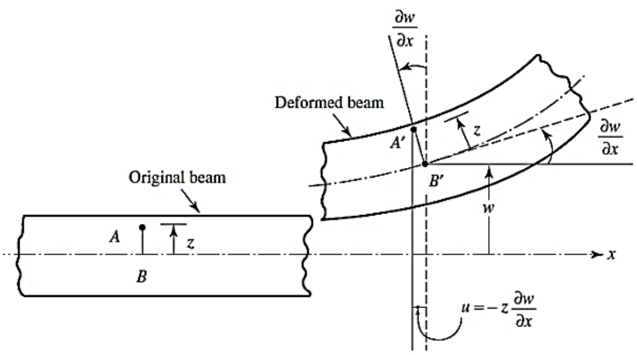

Fig. 1. The deformation of a distinctive transverse perpendicular line in CBT [34]

Generally, the characteristics of the material of the functionally graded sandwich beam elements follow one of the mathematical idealizations, such as the law of exponents, the law of sigmoid type, or the law of power index. In the present investigation, it's supposed that the functionally graded sandwich beam is comprised of metal and ceramic and follows the power-law index, which can be defined by [35]

$$V_c(z) = \left(\frac{z+\frac{h}{2}}{h}\right)^k \tag{9}$$

The constituent volume fraction of the functionally graded material beam is assumed to change unceasingly alongside the direction of thickness and obey the distribution of power-law as in the following,

$$P(z) = (P_c - P_m)\left(\frac{z+\frac{h}{2}}{h}\right)^k + P_m \tag{10}$$

In Equation (10), Pc and Pm are the material characteristics values of the ceramic and the metal, correspondingly. The sum of volume fractions of the ceramic and metal is stated as, : $V_m(z) + V_c(z) = 1$ where V_m and V_c are the metal and ceramic volume fractions, respectively, k is the variation index of a power law, which is a positive variable in the range of $[0, \infty)$. It can be noticed from Equation (9) that at $z = -h/2$, all mechanical properties represent metal constituent, while at $z = h/2$, the ceramic properties will control the beam surface. For our current formulation, in addition to the Poisson's ratio (ν) constant, the other characteristics of the material, like the modulus of elasticity (E) and the density of mass (ρ) being varying along the thickness direction.

3. Functionally graded sandwich beam

The wide applications of FG sandwich structures in the marine and aerospace applications, transportation as well as aviation companies have attracted many considerations, and certain researchers have performed continuous static and dynamic inspections on them. Due to the special performance of the stiffness-to-weight ratio, the use of such systems in various industries is constantly developing. Therefore, in a wide range of FGM material uses, it's crucial to explore the static and dynamic conduct of auxiliary personnel with FGM, such as beams and plates. The plate is considered to have a homogeneous hard core and FGM face sheets.

In this study, an FG symmetric sandwich beam has been considered. The beam has a rectangular cross-section of length (L), width (d), thickness (b), also the FG core is comprised of a mix of an aluminium matrix and the alumina

(Al_2O_3) phases, as shown in Figure 2. The higher part of the beam core is made of pure ceramic (Al_2O_3), and the lower part being pure aluminium, while the area between them is made of FGM. Furthermore, the face sheets are made of homogenous metal [36].

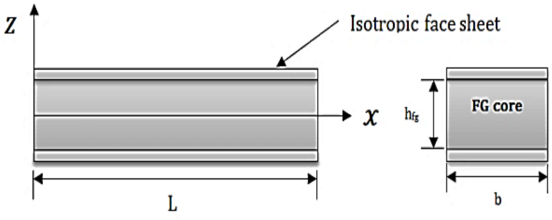

Fig. 2. Geometry configurations of the model

The governing differential equation for the functionally graded beam free vibration can be defined in the framework of the general theory of Euler-Bernoulli beam (CBT) as following,

$$E(z) = (E_c - E_m)\left(\frac{z+\frac{h}{2}}{h}\right)^k + E_m \quad (11)$$

$$\rho(z) = (\rho_c - \rho_m)\left(\frac{z+\frac{h}{2}}{h}\right)^k + \rho_m \quad (12)$$

$$D = \int_{-h/2}^{h/2} E(z)\cdot z^2 dz \quad (13)$$

$$D = (E_c - E_m)h^3\left\{\begin{matrix}\frac{1}{k+3} - \frac{1}{k+2} \\ + \frac{1}{4(k+1)}\end{matrix}\right\} + \frac{E_m h^3}{12} \quad (14)$$

where, I_o is the moment of inertia of the FGM sandwich beam, which can be expressed in term of the volume fraction index as,

$$I_o = \int_{-\frac{h}{2}}^{\frac{h}{2}}\left((\rho_c - \rho_m)\left(\frac{z}{h}+\frac{1}{2}\right)^k + \rho_m\right)dz = \\ = \left(\begin{matrix}\int_{-\frac{h}{2}}^{\frac{h}{2}}\left((\rho_c - \rho_m)\left(\frac{z}{h}+\frac{1}{2}\right)^k\right)dz + \\ \int_{-\frac{h}{2}}^{\frac{h}{2}}\rho_m dz = \frac{(\rho_c-\rho_m)h}{(k+1)} + \rho_m h\end{matrix}\right) \quad (15)$$

Substitute Equations 12, 13 in Equation 9, the following equation can be obtained,

$$\left(\begin{matrix}\left((E_c - E_m)h^3\left(\begin{matrix}\frac{1}{k+3} - \frac{1}{k+2} \\ + \frac{1}{4(k+1)}\end{matrix}\right) + \frac{E_m h^3}{12}\right)\left(\frac{\partial^4 w}{\partial x^4}\right) \\ + \left(\frac{(\rho_c-\rho_m)h}{(k+1)} + \rho_m h\right)\frac{\partial^2 w}{\partial t^2}\end{matrix}\right) = 0 \quad (16)$$

For the present investigation, it is assumed that the beam core is an FGM, whereas the upper, as well as lower skins, are made of the same homogenous material, so the modulus of elasticity $E_2 = E_3$ and the mass density $\rho_2 = \rho_3$. The general representation for the flexural rigidity and inertia for sandwich beam can be written as,

$$D_t = \left(\begin{matrix}\int_{-\left(\frac{d_1}{2}\right)}^{\left(\frac{d_1}{2}\right)}\left((E_c - E_m)\left(\frac{2z+h}{2h}\right)^k + E_m\right)z^2 dz \\ + \int_{-\left(\frac{d_1}{2}+d_2\right)}^{\left(-\frac{d_1}{2}\right)} E_1 z^2\, dz + \int_{\left(\frac{d_1}{2}\right)}^{\left(\frac{d_1}{2}+d_3\right)} E_2 z^2\, dz\end{matrix}\right) \quad (17)$$

For simplicity let $\rho_2 = \rho_3 = \rho_s$, $d_2 = d_3 = d_s$, $E_1 = E_2 = E_s$, then we get,

$$D_t = \left(\begin{matrix}(E_c - E_m){d_1}^3\left(\frac{1}{k+3} - \frac{1}{k+2} + \frac{1}{4(k+1)}\right) + \\ \frac{E_m {d_1}^3}{12} + E_s\left(\frac{2(\frac{d_1}{2}+d_s)^3}{3} - \frac{{d_1}^3}{12}\right)\end{matrix}\right) \quad (18)$$

Following the same procedure, the inertia coefficient for the whole beam is,

$$I_{SP} = \frac{(\rho_c-\rho_m)d_1}{(k+1)} + \rho_m d_1 + 2\rho_s d_s \quad (19)$$

The governing equation of the sandwich beam now becomes,

$$D_t\left(\frac{\partial^4 w}{\partial x^4}\right) + I_0\frac{\partial^2 w}{\partial t^2} = 0 \quad (20)$$

$$\left(\begin{matrix}\left(\begin{matrix}(E_c - E_m){d_1}^3\left\{\frac{1}{k+3} - \frac{1}{k+2} + \frac{1}{4(k+1)}\right\} + \\ \frac{E_m {d_1}^3}{12} + E_s\left(\frac{2\left(\frac{d_1}{2}+d_s\right)^3}{3} - \frac{{d_1}^3}{12}\right)\end{matrix}\right)\left(\frac{\partial^4 w}{\partial x^4}\right) \\ + \left(\frac{(\rho_c-\rho_m)d_1}{(k+1)} + \rho_m d_1 + 2\rho_s d_s\right)\frac{\partial^2 w}{\partial t^2}\end{matrix}\right) = 0 \quad (21)$$

To evaluate the beam natural frequency, the general motion equation solution of a beam having a free vibration consistence section and a fixed modulus is as follows,

$$W(x,t) = W_n(x) \times W_n(t) \quad (22)$$

Then, by substituting Equation 21 in Equation 19 and solving them will get,

$$W(x) = \left(\begin{matrix}A_1\cos\psi x + A_2\sin\psi x + \\ A_3\cosh\psi x + A_4\sinh\psi x\end{matrix}\right) \quad (23)$$

where, A1, A2, A3, and A4 are constants and can be obtained via applying the required beam boundary conditions. Comparing Equation (21) with the general equation of SDF motion for the structure free vibration as following,

$$\omega_{mn}^2 w(t) + \frac{\partial^2 w(t)}{\partial t^2} = 0 \tag{24}$$

$$\omega_n = (\psi_n)^2 \sqrt{\frac{(k+1)\left((E_c - E_m){d_1}^3\left(\frac{1}{k+3} - \frac{1}{k+2} + \frac{1}{4(k+1)}\right) + \frac{E_m {d_1}^3}{12} + E_s\left(\frac{2(\frac{d_1}{2}+d_s)^3}{3} - \frac{{d_1}^3}{12}\right)\right)}{(\rho_c + k\rho_m)d_1 + 2(k+1)\rho_s d_s}} \tag{25}$$

This form is equivalent to the following equation of a homogenous uniform beam,

$$\omega_n = (\psi_n l)^2 \sqrt{\frac{EI}{\rho A l^4}} \tag{26}$$

where, E: Young's modulus of the beam, I: Moment of inertia of, beam, ρ: Density of the beam, A: Cross-sectional area of the beam, *l*: the beam length, *n*: the no. of beam modes.

Thus, for evaluating the value of natural frequency, suitable value boundary conditions should be chosen. In this study, three boundary conditions kinds being selected, as revealed in Table 1 [37]. For convergence, the frequency parameter is,

$$\lambda = \frac{\omega L^2}{h} \sqrt{\frac{\int_{-h/2}^{h/2} \rho(z)\, dz}{\int_{-h/2}^{h/2} E(z)\, dz}} \tag{27}$$

The material characteristics of the functionally graded sandwich beam and the face sheets being presented in Table 2 [38]. The dimensions of beams are W=0.1 m, the considered thicknesses of the FG core and face sheets are (5,10, 20, 50, and 100 mm), and (1, 1.5, 2, and 2.5 mm), respectively. The power-law distribution, k = 0, 0.1, 0.3, 0.5, 1, 2, 5, 10.

4. Numerical investigation

The FEM is implemented to solve many static and dynamic responses of structures, such as structures composed of different beam elements. The numerical method is a technique used to evaluate the approximate solutions of problems [39]. The considered FG sandwich beam was modelled using commercial software (ANSYS version 2021 R1). The ANSYS Design modeler was used to generate the FG composite sandwich beam model. The beam model dimensions were considered based on the ASTM standard, and it is built with an 8-node SOLID186 element and meshed with a 3 mm size element [39-47]. Figure 3a explains the 3D beam model in the ANSYS software for the present analysis. Also, the mesh of beam structure required selected the best element and node required number by using mesh generation technique, as shown in Figure 3b [48-55]. Then, after mesh the beam structure supported the bean with selected boundary condition, and then calculating the required vibration characterizations [56-64]. The modal analysis is carried out to find the natural frequencies for the selected models. At the connection area of the layers and amongst layers and skins of the sandwich glue, stipulations are connected to prevent the genealogical development of layers with each other's deference.

Where, three boundary condition, as simply supported (S-S); clamed-free supported (C-F) and clamped supported (C-C) beam, were analysis by numerical technique.

Table 1.
Kinematic boundary conditions for different beam supports

BC's	At x=0	At x=L	$\psi_n \ell$
S-S	$W = 0$ $\frac{\partial^2 W(x)}{\partial x^2} = 0$	$W = 0$ $\frac{\partial^2 W(x)}{\partial x^2} = 0$	$\Psi_1\ell = 3.1415$ $\Psi_2\ell = 6.2831$ $\Psi_3\ell = 9.4247$ $\Psi_4\ell = 12.5663$
C-F	$W = 0$ $\frac{\partial W(x)}{\partial x} = 0$	$\frac{\partial^2 W(x)}{\partial x^2} = 0$ $\frac{\partial^3 W(x)}{\partial x^3} = 0$	$\Psi_1\ell = 1.875104$ $\Psi_2\ell = 4.694$ $\Psi_3\ell = 7.85471$ $\Psi_4\ell = 10.9955$
C-C	$W = 0$ $\frac{\partial W(x)}{\partial x} = 0$	$W = 0$ $\frac{\partial W(x)}{\partial x} = 0$	$\Psi_1\ell = 4.7300$ $\Psi_2\ell = 7.85322$ $\Psi_3\ell = 10.9956$ $\Psi_4\ell = 14.1371$

Table 2.
Trial properties employed in the FG sandwich Beams [38]

Materials		Property		
		E, GPa	P, Kg/m^3	ν
Alumina-Aluminium	Al_2O_3	380	3800	0.3
	Al	70	2702	0.3
Silicon nitride-Stainless steel	Si_3N_4	322	2370	0.3
	SUS304	207	8166	0.3
Zirconia (ZrO_2)-Titanium	ZrO_2	168	3000	0.3
	Ti-6Al-4V	105	4429	0.3
Face Sheets (Mild steel)		210	7800	0.3

a) b)

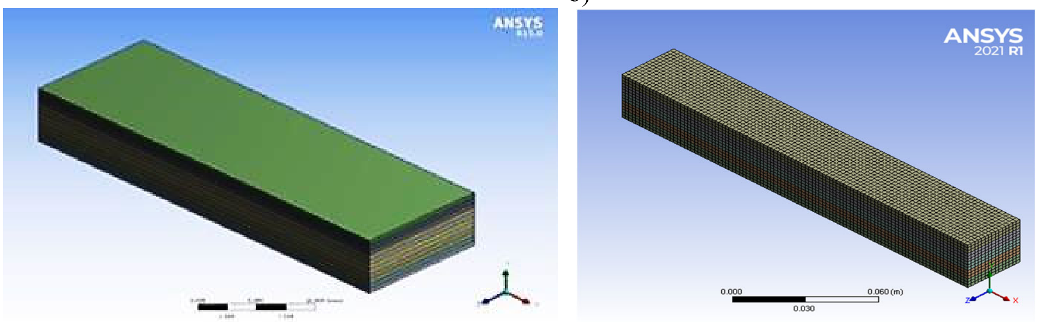

Fig. 3. The model of the FGM sandwich beam without (a) and with mesh (b)

5. Results and discussion

In the present work, the analytical investigation being carried out for analysing the free vibration problem of the FGM sandwich beam structure. The evaluation results include the natural frequency and the frequency parameters of the FG sandwich beams, thorough different parameters, such as FGM core metal, skin thickness, slenderness ratio, aspect ratio, and gradient index. It is assumed that the materials being distributed to the upper and lower parts of the plate, also the FGM part consists of ceramic and materials. Numerical investigation using ANSYS version 2021 R1 for verification purposes is also employed. The obtained results are tabulated and drawn with multiple curves. Material characteristics of the functionally graded beam and face sheets being presented in Table 2 [38].

In order to facilitate the presentation, the dimensionless frequency parameter of the FG sandwich beam is introduced as follows:

$$\lambda = \frac{\omega L^2}{h}\sqrt{\frac{\rho_o}{E_o}} \tag{28}$$

where ω is the natural frequency, ρ_o is equal to (1 Kg/m^3) and E_o is equal to (1 GPa) [38].

Using CBT, natural frequencies for the sandwich beams are presented in Tables 3, 4, 5, and 6. For verifying the suggested mathematical model accuracy in the prediction of the natural frequency of the made functionally graded sandwich beams, Table 3 presents the analytical and numerical results for the 1^{st} non-dimensional frequencies (λ) of a simply supported sandwich beam having different sheet thicknesses (1, 1.5, 2 and 2.5 mm) for different indices of power-law k = 0, 0.5, 1, 2, 5, 10) with FGM core thickness of 10 mm using Eq. (28).

From the results, one can conclude that the natural frequency reduces by raising the exponent of power-law and increases by raising the thickness of the face sheet owing to an increase in material rigidity. Table 4 presents a brief comparison between the analytical and numerical convergence of the initial non-dimensional frequencies of a functionally graded core made from silicon nitride (Si_3N_4)/ stainless steel (SUS304), with the same characteristics of the face sheet beam with (*L/h* = 50) and using CBT for S-S boundary condition. The results for FG sandwich beams are indicated in Table 5; similar analyses are done. The sandwich beam with FG core comprises mainly from Zirconia (ZrO_2)/ Titanium and same skins properties. The behaviour of frequencies of the beams made up of 3 types of constitutes depends upon the mechanical properties values of the constituents.

Table 3.
The 1st non-dimensional frequencies (λ) of a simply-supported sandwich plate type for various power-law indices with (Al_2O_3/Al) FGM core thickness (10 mm)

k	Mild steel face sheet thickness, mm											
	1			1.5			2			2.5		
	Ana.	Num.	Error, %	Ana.	Num.	Error, %	Ana.	Num.	Error, %	Ana.	Num.	Error, %
0	3.76	3.76	0.00	3.53	3.50	0.85	3.36	3.38	0.60	3.23	3.20	0.93
0.5	3.45	3.46	0.29	3.29	3.31	0.61	3.17	3.19	0.63	3.08	3.05	0.97
1	3.35	3.34	0.30	3.22	3.29	2.17	3.12	3.16	1.28	3.038	2.98	1.91
2	3.27	3.27	0.00	3.16	3.18	0.63	3.08	3.04	1.30	3.01	2.96	1.66
5	3.16	3.15	0.32	3.09	3.04	1.62	3.02	2.98	1.32	2.97	2.93	1.35
10	3.12	3.18	1.92	3.06	3.03	0.98	3.01	2.96	1.66	2.96	2.91	1.69

Table 4.
Comparison between the analytical and numerical convergence of the initial non-dimensional frequencies of functionally graded Silicon nitride/Stainless steel, for beam (L/h = 50) and using CBT for S-S boundary

k	sheet thickness=1 mm			sheet thickness=1.5 mm			sheet thickness=2 mm			sheet thickness=2.5 mm		
	Ana	Num.	Error, %	Ana	Num.	Error, %	Ana	Num.	Error, %	Ana	Num.	Error, %
0	4.15	4.19	0.96	3.84	3.91	1.82	3.62	3.70	2.21	3.45	3.49	1.16
0.5	3.24	3.28	1.23	3.11	3.19	2.57	3.01	3.11	3.32	2.94	3.01	2.38
1	2.95	3.01	2.03	2.87	2.90	1.05	2.80	2.80	0.00	2.75	2.78	1.09
2	2.72	2.76	1.47	2.67	2.65	0.75	2.63	2.64	0.38	2.60	2.61	0.38
5	2.52	2.56	1.59	2.50	2.51	0.40	2.48	2.51	1.21	2.46	2.49	1.22
10	2.43	2.40	1.23	2.42	2.46	1.65	2.41	2.40	0.41	2.40	2.42	0.83

Table 5.
Comparison between the analytical and numerical convergence of the initial non-dimensional frequencies of functionally graded Zirconia /Titanium sandwich beam having (L/h = 50) and using CBT for S-S boundary

k	sheet thickness=1 mm			sheet thickness=1.5 mm			sheet thickness=2 mm			sheet thickness=2.5 mm		
	Ana.	Num.	Error, %	Ana.	Num.	Error, %	Ana.	Num.	Error, %	Ana.	Num.	Error, %
0	3.16	3.19	0.95	3.08	3.12	1.30	3.02	2.99	0.99	2.97	2.94	1.01
0.5	2.90	2.91	0.34	2.87	2.90	1.05	2.84	2.79	1.76	2.82	2.87	1.77
1	2.79	2.81	0.72	2.79	2.75	1.43	2.77	2.73	1.44	2.76	2.79	1.09
2	2.70	2.75	1.85	2.71	2.70	0.37	2.71	2.75	1.48	2.70	2.72	0.74
5	2.61	2.66	1.92	2.64	2.67	1.14	2.64	2.69	1.89	2.64	2.69	1.89
10	2.56	2.58	0.78	2.59	2.58	0.39	2.61	2.63	0.77	2.61	2.62	0.38

Table 6.
Comparison between the analysis and numerical convergence of the initial 1st dimensionless frequencies of beams having slenderness ratio (L/h = 100) and surface thickness (5 mm) under different boundary using CBT

k	C-C			S-S			C-F		
	Ana.	Num.	Discrepancy, %	Ana.	Num.	Discrepancy, %	Ana.	Num.	Discrepancy, %
0	8.25	8.27	0.24	3.64	3.64	0.00	1.30	1.32	1.54
0.5	7.64	7.64	0.00	3.37	3.38	0.30	1.20	1.22	1.67
1	7.43	7.45	0.27	3.28	3.31	0.91	1.17	1.17	0.00
2	7.28	7.28	0.00	3.21	3.29	2.49	1.14	1.16	1.75
5	7.08	7.11	0.42	3.12	3.21	2.88	1.11	1.09	1.80
10	7.00	6.87	1.86	3.09	3.11	0.65	1.10	1.05	4.54

However, a beam made from metals having less mechanical properties values (ZrO_2/Ti-6Al-4V) may be particularly sensitive and show low-frequency behaviour at the same time as the Si3N4/SUS304 beam indicates a large decrease in the frequency parameter with the index increase of the power-law due to its changes in mechanical properties.

Table 6 shows a detailed comparison between analytical and numerical convergences of the initial non-frequency coefficients of the functionally graded sandwich beams having a slenderness ratio of (L/h = 100) and a face thickness of (5 mm), using CBT for different boundary conditions. The outputs revealed that when the beam constraint raises, the natural frequency raises, and when the index of gradient index raises, the metal volume fraction raises, which reduces the plate's overall rigidity and affects the frequency parameter value. As the power-law exponent raises, the metal volume fraction raises, reducing the beam's overall rigidity and affecting the frequency parameter value (λ). From all the results, it can be found that the frequency parameter (λ) of all beams decreases with increasing the material volume power-law index (k).

Therefore, from Table 3 to 6 can be shows that the very good argument get between analytical and numerical techniques used to calculate the natural frequency for beam structure. So, the maximum error between the results evaluated did not exceed about (4.5%).

The is because that when the parameter (k) increases, the concentration of the ceramic phase harder than the metal phase decreases. In all tabulated results, it is found that there is a sensible agreement between numerical and analytical results with a maximum error percentage of (5%). Graphical representations of the natural frequency relationships for the functionally graded sandwich beams subjected to different boundary conditions are shown in Figures 4-9. Figure 4 shows the dimensionless fundamental frequencies outputs of the functionally graded sandwich beams having different boundary conditions (face sheet thickness 2 mm, k=0.5). It's found that the natural frequency raises by raising the ratio of thickness, and a beam with C-C type is more sensitive to the change in this ratio compare to S-S and C-F.

Figure 5 shows the dimensionless essential frequencies of the functionally graded sandwich beams having S-S boundary conditions, and the face sheet thickness is 2 mm. It is clear that the frequency parameter increases with increasing the FGM thicknesses and face sheet due to the increase in plate stiffness. The analytical results of the natural frequency of the simply supported functionally graded sandwich beams at (L/h=50), for various power-law indices, are represented in Figure 6, while in Figure 7, the dimensionless essential frequencies of the functionally graded sandwich beams with different boundary conditions (L/h=100) is drawn.

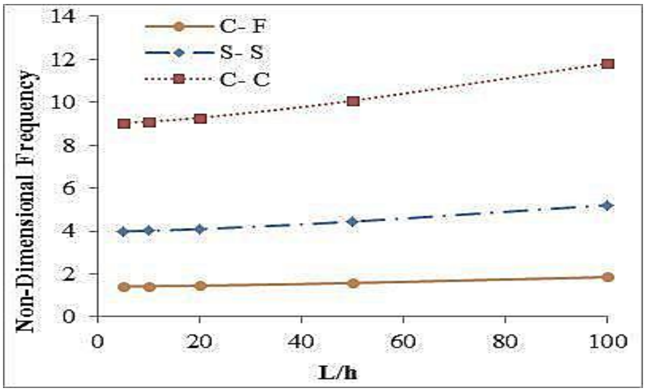

Fig. 4. Frequency parameter under various B.Cs. and at (2 mm) face thickness and k=0.5

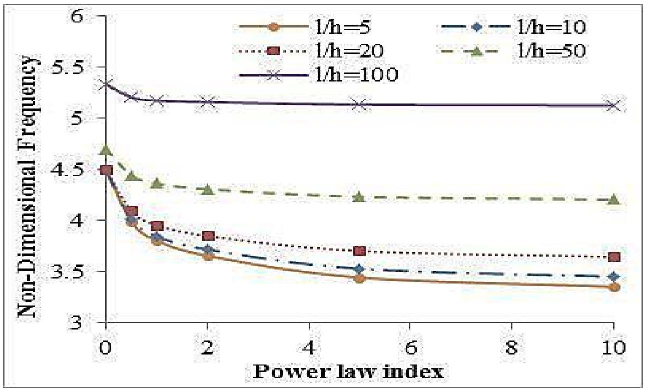

Fig. 5. Frequency parameter for S-S beam, face sheet thickness (2 mm)

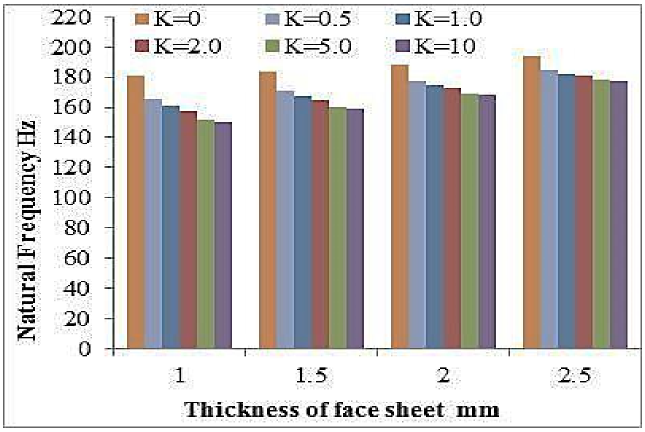

Fig. 6. Analytical results of the natural frequency of (S-S) beams at L/h=50, for various values of the power-law index

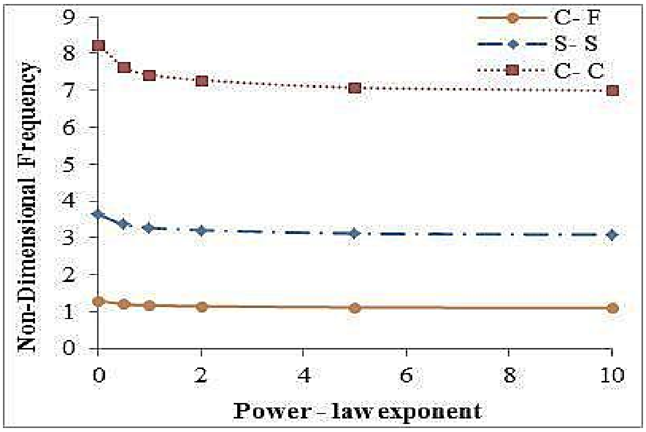

Fig. 7. Frequency of beams having different boundary conditions at slenderness ratio =100

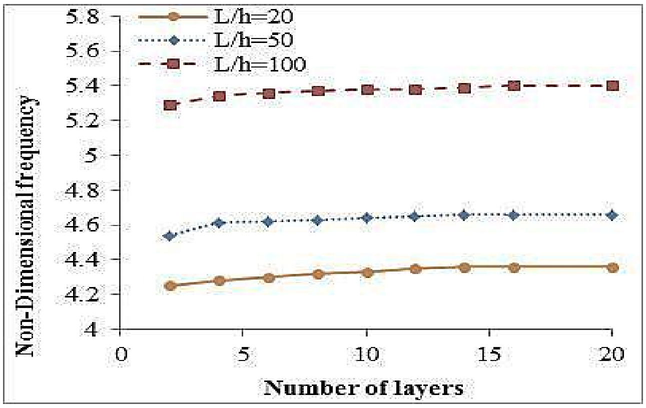

Fig. 8. Dimensionless essential frequencies of the sandwich beams for different layers numbers with various ratios of thickness at k= 0.3

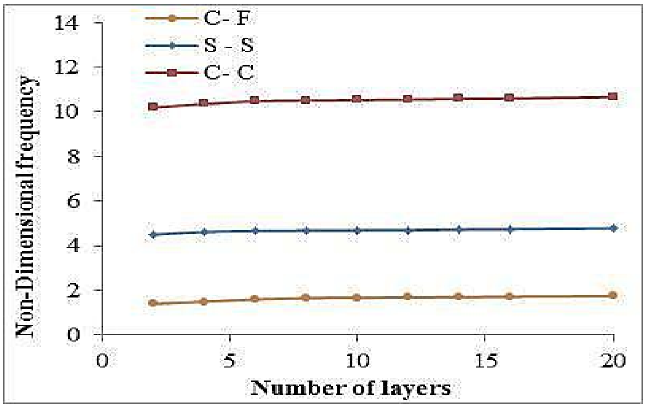

Fig. 9. Dimensionless essential frequencies of the FG sandwich beam various layers numbers with different thickness BC's at k= 0.1

It's concluded that the essential frequency raises by increasing the constraints into the structure. The influence of layers number upon the FGM sandwich beam's natural frequency based on different parameters and for various boundary conditions is drawn in Figures 8 and 9. Figure 8 depicts the analytical results of the dimensionless essential frequencies of FG sandwich beams with different thickness ratios at k= 0.3, while Figure 9 evinces the dimensionless essential frequencies of the functionally graded sandwich beams with different thicknesses and boundary conditions at (k= 0.1). Raising the layer number was found to increase the results' accuracy. From the obtained results, the highest value of the natural frequency occurs when k=0, and with increasing the power index value from metal to ceramic, the stress is decreased, and it is increased when reaching the ceramic layer; this is due to the variation of mechanical properties through different power index until reaching the isotropic status.

6. Conclusions

The free vibration of FG sandwich beams comprised of the functionally graded core and face sheets with an isotropic uniform material is investigated based on the Euler-Bernoulli beam theory and the Finite element method. The material properties are assumed to vary through the thickness direction according to power-law functions. Governing differential equations are derived in closed form. Then, the characteristic frequency equations of beams are obtained. The accuracy of the derived formulation was validated by comparing the numerical result obtained by Ansys software, when calculate discrepancy between the analytical and numerical technique did not exceed about (4.5%). From the above results, it can be concluded that:

1. The comparison between analytical, using drive for general equation of beam motion, and numerical solution, using finite element technique, given a good argument for results calculated with discrepancy for results did not exceed about (4.5%)
2. The natural frequencies increase by raising the thickness ratio, while they decrease as the exponent of power-law raises for all the types of boundary conditions. When the change for frequency lead to about (26%).
3. The developed method that's primarily based totally on Euler-Bernoulli beam theory may be carried out to all kinds of FG materials and in addition to the free structural vibration, it is able to be used easily to investigate the forced structural vibration subjected to a given loading.

4. When the number of layers increases to 25 layers, more accurate natural frequency results for the FGM sandwich beam are obtained for all models, due to modified for the beam stiffness to weight ratio.
5. The sandwich beam frequency parameter increases with the increase in the constraints to the boundary conditions; for example, the frequency parameter for (C-C) is higher than (S-S), and this condition is more than (C-F), and so on, as listed in Table 6.

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