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Stress-strain state simulation of non-uniformly heated elements of components and assemblies of automotive

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ABSTRACT

Purpose: Develop a method for determining and evaluating the stress-strain state, particularly the distribution of thermomechanical stresses in the materials of individual rotating parts of vehicles.

Design/methodology/approach: The proposed method is based on the principle of gradual approximations of the solution when the boundary conditions are satisfied on the curvilinear limiting surfaces of the disk body.

Findings: The proposed method of determining and estimating the distribution of thermomechanical stresses in the disk material makes it possible to take into account the variable geometry: thickness and presence of a hole in the central part of the disk, also correctly determine stress-strain state at any point of unevenly heated rotating axial body.

Research limitations/implications: The work uses generally accepted assumptions and limitations for thermomechanical calculations.

Originality/value: It is proved that in real disks, the stress-strain state is spatial, and the well - known method based on the hypotheses of the plane-stress state does not provide the possibility of calculating the values of stresses in the thickness of the disk. The obtained results can be used to improve the methodology of auto technical examination of road accidents. In addition, they can be taken into account by bus drivers on urban routes when choosing a safe distance in heavy traffic, as well as design engineers of car brake systems.

Keywords: Mathematical model, Elasticity theory, Axisymmetric problem, Critical infrastructure metal structures, Stress-strain state, Displacement function, Thermal deformation, Thermodynamically inverse process, Cylinder of finite length, A disk of constant thickness

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ANALYSIS AND MODELLING

1. Introduction

By analyzing the operating conditions of critical infrastructure facilities, road transport, means of transportation, it has been established that the main factors destroying their elements and assemblies (bearing metal structures of infrastructure facilities, assemblies of vehicles, and individual parts) are sign loads, complex effects of environments, in particular temperature, etc. [1-9]. The impact of load and temperature factors' combinations causes a change in physical and mechanical properties; the possible damage with subsequent destruction and typical ones can be [2-9] fatigue damage, corrosion-mechanical wear, and cracking. The study [2-12] of the state of transportation and infrastructure facilities means after several years of operation concludes that up to 80% of the constituent units and machine parts experience the adverse effects of operating environments to a certain degree. Consequently, fatigue failure is an objective and natural result of the impact of operational load and thermodynamic instability of metallic materials which the metal structures elements of the understudy objects were made. Tiresome destruction leads to failure, significantly intensifies the wear processes, worsens surface roughness, and reduces the fatigue strength of metal materials. As a result, reliability, durability, and resource decrease, and repair costs significantly increase. The service life of metal structures resulting from destruction is often significantly reduced [1, 2, 5, 7-9].

Consequently, during critical infrastructure facilities and road transport operations, complex combinations of force and temperature factors act on the elements of metal structures, which is evident for the understudy objects due to the random nature of disturbance sources (operation modes, temperature effects, etc.). Therefore, solving these problems with sufficient accuracy requires the correct processes' formalization occurring in the elements' materials of infrastructure or vehicles. Applying the elasticity theory principles would be advisable because this will ensure the correct SSS modelling. The subject of the main topic is the determination and estimation of the thermomechanical stress distribution in the material of the metal body of rotation with the presence of a hole in the geometry.

2. Methodology

Thermal and thermo-mechanical stresses significantly impact the overall stress-strain state of metal structures of critical infrastructure facilities and road transport facilities [1-3, 5]. The formation and growth of defects such as cracks often cause a sudden failure of these metal structures [5-12]. Axisymmetric problems of the elasticity theory belong to the class of spatial problems, the solution of which experiences significant mathematical difficulties. The dimension finiteness of the understudy elements causes additional difficulties associated with the need to fulfil the boundary conditions on the side surfaces and ends of the rotation bodies. It is vital to improve the SSS modelling theory to understand better temperature processes in metal structures of critical infrastructure and road transport elements.

3. Results and discussion

There is no exact solution to the problem of elasticity theory for a non-uniformly heated cylinder of finite length or a disk of constant thickness in the literature [13]. The problem for a long cylinder or a solid shaft is solved as a plane deformation problem and for a thin disk of constant thickness, based on the hypotheses of the plane stress state [14]. In this case, the law of temperature change is established by special calculations or experimentally [15]. It should be noted that the same law of temperature change for rotation bodies has not yet been found [16].

In the given paper, we consider the exact solution of the axisymmetric problem of the thermoelasticity theory for a non-uniformly heated cylinder of finite length or a disk of constant thickness. Consider a cylinder with a central hole having a length $z(-\frac{h}{2} \le z \le \frac{h}{2})$ (Fig. 1) and located in the temperature field $\theta = \theta\ (r,z)$

Differential equilibrium equations in cylindrical coordinates look like this [13,19]:

$$\Delta u_1 - \frac{u_1}{r^2} + \frac{e_{,1}}{1-2v} - 2\frac{(1+v)}{1-2v}\ \beta\theta_{,1} = 0;$$

$$\Delta u + \frac{e_{,3}}{1-2v} - 2\frac{(1+v)}{1-2v}\ \beta\theta_{,3} \qquad (1)$$

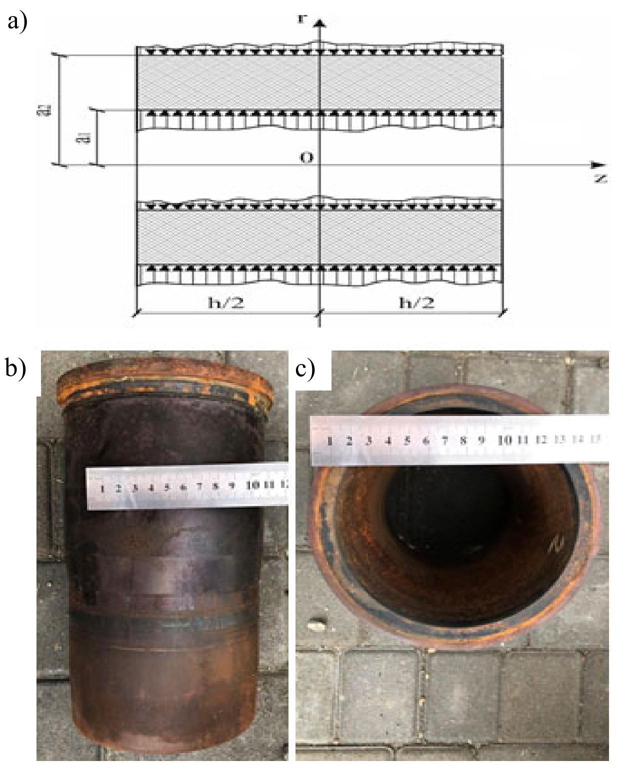

Fig. 1. a) cylinder sleeve scheme, b) cylinder sleeve Volvo D12C engine, c) a fragment of the cylinder sleeve damaged during operation

strain

$$\sigma_{11,1} + \sigma_{13,1} + \frac{\sigma_{11} - \sigma_{22}}{r} = 0,$$

$$\sigma_{13,1} + \sigma_{33,3} + \frac{1}{r}\sigma_{13} = 0. \qquad (2)$$

In equations (1) and (2) the index after the decimal point means the partial derivative for the corresponding coordinate r or z; u_1 i u_3 are the components of the radial and axial displacements, respectively; Δu_1 – the Laplace operator of displacements u_i (i =1,3); $\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{13}$ – the components of the radial, circumferential, axial and shear stresses, respectively.

Strain components are determined by Hooke's law [17]:

$$\sigma_{ij} = 2G\left[e_{ij} + \frac{v}{1-2v} e\,\delta_{ij} - \frac{1+v}{1-v}\beta\theta\,\delta_{ij}\right], (ij = 1,2,3). \qquad (3)$$

with known dependencies between deformations and displacements:

$$e_{11} = u_{1,1};\ e_{22} = \frac{1}{r}u_1;\ e_{33} = u_{3,3};\ 2e_{13}. \qquad (4)$$

where δ_{ij} – Kronecker symbol;
β i v – coefficients of linear thermal expansion and Poisson;
$e = e_{11} + e_{22} + e_{33}$ – volume expansion;
$G = \frac{E}{2(1+v)}$ – shear modulus;
E – elasticity modulus of the first kind.

The limiting conditions on the surface of a rotation body will be as follows [18]:

$$P_r = \sigma_{11}\cos(r,n) + \sigma_{13}\cos(z,n);\ P_z = \sigma_{13}\cos(r,n) + \sigma_{33}\cos(z,n), \qquad (5)$$

P_r and P_z – intensity projections of surface loads on the directions r and z; n – the normal to the surface of the rotation body; (r,n) and (z,n) – the angles between the normal and the directions of the coordinate axes. To close the systems of equations (1), it is necessary to add the heat conduction equation [19]:

$$\Delta\theta + \frac{1}{\lambda}W = 0, \qquad (6)$$

W – the amount of heat given off or absorbed per unit of body volume per unit of time;
λ – coefficient of thermal conductivity;
Δ – Laplace operator for temperature change θ.

Based on the same reasoning and the method for solving the system of differential equilibrium equations (1) described in the works [20-23], we find the displacement components in the radial $u_1(r,z)$ and axial $u_3(r,z)$ directions:

$$u_1(r,z) = u(r) - \frac{z^2}{2}(1+v)\beta\theta_{1,1}\,;$$

$$u_3(r,z) = -\frac{v}{1-v} z \frac{1}{r}(ur)_{1,1} - \frac{v}{1-v}\frac{z^3}{6}(1+v)\beta\Delta_1\theta_1 + + \frac{1+v}{1-v}\beta\int_0^z \theta(r,z)dz\,. \qquad (7)$$

In (7) $u(r)$ – the radial displacement of the points of the plane z =0, which is equal to:

$$u(r) = (1+v)\beta\frac{1}{r}\int_{a_1}^{r} r\theta_1 dr + A_1\frac{r}{2} + A_2\frac{1}{r} \qquad (8)$$

a_1 – the radius of the central hole;
A_1 and A_2 – random stable integrations (if the rotation body is solid ($a_1 = 0$), then A_2 must be considered equal to zero for finite displacements u (r)).

$$\Delta_1\theta_1 = \theta_{1,11} + \frac{1}{r}\theta_{1,1}\,,$$

$\theta_1 = \theta_1(r)$ – external temperature field determined from the usual differential equation $(\Delta_1\theta_1)_{11} = 0$ [20-23], which after integration, takes the following form:

$$\theta_1(r) = B_1\frac{r^2}{4} + B_2 \ln r + B_3\,, \qquad (9)$$

B_1, B_2, B_3 – random stables.

$\sigma_{11} = 0$ when $r = l_{(z)}$,
where $l_{(z)}$ – disc profile equation (Fig. 2).

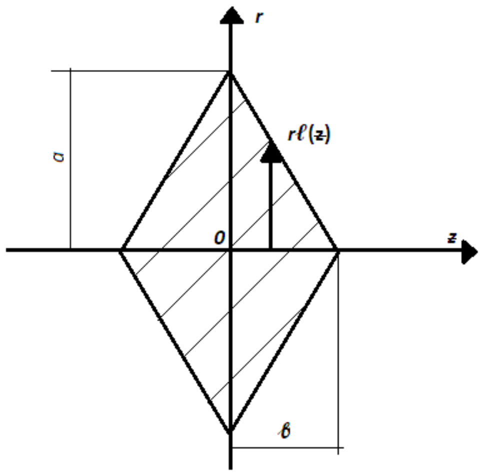

Fig. 2. Profile parameters determination

According to the displacements found, taking into account equations (8) and (9), according to Hooke's law [17,18], we determine the strain, and the check shows that σ_{13} and σ_{33} are zero everywhere, and for σ_{11} and σ_{22} we have:

$$\sigma_{11} = \frac{2G}{1-v}\Big[-(1-v^2)\beta B_1\frac{r^2}{16} - (1+v)\beta B_3 + A_1\frac{1+v}{2} - A_2\frac{1-v}{r^2} - \frac{z^2}{4}(1+v)^2\beta B_1 - (1+v)\beta\theta_2(z)\Big],$$

$$\sigma_{22} = \frac{2G}{1-v}\Big[-(1-v^2)\beta B_1\frac{3r^2}{16} - (1+v)\beta B_3 + A_1\frac{1+v}{2} + A_2\frac{1-v}{r^2} - \frac{z^2}{4}(1+v)^2\beta B_1 - (1+v)\beta\theta_2(z)\Big]$$

$$\sigma_{13} = \sigma_{33} = 0. \quad (10)$$

Formulas (7) and (10) are exact solutions of the equilibrium equations (1) and (2), since after substitution, they turn the latter into identities.

It is known that the deformation of an elastic body is inextricably linked with changes in its temperature [24]. Thus, a change in the deformation field leads to a change in the temperature field and vice versa. Let the temperature of an unstrained cylinder or disk at time t=0 be equal to some constant T_0. When loading a rotation body with external loads, not only a displacement field appears in it, but also a temperature field that differs from T_0 and does not depend on whether the body is heated or not. As a result of external loads, the body will deform, and its temperature will change. The temperature change will be $\theta = T - T_0$, where T – is the absolute temperature of the point of the body. Consequently, during deformation, the temperature of a point of the body changes. As a result, heat can be absorbed or released by an elastic non-insulated body when it interacts with the environment [25].

The change in temperature in the body is due to two processes [26], such as the action of an external load and an external temperature action leading to deformation, which lead to the temperature change θ_2. The compression of the body is accompanied by its heating, and the stretching is accompanied by cooling.

If the deformation of the body is small enough, then after the termination of the action of external forces causing deformation, the body returns to its original under-formed state. Such deformations are called elastic. In this case, the deformation process occurs very slowly, i.e., it will be thermodynamically reversible [27]. Further, we will assume that the change in temperature θ_2 is little and does not lead to significant changes in the elastic and thermal constants of the material, i.e. physical and mechanical characteristics G; E; v; β. Moreover, we assume that the latter do not depend on temperature.

The accepted assumption about the smallness of the temperature change can be written as follows [28,29]:

$$\left|\frac{\theta}{T_0}\right| = \left|\frac{T-T_0}{T_0}\right| \ll 1. \quad (11)$$

The detailed studies for non-uniformly heated rotated bodies require setting the temperature in the form [30, 31]:

$$\theta(r,z) = \theta_1(r) + \theta_2(z) = B_1\frac{r^2}{4} + B_2 lnr + B_3 + \theta_2(z), \quad (12)$$

$\theta_1(r) = B_1\frac{r^2}{4} + B_2 lnr + B_3$ – external temperature load;

$\theta_2(z)$ – temperature change, which is formed in the rotation body due to deformation under the influence of external loads.

For the case under consideration, we take the law of temperature distribution along the radius of the rotation body quadratic [30-32], i.e. we consider $B_2 = 0$:

$$\theta_1(r) = B_1\frac{r^2}{4} + B_3. \quad (13)$$

The strain is determined from the system of equations (10) under the fulfilment of the limiting conditions (5), which, due to the equality of the strains σ_{13} and σ_{33} to zero, are simplified and for a cylinder or disk with a central hole take the following form:

$$\sigma_{11} = 0, if\ r = a_1, and\ \sigma_{11} = 0, if\ r = a_1. \quad (14)$$

Here a_1 and a_2 – inner and outer radii of the rotation body.

Satisfying the limit conditions (14), we have for random stable integrations:

$$A_1 = \frac{1-v}{8}\beta B_1(a^2{}_1 + a^2{}_2) + 2\beta B_2\ ;$$

$$A_2 = (1+v)\beta B_1\frac{{a_1}^2{a_2}^2}{16}. \quad (15)$$

In addition, we find the temperature change $\theta_2(z)$, formed due to the action of the external temperature field $\theta_1(r)$:

$$\theta_2(z) = -\frac{z^2}{4}(1+v)B_1 . \quad (16)$$

After substituting expressions (15) and (16) into the system of equations (10), we have for strains:

$$\sigma_{11} = \frac{\beta E B_1}{16}\left(a_2{}^2 + a_1{}^2 - \frac{a_1{}^2 a_2}{r^2} - r^2\right) ;$$

$$\sigma_{22} = \frac{\beta E B_1}{16}\left(a_2{}^2 + a_1{}^2 + \frac{a_2{}^2 a_1{}^2}{r^2} - 3r^2\right) ;$$

$$\sigma_{13} = \sigma_{33} = 0. \quad (17)$$

If a disk of constant thickness or a cylinder of finite length with a central hole has a constant temperature T_1 and T_2 on its inner $(r = a_1)$ and outer $(r = a_2)$ cylindrical surfaces, then in this case the equation (13) becomes:

$$\theta_1(r) = T_1 + \frac{T_2 - T_1}{a_2{}^2 - a_1{}^2}(r^2 - a_1{}^2) , \quad (18)$$

where

$$B_1 = \frac{4(T_2 - T_1)}{a_2{}^2 - a_1{}^2} : B_3 = T_1 - \frac{T_2 - T_1}{a_2{}^2 - a_2{}^2} a_1{}^2 . \quad (19)$$

Substituting in (17) the value of B_1 from (19) we have:

$$\sigma_{11} = \frac{\beta E}{4}\frac{T_2 - T_1}{a_2{}^2 - a_1{}^2}\left(a_2{}^2 + a_1{}^2 - \frac{a_1\, a_2{}^2}{r^2} - r^2\right) ;$$

$$\sigma_{22} = \frac{\beta E}{4}\frac{T_2 - T_1}{a_2{}^2 - a_1{}^2}\left(a_2{}^2 + a_1{}^2 + \frac{a_2{}^2 a_1{}^2}{r^2} - 3r^2\right) ;$$

$$\sigma_{13} = \sigma_{33} = 0 \quad (20)$$

For an unevenly heated solid $(a_1 = 0;\ A_1 = 0)$ disk or cylinder, the strains look like this:

$$\sigma_{11} = \frac{\beta E}{4}\frac{a_2{}^2 - r_2}{a_2{}^2 - a_1{}^2}(T_2 - T_1) ;$$

$$\sigma_{22} = \frac{\beta E}{4}\frac{a_2{}^2 - 3r^2}{a_2{}^2 - a_1{}^2}(T_2 - T_1) ;\ \sigma_{13} = \sigma_{33} = 0. \quad (21)$$

For the temperature distribution $\theta(r, z)$ from the equation (12), where $B_2 = 0$, taking into account equations (16) and (19), we have:

$$\theta(r, z) = T - T_0 = T_1 + \frac{T_2 - T_1}{a_2{}^2 - a_1{}^2}[r^2 - a_1{}^2 - (1+v)z^2]. \quad (22)$$

Hence, the absolute temperature of a point of an unevenly heated rotation body with a central hole will be:

$$T = T_0 + T_1 + \frac{T_2 - T_1}{a_2{}^2 - a_1{}^2}[r^2 - a_1{}^2 - (1+v)z^2], \quad (23)$$

T_0 – the initial body temperature of the disk or cylinder.

For a solid $(a_1 = 0)$ non-uniformly heated body, the rotation is:

$$T = T_0 + T_1 + \frac{T_2 - T_1}{a_2{}^2}[r^2 - (1+v)z^2]. \quad (24)$$

Substituting the temperature value from (12), where $B_2 = 0$, into the heat conduction equation (6), we find the heat sink power per volume unit of the cylinder body with a central through hole (or ring):

$$W = -\frac{\lambda(1-v)}{2}B_1 = -\frac{2\lambda(1-v)}{a_2{}^2 - a_1{}^2}(T_2 - T_1), \quad (25)$$

which disappears when $B_1 = -\frac{4(T_2 - T_1)}{a_2{}^2 - a_1{}^2} = 0$, i.e. after the external heating stops.

The temperature change $\theta_2(z)$, which is formed in an unevenly heated cylinder with a central hole during its deformation due to the action of an external temperature field $\theta_1(r)$, is found from equation (16) taking into account (19):

$$\theta_2(t) = -\frac{z^2(1+v)}{a_2{}^2 - a_1{}^2}(T_2 - T_1). \quad (26)$$

In this case, as mentioned above, conditions (11) must be satisfied, i.e.:

$$\left|\frac{\theta}{T_0}\right| = \left|\frac{z^2(1+v)}{a_2{}^2 - a_1{}^2}(T_2 - T_1)\right| \ll 1. \quad (27)$$

The value of θ_2 is very small, which is confirmed by the following numerical example. Let investigate a solid $(a_1 = 0)$ unevenly heated disk of constant thickness. It should be emphasized that the dimensions and temperature of the disk were taken for the case of a perfectly elastic body. Let the outer radius and thickness of the disk be a_2=0.2 m and h=0.05 m, respectively; material – steel 12X18H10T (AISI 321); v = 0.3: T_2 = 673 K and T_1 = 573 K. We find the value of θ_2 from (26):

$$\theta_2(t) = -\frac{0.0025 \cdot 1.3}{4 \cdot 0.04}(673 - 573) \approx 2.03\ \text{K},$$

in which z = h / 2.

For conditions (27), where T_0 = 293 K, we have:

$$\left|\frac{\theta}{T_0}\right| = \left|\frac{2.03}{293}\right| \approx |0.0069| \ll 1.$$

Therefore, the correctness of the model has been proven: the requirements (27) are satisfied, i.e. the deformation process will be thermodynamically reversible.

4. Conclusions

When calculating the stress-strain state of metal structures of unevenly heated elements of critical infrastructure and road transport, it is rational to use the approaches of the elasticity theory, which ensures good accuracy and correctness in the formalization of processes when performing mathematical modelling. Analytical modelling proposed a solution to the problem and substantiated formulas for calculating strains and temperature fields under random load and temperature change laws. Studies of metal structure elements that can be formalized during modelling as non-uniformly heated rotation bodies or disks of constant thickness are of sufficient practical value. The authors have proposed a method based on the principle of graded approximations for determining and estimating the thermomechanical stress distribution in a cylinder material, namely thickness and the presence of a hole in the geometry. The approaches and methodologies adopted to determine and evaluate the stress-strain state, in particular, the thermo-mechanical stress distribution in materials of components and assemblies of automotive, are suggested.

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