DOI: 10.5604/01.3001.0016.1480
Volume 114 • Issue 1 • September 2022

Journal of Achievements in Materials and Manufacturing Engineering

International Scientific Journal published monthly by the World Academy of Materials and Manufacturing Engineering

Probability-statistical estimation method of feed influence on the tangential cutting force under turning

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ABSTRACT

Purpose: This research aims to develop the mathematical model and propose a method for estimating the feed stochasticity impact on the tangential cutting force during turning. The main reason for this research is that the existing models for determining the tangential component of the cutting force do not take into account the stochasticity of the feed rate.

Design/methodology/approach: Measurements of tangential cutting force during turning on general-purpose lathes with known feed dispersion parameters were made. The mathematical model was developed, and dispersion characteristics (mean value, dispersion and mean square deviation) of the tangential cutting force component depending on the corresponding dispersion characteristics of the feed rate were obtained. The method of assessing the impact of stochasticity of the feed rate on the tangential cutting force is proposed.

Findings: As the result of the carried-out investigations, it is proved that the stochasticity of the feed rate affects the dispersion of the tangential cutting force during turning. For specific conditions, the share of feed stochasticity in the dispersion of tangential cutting force component is from 40 to 60% and should be taken into account while prescribing rational cutting modes.

Practical implications: The obtained results make it possible to adjust the cutting modes, particularly the amount of feed, under the conditions of real equipment to ensure certain power characteristics of the cutting process to prevent overloads during cutting. This investigation benefits to the establishment of additional factors affecting oscillations in the cutting process.

Originality/value: The probabilistic-statistical approach is used in this investigation in order to prove that the stochasticity of the feed rate affects the dispersion of the tangential cutting force component.

Keywords: Tangential cutting force, Cutting depth, Stochasticity feed, Distribution law, Distribution characteristics

Reference to this paper should be given in the following way:

V. Krupa, N. Tymoshenko, V. Kobelnyk, I. Petrechko, Probability-statistical estimation method of feed influence on the tangential cutting force under turning, Journal of Achievements in Materials and Manufacturing Engineering 114/1 (2022) 22-31. DOI: https://doi.org/10.5604/01.3001.0016.1480

ANALYSIS AND MODELLING

Legend

t – cutting depth, mm;
s – feed, mm/rev;
$\bar{s}$ – mean value of feed, being approximately equal to the mathematic expectation, mm/rev;
$\sigma(s)$ – is the mean quadratic deviation of the feed value mm/rev;
V – cutting speed, m/min;
Pz – tangential component of the cutting force, N;
$\overline{Pz_i}$ – mean value of Pz, N;
$M(Pz)$ – the mathematical expectation of Pz, N;
$\sigma(Pz)$ – is the mean quadratic deviation of Pz, N;
$D(Pz)$ – dispersion of Pz, N^2;
Pz_{max} – maximum value of Pz, N;
Pz_{min} – minimum value of Pz, N;
k_r, k_m, k_f, k_γ, k_λ – coefficients taking into account the radius of the cutter tip, the quality of the material being processed, top planner angle, front cutter angle and main cutting edge inclination angle, respectively, on Pz;
x_{Pz}, y_{Pz}, n –are degrees that take into account the effect of cutting depth t, feed s and cutting speed V on Pz, respectively;
a, b – are the thickness and the width of the cutting layer, respectively;
τ_d– are shearing stress tangents;
ψ– is friction angle on the front surface;
γ and θ – are the cutting tool primary angle of the cutting edge and the shearing area deviation angle, respectively.
σ_{st} – is the mean yield stress of the turning material in the area of plastic deformation, which corresponds to the average temperature in the deformation area;
u – is the specific energy of deformation;
μ – is the coefficient of friction in the yield stress portions;
μ_1 and μ_2 – are coefficients of friction from the friction forces at the front and back surfaces of the cutting tool;
k_C – is the coefficient of the chip thickening;
l_3 – is the value being specified by the assumed cutting tool wearing criterion on the back surface in the particular calculation;
φ – primary angel at the plane.

1. Introduction

One of the most important elements of most machine parts is the cylindrical outer and inner surfaces, which are made to the required accuracy grade with a given roughness class. In order to solve the whole complex of technical problems that arise in the treatment of such surfaces, the design of metal-cutting machines and technological equipment, there is a need for methods for determining the tangential cutting force, which would cover the effect of the elements of the cutting mode: cutting depth t, feed s, cutting speed V on the magnitude of the tangential cutting force. The importance of developing such methods is confirmed by the large number of recognized scientists who have proposed their methods and formulas for determining the tangential cutting force Pz.

Probabilistic-statistical methods are widely used in the investigation of cutting processes. They are most frequently used while solving optimization problems, as well as in the investigation of cutting process characteristics. For example, [1] the problem of multicriteria optimization of the turning process using the probabilistic-statistical approach is solved, [2] optimization of the turning process of hardened steel AISI 52100 using stochastic programming is carried out, and [3] the stochasticity of various criteria on the optimization process is taking into account in this paper. A great number of papers are devoted to the analysis of stochasticity of cutting forces occurring during metal layer cutting in turning [4] and milling [5,6]. In the paper [7] the methods of probability theory and the theory of Markov processes – to obtain the limit distribution of the random process of dynamic impact on the employee of negative factors over time were applied and main rates against which the level of occupational risks within the "man-machine-environment", a methodology for the assessment elastic properties of the composite material were obtained [8].

According to the analysis of the literature, the determination of the Pz cutting force can now be made by four methods: tabular procedure [9], computer simulation, and using empirical [12-15] and analytical [16-19] dependencies. The tabular method, despite its simplicity and accessibility, offers low accuracy, being its major disadvantage, as it is impossible to take into account all the specific values of the factors affecting the cutting force. The computer simulation method of determining the Pz cutting force is based on SolidWorks, Autodesk Inventor, T-Flex CAD, LS-DYNA, etc. software and uses the finite element method. In computer simulation it is used an updated Lagrangian approach with dynamic refinement [10], program LS-DYNA [11], differently texturized cutting tools [12], friction coefficients [13], etc. Its disadvantage is that it is quite complicated to be applied at production sites and has a considerable number of tolerances. The accuracy of its resultsdepends on the shape and the number of elements themselves.

The most commonly used empiric formulas, which were obtained from the references [14] respectively, or the similar ones, [15,16] in particular

$$Pz = 10 \cdot C \cdot t^{x_{Pz}} \cdot s^{y_{Pz}} \cdot V^n \cdot k_V^n \cdot k_m \cdot k_f \cdot k_\gamma \cdot k_\lambda \quad (1)$$

The main disadvantage of this method is that the results are not always reliable enough, as the factors of different authors in these formulas for similar materials and cutting modes differ many times.

Analytical formulas are also often used. For example, dependence is given in [17,18]:

$$Pz = \frac{a \cdot b \cdot \tau_d \cdot cos(\psi - \gamma)}{sin\,\theta \cdot cos(\theta + \psi - \gamma)} \tag{2}$$

The analysis of formulas shows that their practical application is problematic, as for the particular case, some values are not known; for example, the deviation angle value of the shearing area is not available.

One of the most accurate theoretical formulas for cutting force determination [19]:

$$Pz = 1{,}155\sigma_{\text{st}} ust_r \left\{\left[1 + \mu_1(1 - tg\gamma) + \frac{(0.5+\mu)\cdot u}{2k_C}\right] \times \times cos\,\gamma + \frac{k_C}{4u\,cos\,\gamma} + \mu_1\,sin\,\gamma + \frac{\mu_2 \cdot l_3}{us\,sin\,\phi} + \frac{k_S \cdot s \cdot sin^2\,\varphi}{4u\,cos\,\gamma}\right\} \tag{3}$$

The difficulty of determining the individual components of the formula for specific types of materials.

A large number of investigations are dealing with specific problems solution concerning the investigation of cutting forces in the processing of individual materials, including composite ones [20], clarification of various parameters impact on cutting forces, such as angles in the plane and other geometric parameters of the tool, cutting speeds, thickness and width of the cut layer [21], and development of software to estimate cutting forces in turning [22]. The direction of software development for determining the cutting forces, particularly due to the array of defining parameters, is advantageous [23].

In general, almost all investigations prove that one of the determining factors influencing the magnitude of the cutting force is the magnitude of the cutting depth and feed.

The objective of this paper is to investigate the effect of feed rate dissipation on The data presented above to testify to the dissipation of tangential cutting force as in the case of turning.

The existing theoretical and empirical dependences for finding the tangential cutting forces do not consider certain factors that significantly affect the results' reliability. Firstly, they do not take into account sufficient differences in the physical and mechanical properties of the alloy grades of the same class.

Secondly, in theoretical and empirical formulas, such element of the cutting mode as the feed is given in the deterministic version, although for general-purpose universal turning, boring, drilling and milling machines, the feed s is a random variable with a normal distribution law [24,25]. For example, for a 16K20 lathe machine tool, according to [24], the relation s_{max_i}/s_{min_i} within the range of passport feed values is 0.05-0.2 mm/rev. varies in the range of 1.27-1.58. Dispersion of the feed value is present even in new machine tools due to a certain length of the kinematic chain. In real production, the term of operation of the machine tool is at least ten years. These factors are not taken into account when conducting studies to find the influence of cutting mode elements on cutting force for a new tool and turning materials [26-28] in the study of mild steel cutting process by using the plasma arc method [29] as well as for modelling cutting force based on tool wear and cutting tip radius augmentation in the process [30]. In the research paper [31], the effect of stochastic cutting force on loads of lathe elements is considered based on a probabilistic approach. In some works, the influence of stochasticity of processing modes on the quality of surfaces is given [32-34] and to substantiated experimentally the choice of material for the tool's cutting edge and the method of surfacing it on the cutter In [35] reserched of a hydraulic tool, operating under the simultaneous influence of friction and cyclic loading.

The data presented above testify that the tangential cutting force is of the stochastic nature. That is why the creation of the probability statistical method of estimation the influence of feed on the force *Pz*, under turning the cylinder surface, will improve the reliability of the obtained results and is undoubtedly an urgent task.

2. Experimental studies

The purpose of the experimental research is to record the graphs of changes in a tangential component of the cutting force in real time on different kinematic feed chains, as well as to show the dispersion of the tangential cutting force on machines with different wear.

Experimental studies on the tangential component of the cutting force during turning were performed on a 16K20 lathe machine tools, with different service life and, accordingly, the wear of components and parts. Photo and scheme of the installation for the investigations are shown in Figure 1. A special piezoelectric dynamometer with a piezo-electric sensor was installed in the cutter. A torch cutter was installed and fixed in the dynamometer. The dynamometer was connected to the power supply unit and the tensor station. A pre-cut workpiece with a diameter of 80 mm made of 45 steel (ISO P) (carbon content 0.45%) was installed in the cartridge of the machine. The support was fitted with a special strain gauge dynamometer mounted on piezoelectric feeder with a power supply unit and a fixed cutter. The material of the cutting part of the cutter is T15K6 hard alloy (Co – 6%, TiC – 15%, WC – 79%) without coating. The geometrical parameters of the cutter: the main angle in the plan – φ=45°, the auxiliary angle in the plan φ'=15°, the main back angle α=8°, the main front angle γ=8°. The experiment was performed without the use of a lubricant-cooling process medium.

The dynamometer was connected to the computer and analyzed by special software. After calibration of the device, the process of turning was performed and the values of the cutting force were recorded for each of the planned kinematic feed chains. The results of records of cutting force at feed values s = 0.06 mm/rev, s = 0.1 mm/rev, s = 0.15 mm/rev, and s = 0.2 mm/rev are presented in Figure 2.

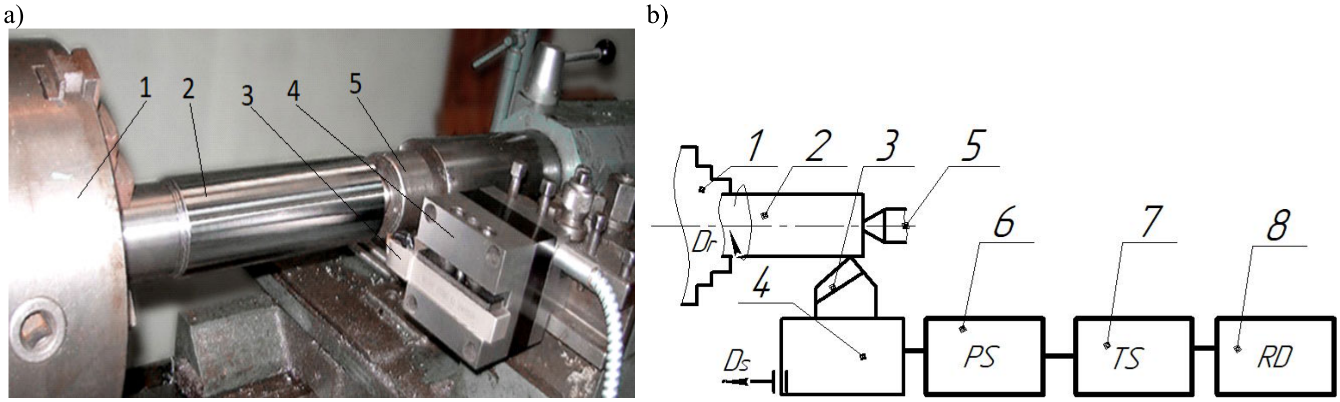

Fig. 1. Experimental setup for determining the tangential component of the cutting force while turning on the basis of 16K20 lathe machine tool: a) photography; b) block diagram of the installation; 1 – lathe chuck; 2 – workpiece; 3 – cutter; 4 – support with dynamometer; 5 – back center; 6 – power supply; 7 – tensor station; 8 – the recording device

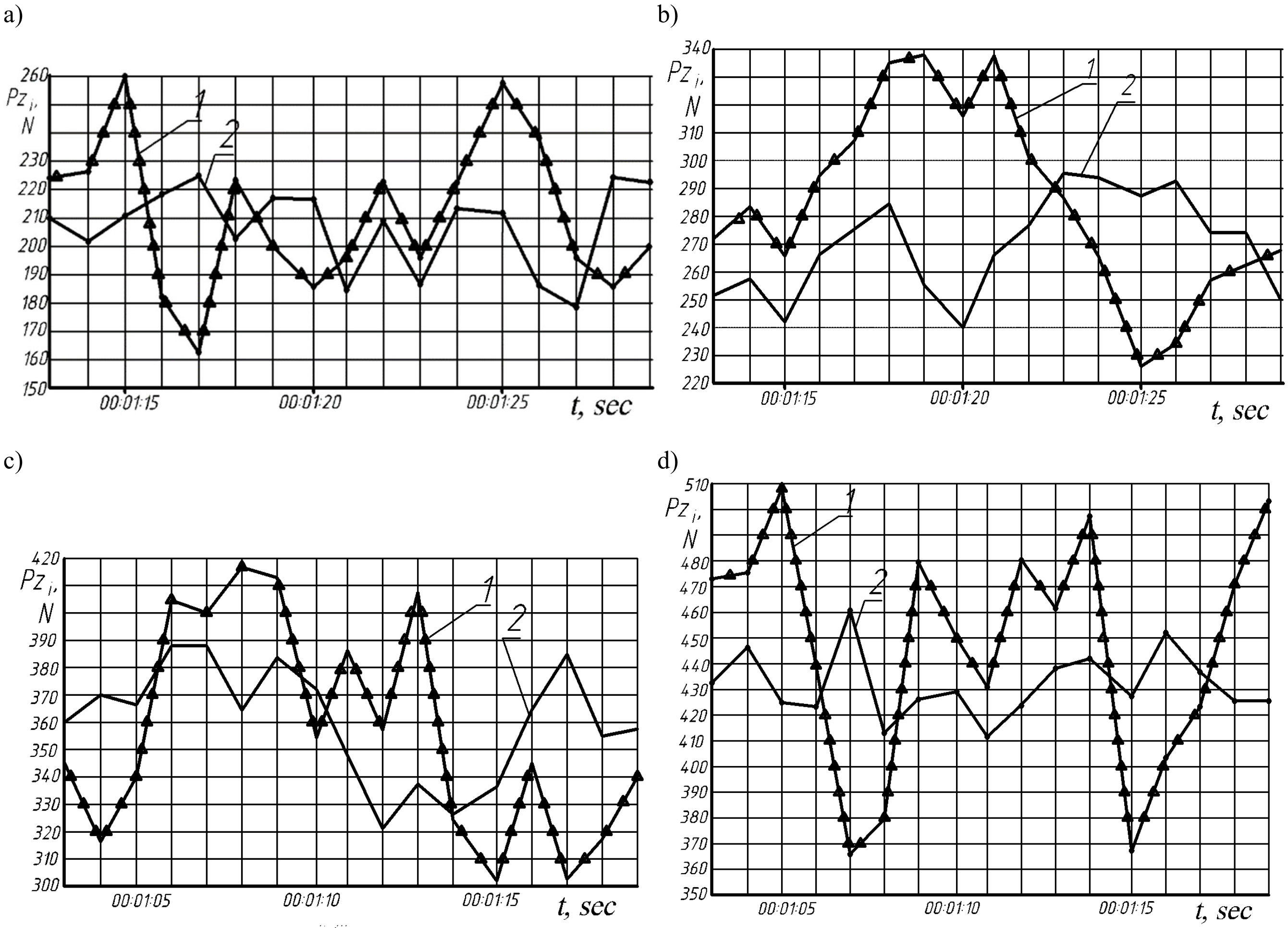

Fig. 2. Fragments graph of change of values of tangential cutting force at turning: a) for the value of feed s=0.06 mm/rev; b) for the value of feed s=0.1 mm/rev; c) for the value of feed s=0.15 mm/rev; d) for the value of feed s=0.2 mm/rev; 1 – on the shopworn machine tool (machine tool No 1); 2 – on the machine tool with short service life (machine tool No 2)

Analyzing the obtained graphs, we can see that on a heavily worn machine, the dispersion of the force tangential component value is smaller than on a less worn one. This tendency is observed in all investigated kinematic feed chains. In addition, it can be seen from the graphs that if the feed rate increases, the dispersion of the tangential cutting force increases. As the feed rate increases, its scattering dispersion increases [24]. Therefore, there is a relationship between the dispersions of feed scattering and the scattering dispersion of the tangential cutting force component.

The measurement results are presented in Table 1.

Table 1.
Dissipation characteristics of the experimental values of *Pz*, maximum and minimum cutting forces and criteria calculation results for machine tool №1 and №2

Dissipation characteristics	Passport feed values, mm/rev					
	0.06	0.1	0.125	0.15	0.175	0.2
Machine tool №1						
$\overline{Pz_i}$, N	205.5	283.1	318.3	361.1	429.2	493.7
*D_{ex} (Pz), N^2	179.6	249.6	295.8	338.6	479.6	595.4
*σ_{ex} (Pz), N	13.4	16.1	17.2	18.4	21.9	24.7
$Pz_{max0.95}$, N	244.2	329.7	368.1	414.3	492.6	564.3
$Pz_{min0.05}$, N	166.7	236.5	268.5	307.8	365.8	423.1
$Pz_{max0.95}/\overline{P_{zi}}$	1.19	1.16	1.15	1.14	1.12	1.14
t_k	52.9	21.3	24.1	33.7	27.8	
$P(t_k)$	<0.0001					
$F(F_{tabl}=1.14)$	1.39	1.185	1.144	1.417	1.241	
Machine tool №2						
$\overline{Pz_i}$.N	215.2	270.3	315.1	357.7	400.2	483.2
*D_{ex} (Pz), N^2	68.2	94.8	112.1	130.2	180.1	208.6
*σ_{ex} (Pz), N	8.26	9.74	10.59	11.41	13.42	14.44
$Pz_{max0.95}$, N	235.97	299.51	346.86	391.93	440.46	526.53
$Pz_{min0.05}$, N	186.43	241.09	283.34	323.47	359.94	439.87
$Pz_{max0.95}/\overline{Pz_i}$	1.12	1.11	1.10	1.10	1.10	1.09
t_k	65.46	44.05	38.7	34.12	59.54	
$P(t_k)$	<0.0001					
$F(F_{tabl}=1.14)$	1.39	1.18	1.16	1.38	1.16	

For each of the feeds, the mean $\overline{Pz_i}$, the dispersion of $D_{ex}(Pz_i)$, and the mean squared deviation $\sigma_{ex}(Pz_i)$ of the tangential component of the cutting force, which are summarized in Table 1, were experimentally measured. For the obtained characteristics, $\overline{Pz_i}$, $D_{ex}(Pz_i)$, $\sigma_{ex}(Pz_i)$, the Student criterion was used to find the significance of differences in the dissipation values of the tangential component of the cutting force, measured on adjacent kinematic feed chains, using the dependence [29]:

$$t_k = \frac{|\overline{Pz}_{i+1} - \overline{Pz}_i|}{\sqrt{\sigma(Pz)^2_{i+1} + \sigma(Pz)^2_i}} \cdot \sqrt{n} \tag{4}$$

where $\sigma(Pz)_{i+1}$, $\sigma(Pz)_i$ – are the corresponding root mean square deviations (i = 1…6, sequence number of the kinematic supply chain); n = 200 (number of cutting force values selected (every 1 sec. for 200 seconds);

The probability $P(t_k)$ was determined based on a significance level of 0.05 [36].

The significance of the difference in the variances was found by Fisher criterion *F* using the formula:

$$F = \frac{\sigma^2_{i+1}}{\sigma^2_i} \tag{5}$$

The Student and Fisher criteria calculation results are presented in Table 1.

Since in all cases, the value of $P(t_k)$<<0.05, we conclude that by the Student criterion, the difference in the results is significant.

According to the Fisher criterion, it has been found that with increasing the dispersion of the feed, the dispersion of the cutting force increases significantly.

It is evident that on the shopworn machine, the cutting force dissipation is much greater than on the unworn one. Since the production uses equipment with different service life, there is a need to develop a model which takes into account the degree of equipment wear when predicting the cutting force.

3. Mathematical modelling

The proposed probabilistic statistical method deals with the determination of the distribution law of the value *Pz* as a random value depending on the stochastic feed. To illustrate this method, we use the known formulas, empirical (1) and analytical (3), to determine the tangential cutting force depending on the elements of the cutting mode, having assumed that all other elements of the mode are constant, except that of feed. In general-purpose metal-cutting machines, as has been proved in [27,28], which are equipped with the feed and speed boxes, the real feed values are random values with the normal law of distribution, mathematic expectation *M(Pz)* and the dissipation dispersion $D(Pz)=\sigma^2(Pz)$. In this case, the value *Pz* will be treated as random with the certain distribution law and on its basis, except its mathematic expectation, let us find its maximum value Pz_{max}, and, being risky at 5%, the value of the interval $[Pz_{min\ 0.05}; Pz_{max\ 0.05}]$.

The cut point density (differential distribution function) of the random value S [24] is:

$$f(s) = \frac{1}{\sqrt{2\pi}\sigma(s)} e^{\frac{-(s-\bar{s})^2}{2\sigma(s)^2}} \tag{6}$$

If the differential functions $f(s)$ of the random valuess_i are known, the corresponding differential functions $g(Pz) = g(y)$ of the random values $Pz_i = \varphi(s_i)$ are found from the equation [37,38]:

$$g(Pz) = f[\psi(Pz)] \cdot \psi'(Pz), \qquad (7)$$

where $\psi(Pz_i)$ – are the corresponding inverse functions of the function $Pz_i = \varphi(s_i)$; $(i = \overline{1, 6})$

Dependence (1) at *t*=const and *V*=const will be transformed into the formula

$$Pz = C \cdot s^y, \qquad (8)$$

where $C = C_P \cdot t^x \cdot V^n \cdot k_p$, y – are constant values; s – is a random value according to the Gauss distribution law.

The inverse function $\psi(Pz_i) = \psi(z)$, taking into account the fact,that the value Pz_i is always positive, and its derivative $\psi'(Pz_i) = \psi'(z)$ look like respectively:

$$\psi(z) = \sqrt[y]{z/C} \qquad (9)$$

$$\psi(z') = \frac{1}{C \cdot y} \sqrt[y]{\left(\frac{z}{C}\right)^{1-y}} \qquad (10)$$

Having used the dependences (7, 9 and 10), the differential function of the random value distribution *Pz* $g(Pz) = g(z)$ is obtained:

$$g(Pz) = \frac{1}{\sigma(s) \cdot \sqrt{2 \cdot \pi}} \cdot \frac{1}{C \cdot y} \sqrt[y]{\left(\frac{z}{C}\right)^{1-y}} \cdot e^{-\frac{\left(\sqrt[y]{\frac{z}{C}} - \bar{s}\right)^2}{2\sigma(s)^2}} \qquad (11)$$

The formula (3) will be presented as follows:

$$Pz = As^2 + Bs + C \qquad (12)$$

where Pz, s – are random values, $A > 0$, $B > 0$, $C > 0$ – are constant values and

$$A = \frac{1.155\sigma_{st} \cdot k_C \cdot \sin^2\varphi}{4 \cdot \cos\gamma};$$

$$B = 1.155 \cdot \sigma_{st} \cdot u \cdot t_r \cdot \cos\gamma \left[1 + \mu_1(1 - tg\gamma) + \right.$$
$$\left. + \frac{0{,}5 + \mu}{2k_c} + \frac{k_c}{4 \cdot u \cdot \cos^2\gamma} + \mu \cdot tg\gamma\right];$$

$$C = \frac{1{,}155 \cdot \sigma_{st} \cdot t_r \cdot \mu_2 \cdot l_3}{\sin\varphi}.$$

As the function $Pz = As^2 + Bs + C$ is a differential one and steadily increasing, the formula (7) can be used for finding the distribution function $g(Pz)$ of the random value Pz.

The reverse function $\psi(Pz)$ looks like:

$$\psi(Pz) = \frac{\sqrt{\Delta + 4A \cdot Pz} - B}{2A}, \text{ where } \Delta = B^2 - 4AC,\ Pz > C.$$

Given the density of distribution of the random variable ***s*** from (6) obtained

$$f[\psi(Pz)] = \frac{1}{\sqrt{2\pi}\sigma(s)} e^{-\frac{\left(\frac{\sqrt{\Delta + 4A \cdot Pz} - B}{2A} - \bar{s}\right)^2}{2\sigma(s)^2}}. \qquad (13)$$

From equation (13) we find a derivative of the inverse function in:

$$\psi'(Pz) = \frac{1}{\sqrt{\Delta + 4A \cdot Pz}}. \qquad (14)$$

Substituting expressions (13) and (14) into equality (7), btained the required density of distribution for a random variable:

$$g(Pz) = \frac{1}{\sqrt{2\pi}\sigma(s)} \frac{1}{\sqrt{\Delta + 4A \cdot Pz}} e^{-\frac{\left(\frac{\sqrt{\Delta + 4A \cdot Pz} - B}{2A} - \bar{s}\right)^2}{2\sigma(s)^2}}. \qquad (15)$$

For convenience of further calculations, we denote a random variable *Pz* by *y*. Then expression (15) is written in the form

$$g(Pz) = \frac{1}{\sqrt{2\pi}\sigma(s)} \frac{1}{\sqrt{\Delta + 4A \cdot Pz}} e^{-\frac{\left(\frac{\sqrt{\Delta + 4Ay} - B}{2A} - \bar{s}\right)^2}{2\sigma(s)^2}}. \qquad (16)$$

The mathematical expectation of a random variable *y* is determined by the formula [37,38]

$$M(y) = \int_{y_1}^{y_2} y \cdot g(y) dy, \qquad (17)$$

where y_1 and y_2 – are the change limits of *y* (i.e. the limits of change *Pz*).

Substituting expression (16) for $g(y)$ into formula (17), obtained

$$M(y) = \frac{1}{\sqrt{2\pi}\sigma(s)} \int_{y_1}^{y_2} y \frac{1}{\sqrt{\Delta + 4Ay}} e^{-\frac{\left(\frac{\sqrt{\Delta + 4Ay} - B}{2A} - \bar{s}\right)^2}{2\sigma(s)^2}} dy. \qquad (18)$$

To calculate the integral, introduced a new variable

$$z = \frac{\frac{\sqrt{\Delta + 4Ay} - B}{2A} - \bar{s}}{\sigma(s)} = \frac{\sqrt{\Delta + 4Ay} - B}{2A\sigma(s)} - \frac{\bar{s}}{\sigma(s)} \qquad (19)$$

From here obtained:

$$\sigma(s) \cdot z = \frac{\sqrt{\Delta + 4Ay} - B}{2A} - \bar{s},$$
$$\sqrt{\Delta + 4Ay} = 2A\sigma(s)z + 2A\bar{s} + B,$$
$$y = A\sigma(s)^2 z^2 + \sigma(s)(2A\bar{s} + B)z + A\bar{s}^2 + B\bar{s} + C,$$
$$dy = \sigma(s)(2A\sigma z + 2A\bar{s} + B)dz.$$

Then with the resulting expressions, formula (18) will have the form

$$M(y) = \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_2} [A\sigma(s)^2 z^2 + \sigma(s)(2A\bar{s} + B)z + (A\bar{s}^2 +$$
$$B\bar{s} + C)] \cdot e^{-\frac{z^2}{2}} dz, \qquad (20)$$

where $z_1 = \frac{\sqrt{\Delta + 4Ay_1} - B}{2A\sigma(s)} - \frac{\bar{s}}{\sigma(s)}$, $z_2 = \frac{\sqrt{\Delta + 4Ay_2} - B}{2A\sigma(s)} - \frac{\bar{s}}{\sigma(s)}$.

Having found the integral $\int_{z_1}^{z_2} z^2 e^{-\frac{z^2}{2}} dz$, $\int_{z_1}^{z_2} z e^{-\frac{z^2}{2}} dz$, $\int_{z_1}^{z_2} e^{-\frac{z^2}{2}} dz$ and having introduced the notations $R_1 = A\sigma(s)^2$, $R_2 = \sigma(2A\bar{s} + B)$, $R_3 = A\bar{s}^2 + B\bar{s} + C$, the mathematical expectation $M(y)$ will look like:

$$M(y) = \frac{1}{\sqrt{2\pi}} \left[(R_1 z_1 + R_2) e^{-\frac{z_1^2}{2}} - (R_1 z_2 + + R_2) e^{-\frac{z_2^2}{2}}\right] +$$
$$(R_1 + R_3)[\Phi(z_2) - \Phi(z_1)] \qquad (21)$$

where $\Phi(z_1)$, $\Phi(z_2)$ are the Laplace functions,

$$D(y) = \frac{1}{\sqrt{2\pi}} \cdot \Big\{ [R_1^2(z_1^3 + 3z_1) + 2R_1R_2(z_1^2 + 2) + (R_2^2 + 2R_1R_3)z_1 + 2R_2R_3] \cdot e^{-\frac{z_1^2}{2}} - [R_1^2(z_2^3 + 3z_2) + 2R_1R_2(z_2^2 + 2) + (R_2^2 + 2R_1R_3)z_2 + 2R_2R_3]\, e^{-\frac{z_2^2}{2}} \Big\} + (3R_1^2 + R_2^2 + 2R_1R_3 + R_3^2)[\Phi(z_2) - \Phi(z_1)] - [M(y)]^2 \quad (22)$$

4. Results and discussion

4.1. The results of theoretical investigations according to the developed model

Results of calculation Pz values for the analytical (3) and empiric (1) formulas are presented in Table 2. Results obtained for feed dissipation values on 16K20 lathe machine tool according to data [24] (mashine tool №1).

Table 2.
Results of calculations of Pz values for analytical and empirical formulas

Formula	Scattering characteristics	Passport feed values, mm/rev					
		0.06	0.1	0.125	0.15	0.175	0.2
Analytical	$M_{an}(Pz)$, N	207.6	277.8	328.2	377.9	434.9	478.9
	$D_{an}(Pz)$, N^2	79.6	111.9	118.6	142.7	217.9	257.5
	$\sigma_{an}(Pz)$, N	8.92	10.58	10.89	11.95	14.76	16.047
Empiric	$M_{emp}(Pz)$, N	190.7	269.8	321.7	369.2	413.3	461.2
	$D_{emp}(Pz)$, N^2	115.9	126.6	117.8	127.7	178.7	193.6
	$\sigma_{emp}(Pz)$, N	10.8	11.3	10.9	11.3	13.4	13.9
$D_{an}(Pz)/D_{експ}(Pz)$		0.443	0.448	0.401	0.421	0.454	0.432
$D_{emp}(Pz)/D_{експ}(Pz)$		0.645	0.507	0.398	0.377	0.373	0.325

Let us analyze the possibility of replacing the obtained theoretical law of distribution of random variable Pz by normal distribution with parameters $M[Pz] = M(y)$ та $D[Pz] = D(y)$ and, accordingly, for both empirical and analytical dependencies.

Taking into account the central limit theorem of probability theory stated by Lyapunov and having used the Kolmogorov criterion, we established the possibility of replacing the obtained functions (11) and (15) by the probability density of the normal law with the parameters $M[Pz] = \overline{Pz}$ and $D[Pz]$. To use the Kolmogorov criterion, the maximum value of λ the difference module between the calculation distribution function $G*(Pz)$ and the corresponding theoretical function $G(Pz)$ and the probability $P(\lambda)$ [36] were found according to the formulas:

$$\lambda = max|G*(Pz) - G(Pz)| \cdot \sqrt{n} \quad (23)$$

$$P(\lambda) = 1 - \sum_{k=-\infty}^{\infty}(-1)^k e^{-2k^2\lambda^2}. \quad (24)$$

Taking into account that the obtained computational functions (11) and (15) are replaced by the probability density of the shortened (to the left) normal law which looks like:

$$h(Pz) = \frac{1}{D(Pz)\cdot\sqrt{2\pi}} \cdot e^{-\frac{(Pz_i - \overline{Pz})^2}{2\cdot D(Pz)}}, \quad (25)$$

The obtained theoretical curves of the distribution density and superposition on them the corresponding curves of the normal distribution density, built for the passport feed values *s*=0.2 mm/rev, are shown on the graphs (Fig. 3).

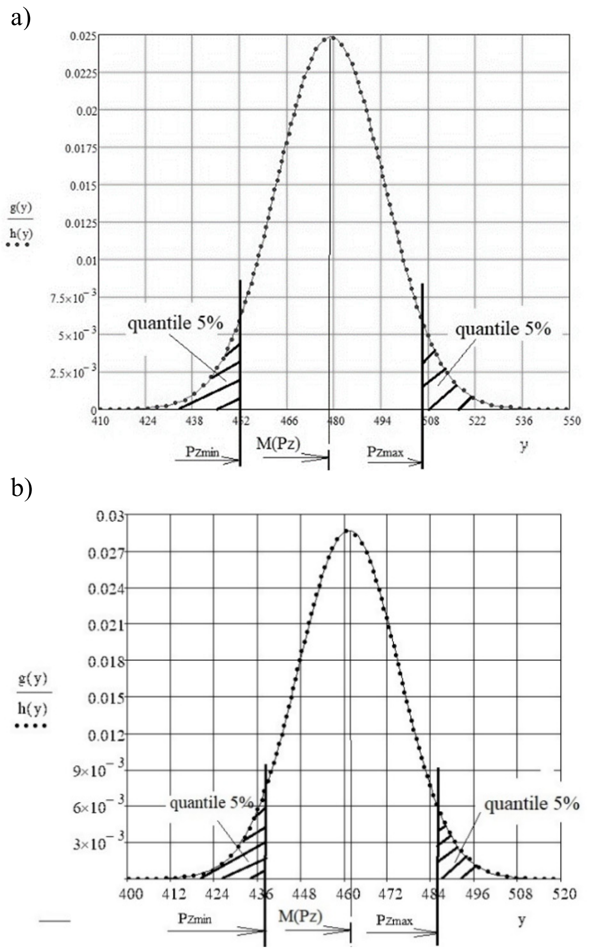

Fig. 3. Density curves of theoretical $g(Pz) = g(y)$ and normal $h(Pz) = h(y)$ distribution of the random value Pz at a feed 0.2 mm/rev: a) obtained from the empiric formula (1); b) obtained from analytical formula (3)

The statistical estimation of the random feed influence on the cutting force by turning or boring was proposed for the random value Pz, the quantities being $0.05 - Pz_{min}$ and $0.95 — Pz_{max}$.

Table 3.
The results of the cutting force calculation taking into account the stochastic feed of the analytical and empiric formulas

Formulas	Value	Passport feed values, mm/rev					
	Pz	0.06	0.1	0.125	0.15	0.175	0.2
Analytical	$M_{an}(Pz)$, N	207.6	277.8	328.2	377.9	434.9	478.9
	$Pz_{max0.95}$, N	222.3	295.2	346.1	397.5	459.2	505.3
	$Pz_{min0.05}$, N	192.9	260.4	310.3	358.3	410.6	452.5
	$Pz_{max}/ M_{an}(Pz)$	1.071	1.063	1.055	1.052	1.055	1.056
	δ,%	14.2	12.5	10.9	10.4	11.2	11.0
Empiric	$M_{emp}(Pz)$, N	190.7	269.8	321.6	369.1	413.3	461.2
	$Pz_{max0.95}$, N	208.5	288.4	339.4	387.8	435.3	484.1
	$Pz_{min0.05}$, N	172.9	251.2	303.8	350.4	391.3	438.3
	$Pz_{max}/ M_{emp}(Pz)$	1.093	1.069	1.055	1.051	1.053	1.05
	δ,%	18.7	13.8	11.1	10.1	10.6	9.9

In our case, for a normal distribution law, it looks like

$$G(h\frac{1}{2}\left\{1+G_0\left(\frac{Pz_{\max}-M(Pz)}{\sqrt{2D(Pz)}}\right)\right\}_{max}, \qquad (26)$$

where $U_P=\frac{Pz_{\max}-M(Pz)}{\sqrt{D(Pz)}}$.

Thus, the quantities of the normal distribution law are found according to formulas, respectively:

$$Pz_{\min}=\overline{P_{z_J}}-U_p\cdot\sigma_{Pz_j} \qquad (27)$$

$$Pz_{\max}=\overline{P_{z_J}}+U_p\cdot\sigma_{Pz_j} \qquad (28)$$

Relative error of the *Pz* value dissipation is found according to the formula:

$$\delta=\frac{Pz_{max}-Pz_{min}}{M(Pz)}\cdot 100\% \qquad (29)$$

4.2. The results of calculations based on the developed model and their analysis

The calculation results of values are presented in Table 3.

The results of the calculations testify, that the relation $Pz_{max}\,/\,M(P_z)$ equals 1.05-1.093, that is, the maximum tangential component of the cutting force exceeds the mean one up by 9.3% and the relative error of the value dissipation of the tangential component of cutting force relatively the mean equals 9.9-18.7%, which is sufficient enough and must be taken into account while designing machine-tool system.

Comparison of the dispersions of the tangential cutting force value dissipation based on experimental and theoretical studies (by analytical and empirical formulas) allows us to establish that the proportion of the effect of stochasticity of the feed on the dissipation value of Pz is from 40 to 60%. The remaining dissipation of the tangential component of the cutting force is caused by other factors that have not been considered in the work, such as stochasticity of other elements of the cutting mode, cutting depth and speed, vibration, heterogeneity of the workpiece material, etc. This requires additional experiments to identify how much influence these factors have on the stochasticity of the tangential and other cutting force components and may be the subject of further studies. The values of feed stochasticity significantly affects the maximum tangential value of the cutting force, increasing it by 14-19% of the calculating average value. This, in turn, causes an overload on the elements of technological system and results in negative consequences. Therefore, depending on the amount of dissipation, it is recommended to reduce the feed rate on each specific kinematic chain during processing by the first value to ensure the given (calculated) maximum value of the cutting force. Based on the results of theoretical and experimental investigations, the following recommendations for the method application are given

1. To determine the value of the machine feed stochasticity, particularly the mean value, dissipation dispersion and mean-square deviation;
2. To determine the average value and dissipation dispersion of the cutting force using dependences (21) and (22);
3. To determine the maximum value of the cutting force $Pz_{max0.95}$, using dependence (28);
4. To adjust the feed rate taking into account the obtained value $Pz_{max0.95}$.

The application of this method will provide sufficient predictability of the cutting process characteristics in real production in terms of partially worn equipment.

5. Conclusions

The dissipation of the cutting force value at processing, turning in particular, is also due to the dissipation of the feed rate. Based on the obtained dependences for finding the density and the characteristics of the value distribution of the tangential cutting force under turning according to the empiric and theoretical formulas, it was determined that the stochastic value *Pz* is subject to a great extent of accuracy, of the normal distribution law.

As a result of the experimental studies, the effect of stochasticity of the feed on the dissipation of the tangential component of the cutting force during turning was confirmed. The effect of the stochasticity of the feed on the dissipation value of Pz is from 40 to 60% for specific processing conditions.

The proposed method for determining the maximum value of the tangential component of the cutting force during turning will make it possible to carry out more precise calculations of the elements of the machine-tool equipment under turning.

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