

A REVIEW OF SURFACE DEFORMATION AND STRAIN MEASUREMENT USING TWO-DIMENSIONAL DIGITAL IMAGE CORRELATION

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Abstract

Among the full-field optical measurement methods, the *Digital Image Correlation* (DIC) is one of the techniques which has been given particular attention. Technically, the DIC technique refers to a non-contact strain measurement method that mathematically compares the grey intensity changes of the images captured at two different states: before and after deformation. The measurement can be performed by numerically calculating the displacement of speckles which are deposited on the top of object's surface. In this paper, the *Two-Dimensional Digital Image Correlation* (2D-DIC) is presented and its fundamental concepts are discussed. Next, the development of the 2D-DIC algorithms in the past 33 years is reviewed systematically. The improvement of 2D-DIC algorithms is presented with respect to two distinct aspects: their computation efficiency and measurement accuracy. Furthermore, analysis of the 2D-DIC accuracy is included, followed by a review of the DIC applications for two-dimensional measurements.

Keywords: surface deformation, strain measurement, two-dimensional digital image correlation.

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1. Introduction

In the 21st century, the man-made structures and machines are getting more complex than before. As a result, the surface deformation and strain measurement becomes ultimately important in many engineering applications and – most of the time – the obtained strain values are used to visualize the strength problems in a structural member. Thus, a precise strain measurement method is needed as misleading results might cause much financial loss to the industries and also put human lives in jeopardy. In order to overcome this situation, different types of surface deformation and strain measurement methods have been invented and improved to cope with the new challenges faced by the engineers. However, each of the surface deformation measurement methods has its own advantages and disadvantages. For example, the scratch strain gauge is valuable in measuring the dynamic events, but it is an expensive method to determine the strain in a single location [1]. The electrical resistance strain gauges, although they provide precise results in the measurement of surface deformation, are difficult in handling which makes them imperfect strain measurement methods. They require especially tedious procedures of fixing the strain gauges to a specimen [2]. The handling difficulty can be solved by introduction of an extensometer as it is easily attached to the specimen [3]. However, attaching the extensometer to the specimen produces unwanted stress concentration at the contact points between the extensometer's arms and the specimen surfaces. Although, in the brittle coating method, the preparatory activities – such as applying a coating on the specimen's surface – are rather simple, the coating itself encounters both flammability

and toxicity problems [4]. As a result, the method is not commonly acceptable as safety precautions against these dangers must be taken into account. The photo-elasticity method is getting less popular as the finite element analysis grows rapidly. In addition, this technique is limited to the transparent materials which exhibit the property of birefringence [5]. Therefore, the photo-elastic coating method is introduced and the above problem is solved as the well-polished surface of the specimen is covered with a thin sheet of photo-elastic material with reflective adhesive [6]. However, this technique faces the same problem as the electrical resistance strain gauges, where a perfect bond between the coating and the specimen is crucial. Attention must be given to selection of the adhesive and preparation the surface. The geometric moiré technique also faces the same problem as the electrical resistance strain gauges. Moreover, this technique is unable to provide accurate results in analysis of the small strain and is also not suitable for the high-temperature surface deformation measurement [7]. The holographic interferometry is an advanced optical strain measurement method which provides full-field deformation measurement results with a high degree of accuracy. This method does not require surface preparation and it is suitable for all types of material surfaces [8, 9]. Nevertheless, a disadvantage of this method is blurring of the recorded hologram if the equipment is not isolated from vibrations.

In the recent years, the *Digital Image Correlation* (DIC) technique is widely used for the displacement measurement in experimental solid mechanics. In comparison with the interferometry methods, the DIC technique has been given substantial attention as it does not require a stringent experimental setup and a complex optical system. Furthermore, the surface deformation measurement using the DIC technique can be carried out effortlessly since it requires neither the fringe processing nor the phase analysis. Even though the theoretical simplicity of the DIC technique is so attractive, the surface deformation and strain measurements still require a huge computational cost. Therefore, the DIC algorithms have been gradually modified and improved in the past three decades in order to increase their computation efficiency and measurement accuracy. Today, numerous successful applications from different areas can be found in the literature so that a review paper systematically explaining the development of DIC technique is needed. In 2009, Pan *et al.* [10] published a paper which extensively reviews the development of DIC algorithms. In their paper, the technical information on the DIC technique was presented in an informative way. However, development of the DIC technique over the years was not presented systematically and chronologically. A more systematic review of the DIC technique in this way seems to be desirable for readers to understand the improvement of DIC technique over the years. Therefore, in this paper a review of the development of the two-dimensional DIC algorithm is presented in a chronological way. The discussions are focused on two distinct aspects of improvement of the 2D-DIC algorithms: their computation efficiency and measurement accuracy. Furthermore, the accuracy analysis of the 2D-DIC is included, followed by a review of applications of the DIC in the two-dimensional measurement.

2. Fundamental concepts of Two-Dimensional Digital Image Correlation

In general, the term *Digital Image Correlation* (DIC) refers to a non-contact strain measurement method that mathematically compares the grey intensity changes of the images captured at two different states: before and after deformation. This can be achieved by choosing two subsets (small aperture for pattern matching) from the reference (undeformed) and the deformed images for correlation. Ideally, the DIC technique is able to correlate many types of patterns, such as grids, dots, lines and random patterns [11]. Nonetheless, the surface patterns must exhibit the isotropic behaviour and do not have a preferred orientation as the repeating textures would lead to problems with faulty registration. Therefore, the random speckle patterns

shown in Fig. 1 are recommended because they are considered to be non-periodic textures. Although the random speckle patterns look like the laser speckle patterns in nature, in the DIC technique they directly adhere to the top of object's surface. Thus, they are being deformed along with the surface. Theoretically, there are several advantages of using a random speckle pattern in the DIC technique. For example, the correlations during the image processing will not be lost even when the object is experiencing large deformations. Besides, the speckle pattern which contains much information is available everywhere on the entire surface, so that it permits the use of subsets during the correlation process [12].

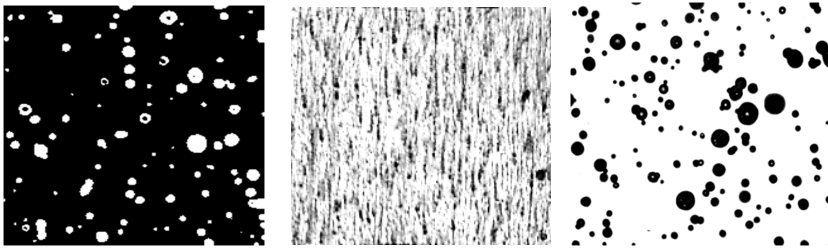


Fig. 1. Random speckle patterns.

In the surface deformation measurement using the *Two-Dimensional Digital Image Correlation* (2D-DIC) technique, special attention must be given to the arrangement of specimen, light sources and camera. This is because the accuracy and consistency of a measurement depends heavily on the imaging system set-up. Fig. 2 shows a schematic diagram of the experimental set-up for the 2D-DIC system. Basically, the specimen with random speckle pattern on its surface must be positioned normally to the optical axis of camera in order to eliminate the out-of-plane displacement. Then, during the entire strain inducing event, a series of images are being captured before and after deformation, followed by storing of the digital images in the computer for further processing. Technically, in the 2D-DIC technique the digital image resolution plays an important role in improving the measurement accuracy. This is because the image resolution represents the pixels (picture elements) of an image. In other words, by using a higher spatial-resolution image, a more accurate result can be obtained since each pixel is representing a more refined quantity of the sample's surface space.

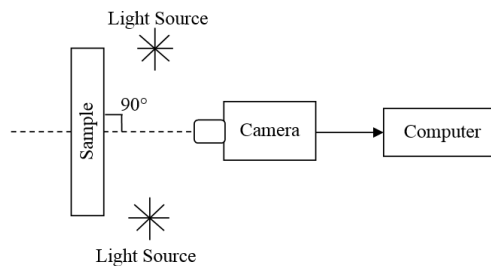


Fig. 2. A schematic diagram of the experimental set-up for 2D-DIC system.

At the beginning of the image matching process, the *Region of Interest* (ROI) in the reference image has to be specified first, followed by determination of the corresponding subsets in the deformed images. Fig. 3 shows matching of the subsets of the reference and the deformed images, followed by determination of the displacement vectors which indicate movements of the correlated subsets. This concept has been successfully applied since the neighbouring points in the reference image are assumed to remain as the neighbouring points even after

the deformation took place. In the DIC technique there is no guideline or rule in determining the optimum size of a subset and therefore it is a very subjective matter. A large subset will require a longer computation time and gives an average result of the displacement field. However, a small subset contains an inadequate number of features and hence in the correlation process it becomes difficult to be distinguished from other subsets. As a consequence, the correlation may not provide a reliable result [13].

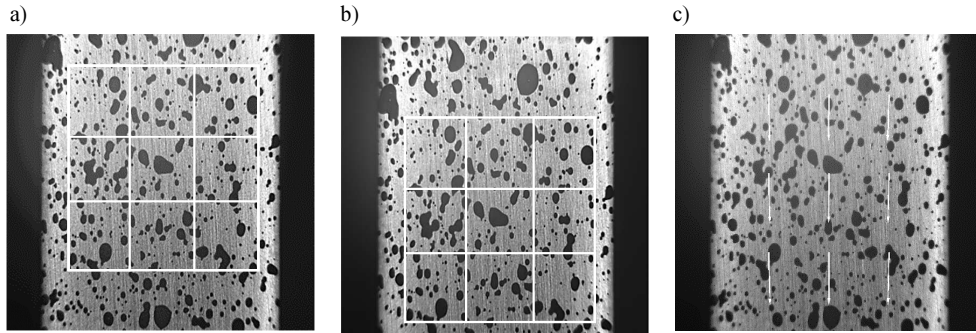


Fig. 3. Determination of the displacement vectors using the digital image correlation: a) the reference image; b) the deformed image; c) the displacement vectors.

Technically, during the image matching process the following correlation criteria are applied to evaluate similarity between the reference and the deformed subsets: the *Cross-Correlation Criterion* (CC), the *Sum of Absolute Differences Criterion* (SAD) and the *Squared Sum Differences Criterion* (SSD). Since the images are captured at two different states, there must be some changes in the intensity values of digitized images. As a result, a similarity between the selected subsets from the reference and the deformed images is significantly reduced, together with the accuracy of the obtained results. Over the years, the correlation algorithms that compensate the variations of intensity values have been developed. For example, the *Zero-Normalized Cross-Correlation Criterion* (ZNCC) and the *Zero-Normalized Squared Sum of Differences Criterion* (ZNSSD) have been reported to be the most robust correlation criteria. This is because the accuracy of the results using the ZNCC and ZNSSD criteria are unaffected by the offset and scale in lighting. Also, the *Normalized Cross-Correlation Criterion* (NCC) and *Normalized Squared Sum of Differences Criterion* (NSSD) are reported to be unaffected by the light scale, but are still sensitive to the offset in lighting. The detailed discussion on these correlation criteria can be found in the literature [10, 14, 15].

In order to determine the average in-plane displacement of the specimen, mapping functions or shape functions [16] are used to locate an initially square subset in the reference image within the next captured image under loading conditions. For the rigid body translation, the zero-order shape functions are used to represent the translation of each point within the selected subset in x and y -directions. However, the zero-order shape functions are not adequate to represent the subsets that are undergoing a combination of translation, rotation, normal strain and shear strain. Therefore, the first-order shape functions are used and expressed in the form of:

$$\zeta_1 = u + \frac{\partial u}{\partial x} \Delta x + \frac{\partial u}{\partial y} \Delta y, \quad (1)$$

$$\eta_1 = v + \frac{\partial v}{\partial x} \Delta x + \frac{\partial v}{\partial y} \Delta y, \quad (2)$$

where: ζ_1 and η_1 are the total displacements of the subset; u and v are the translations; $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ are the normal strains; $\frac{\partial u}{\partial y}$ and $\frac{\partial v}{\partial x}$ are the shear strains; Δx and Δy are the distances from the subset centre to an arbitrary point within the same subset in x and y -directions, respectively. Fig. 4 shows the deformation parameters used to represent the average in-plane displacement of a subset.

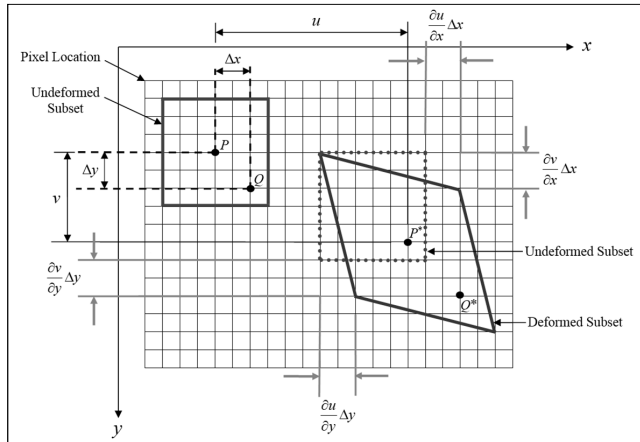


Fig. 4. The deformation parameters used to represent the average in-plane displacement of a subset.

If the sub-pixel accuracy is to be achieved in a measurement, the intensity values within the sub-pixel locations have to be provided since the intensity of images captured with a digital camera is discrete in nature. This can be achieved using a sub-pixel interpolation scheme which is employed to represent the grey level values between the sub-pixel locations before starting the image matching process. For example, bilinear interpolation [17, 18], bi-cubic spline interpolation [19, 20] and bi-cubic B-spline interpolation [21] have been reported in numerous papers. Higher-order interpolation schemes are always recommended in the analysis as they provide results with a higher degree of accuracy. However, the time required for the image correlation in such schemes is substantially longer than in a lower-order interpolation scheme [10].

In the early stage, the commonly used coarse-fine searching scheme was the DIC algorithm employed for determining the deformation parameters of zero-order shape functions [17]. This algorithm would search for a corresponding subset in the deformed images, which maximizes the correlation coefficient with the step size of 1 pixel. The step size is further reduced to 0.1 pixel or 0.01 pixel, depending on the sub-pixel accuracy level. Lastly, the DIC algorithm was used to calculate the displacement parameters. Then, the displacement fields of specimens were determined. In the last three decades, these algorithms were modified and improved in order to simplify the iteration process during determination of the displacement parameters, as well as to better the accuracy of DIC algorithms. The details of the development of DIC algorithms are discussed in the following section.

3. Development of Two-Dimensional Digital Image Correlation algorithms

The *Digital Image Correlation* (DIC) was first introduced by Peters and Ranson [22] in 1980's for the experimental stress analysis. They proposed a digital imaging technique

for measuring the surface displacement of speckle patterns in the reference and deformed images. From the images, subsets from the deformed images were numerically correlated with those from the reference image and the surface displacement was calculated. Lastly, stresses within the structure were determined. In 1983, Sutton *et al.* [17] improved the DIC technique to obtain the full-field in-plane deformations of a cantilever beam. In the experiment, a specimen with white pattern of random speckles on its surface was placed perpendicularly to the optical axis of a video camera. During the strain inducing event, an image was captured in its undeformed state, followed by continuous capturing of the subsequent deformed images. Based on these images, the authors suggested that the light's intensity distribution reflected by the specimen can be stored as a set of grey levels in a computer. The grey levels' values ranged from 0 to 255, where 0 represents the zero light intensity and 255 represents the maximum light intensity. Since the sensors recorded the continuously varying intensity pattern in a discrete form, a surface fit method known as a bilinear interpolation was applied in order to represent the data in a continuous form. For determination of the displacement values using the DIC technique, they introduced a correlation coefficient, C , expressed in the form of:

$$C\left(u, v, \frac{\partial u}{\partial x}, \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}\right) = \iint [f(x) - g(x')]^2 dx, \quad (3)$$

where: u , v , $\frac{\partial u}{\partial x}$, $\frac{\partial v}{\partial y}$, $\frac{\partial u}{\partial y}$ and $\frac{\partial v}{\partial x}$ are the deformation parameters of the first-order shape functions; $f(x)$ and $g(x')$ are the intensity values in the reference and deformed states, respectively. By minimizing the square of the intensity difference between the chosen subsets, the displacement parameters can be determined by a series of iterations. In principle, to determine the deformation parameters of the first-order shape functions, the initial translation values of u and v were first estimated, while the remaining variables – such as $\frac{\partial u}{\partial x}$, $\frac{\partial v}{\partial y}$, $\frac{\partial u}{\partial y}$ and $\frac{\partial v}{\partial x}$ – were set to zero. Basically, the iteration process was stopped once the estimated translation values of u and v gave the minimum correlation coefficient. Next, the normal strain values of $\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y}$ were estimated, while the determined translation values of u and v were retained and the shear strain values of $\frac{\partial u}{\partial y}$ and $\frac{\partial v}{\partial x}$ were set to zero. Again, the iteration process was stopped once the minimum correlation coefficient was obtained. The iteration process remained the same for determination of the shear strain values of $\frac{\partial u}{\partial y}$ and $\frac{\partial v}{\partial x}$. Finally, the deformation parameters of the first-order shape function, u , v , $\frac{\partial u}{\partial x}$, $\frac{\partial v}{\partial y}$, $\frac{\partial u}{\partial y}$ and $\frac{\partial v}{\partial x}$ were determined when the correlation coefficient was minimized.

In 1985, Chu *et al.* [18] further explained the basic theory and assumptions used in the DIC technique. Besides, four experiments: the uniform translation, rigid-body rotation, constant angular-velocity motion and uniform finite-strain test were carried out in order to validate the feasibility of the correlation method in experimental mechanics. Based on the experimental works, they concluded that the DIC technique was very accurate in determining the deformation parameters, such as translations and rotations. However, the correlated results of rotations did not agree with the actual values when the rotations were found to be greater than 8 degrees. It happened as the similarity of the intensity values between the reference and the deformed

subsets decreased drastically when the angle of rotation increased. Basically, this scenario is reported as a decorrelation problem and the intensity values are almost completely uncorrelated when the angle of rotation is approaching 10 degrees. The detailed explanation of this phenomenon can be found in the literature [16].

3.1. Improvement of 2D-DIC algorithms regarding their computational efficiency

In 1986, Sutton *et al.* [23] modified the iterative *Digital Image Correlation* (DIC) algorithm by introducing the Newton-Raphson method with differential corrections to speed up the searching time. The modified algorithm has reduced the strain determination time and still managed to achieve the accuracy equivalent to the previously used coarse-fine iterative technique. In 1989, Bruck *et al.* [19] improved the DIC algorithm by developing a complete model of Newton-Raphson method which provides the corrective feedbacks for the initially estimated six deformation parameters of the first-order shape functions. Next, the determined corrective feedbacks were added to the initial estimation values and the iteration process was terminated once the convergence was obtained. This algorithm had eliminated a number of unnecessary iterations as the initial estimation of the six deformation parameters had been found to converge faster than in the coarse-fine search method. In their work, the Newton-Raphson iteration method was expressed in the form of:

$$P = P_o - \frac{\nabla C(P_o)}{\nabla \nabla C(P_o)}, \quad (4)$$

where: P_o and P are the initial and the next iterative approximates of the six deformation parameters, respectively; $\nabla C(P_o)$ is the Jacobian matrix containing the derivatives of the correlation function and $\nabla \nabla C(P_o)$ is the Hessian matrix containing the second derivatives of the correlation function. Even though the time for determining the deformation parameters has been reduced drastically in comparison with the previously used coarse-fine search method, there are two numerical concerns about the proposed algorithm. First, the corrective feedbacks' values should be small; otherwise the determination time will be increased. Secondly, the proposed DIC algorithm performed efficiently only when the initial estimation values were close enough to the actual values. In the same year, Hovis [24] developed a Centroidal Tracking algorithm which performed the two-dimensional full-field deformation measurements by tracking the displacement of the speckles' centroids. By using this method, the interpolation procedures in smoothing the intensity values of the reference and the deformed images were eliminated. Besides, the initial estimations of the displacement values, u and v , were no longer necessary since the positions of the speckles in the deformed images were determined using the centroidal tracking algorithm. Before the analysing process was carried out, the image processing – such as filtering and edge enhancement – was performed to enhance the surface features. Next, the precise positions of selected speckles' centroids were determined, followed by identification of the change with respect of the distance between the selected speckles in two subsequent images. Referring to Fig. 4, the relative elongation at the point P in the direction of Q , ε_{PQ} , can be expanded in respect the strain components ε_{ij} . Then, the strain components can be determined using the Geometric Approach equation [25], as shown below:

$$\frac{1}{2} \varepsilon_{PQ}^2 + \varepsilon_{PQ} = \varepsilon_{xx} \cos^2 \theta_x + \varepsilon_{yy} \cos^2 \theta_y + 2\varepsilon_{xy} \cos \theta_x \cos \theta_y, \quad (5)$$

where: ε_{xx} and ε_{yy} are the normal strains in x and y -directions, respectively; ε_{xy} is the shear strain in xy -plane; θ_x and θ_y are the orientations of the line element PQ in the reference image.

Even though the Geometric Approach is simple to use, the proposed equation is not suitable for measurement of a large deformation. This is because the term $\varepsilon^2/2$ in (5) was found to still contribute a small value despite the fact that it is treated as negligible when the relative change in the distance between two speckles, ε , is small [26].

In 1993, Chen *et al.* [27] suggested that the full-field displacements, u and v , of a specimen could be determined using a two-step *Fast-Fourier Transform* (FFT) algorithm. In the first step, the complex spectra of selected subsets from the reference and the deformed images were calculated and the resultant spectrum was then determined based on the phase difference of the two spectra. In the second step, the FFT algorithm was applied to process the resultant spectrum and a noticeable signal peak was generated. By using the peak-finding approach, the displacements, u and v , were successfully determined. Even though FFT can be used to determine the full-field displacement in a very fast way, the proposed approach is not suitable for large deformation and rotation. In 1998, Vendroux and Knauss [28] further optimized the Newton-Raphson method in determining the two-dimensional surface deformation. In their work, *Scanning Tunnelling Microscopy* (STM) was used to scan the surface topographies and the out-of-plane displacement, w_o , was extracted together with the in-plane deformation parameters. Next, the out-of-plane displacement was introduced into the correlation coefficient and minimized together with the other six in-plane deformation parameters. The modified approach was named the least squares correlation coefficient. It is expressed as:

$$C = \frac{\Sigma\{f(x) - [g(x') - w_o]\}^2}{\Sigma f^2(x)}, \quad (6)$$

where $f(x)$ and $g(x')$ are the intensity values in the reference and the deformed states, respectively. Besides, an approximated Hessian matrix was proposed; it has simplified the iteration processes of the Newton-Raphson method. By using the proposed algorithm, 25 percent of improvement was achieved in speed, as well as the convergence robustness of the algorithm, provided that the initial estimation values were ± 7 pixels off the actual displacement values.

In 2003, Hung *et al.* [29] presented a peak-finding algorithm known as *Fast and Simple* (FAS) for measurement of the surface deformation in two dimensions. This algorithm was developed to reduce the computational time required by the Newton-Raphson method. Although the proposed algorithm has reduced the determination time of the local deformation, its accuracy is decreased since it does not consider distortion of the deformed subset. In 2006, Zhang *et al.* [30] proposed a novel coarse-fine searching scheme which reduced the computational complexity by as much as 3 percent in comparison with the conventional coarse-fine searching scheme. In their work, an affine transform was used to manually determine the approximate location of each pixel in the reference and the deformed images. Next, the novel coarse-fine searching scheme was applied around the approximated locations in order to obtain the true values of the displacement. Nonetheless, the determination time of the local deformation using the coarse-fine searching scheme is still time-consuming in comparison with the peak-finding algorithm and the Newton-Raphson method.

In 2009, Paeppegem *et al.* [31] further optimized the peak-finding approach in their study on the deformation characteristics of window security film. In their work, the centre of mass of the correlation peak was calculated and then the local deformation with the sub-pixel accuracy was determined. Even though the proposed approach is easy to implement, its accuracy is decreased as the peak-finding algorithm does not consider distortion of the deformed subset. In the same year, Pan [32] proposed a reliability-guided DIC method which uses the *Zero-Normalized Cross-Correlation* (ZNCC) to determine the surface deformation in two dimensions.

In principle, it starts with a seed point in the reference image, followed by searching of the corresponding point in the deformed image. Next, four or eight neighbouring points are analysed and the image matching analysis is performed by selecting a neighbouring point with the highest ZNCC coefficient. By doing this, a reliable image matching path is successfully determined. Between 2011 and 2013, Pan and Li [33] and Pan *et al.* [34] proposed a fast DIC method which efficiently eliminated redundant calculations in the conventional Newton-Raphson method. In the work of Pan and Li, a reliability-guided displacement scanning strategy was adopted to avoid the laborious integer-displacement searching scheme and a pre-computed global interpolation coefficient table was generated to eliminate repetitive interpolation calculation in sub-pixel positions. By using the proposed approach, the determination time of the local deformation has been reduced drastically without influencing the measurement accuracy. Also, in Pan *et al.* work, the authors proposed a more efficient *Inverse Compositional* matching strategy and *Gauss-Newton* (IC-GN) algorithm eliminating the requirement of re-computing the Hessian matrix in each of the iteration processes. In other words, the Hessian matrix was predetermined using the reference image instead of analysing each of the deformed images. Based on their findings, the proposed IC-GN algorithm achieved the same accuracy as the conventional Newton-Raphson method in a shorter determination time.

In 2012, Zhou and Chen [35] introduced a propagation function providing a more accurate initial estimation of the deformation parameters and hence requiring fewer iterations to converge the deformation parameters. In their work, they stated that the direct adoption of the deformation parameters to the next iteration gave a high prediction error, especially when the object was experiencing a large deformation. Therefore, instead of using integer displacement parameters in the next iteration, the predetermined deformation parameters were expressed as a form of function to increase the computational efficiency and the initialization accuracy. Nonetheless, the initialization error of the proposed propagation function increases with the subset's step size. Between 2012 and 2014, Pan *et al.* [36] and Guo *et al.* [37] addressed the decorrelation issue of the DIC technique which occurred in the deformed images due to a large deformation. They proposed the concept of automatic reference image updating scheme to replace the fixed reference image if the determined correlation coefficient was lower than the pre-set threshold value. In other words, the reference image was updated or replaced by the image captured just before the current deformed image when the iteration of correlation failed to converge. By the correlation between the updated reference image and the current deformed image, the accumulated displacement parameters were used to represent a large deformation. In addition, the reference image updating scheme was found to be useful in dealing with the decorrelation issue caused by serious illumination variations.

Recently, due to a greater demand for real-time and high-resolution measurements, several algorithms have been proposed by the researchers to further reduce the computational time of the DIC technique. Jiang *et al.* [38] applied the integral image technique [39] to accelerate the computation processes of Hessian matrix. In principle, this technique is commonly used to simplify the summation of pixel values in an image subset, regardless of the total image size. Since most of the determination time of the Newton-Raphson iteration scheme is dedicated to calculation of Hessian matrix, the proposed integral image technique proved to be effective in reducing the complexity of the Hessian Matrix's calculations. Shao *et al.* [40] suggested a seed point-based parallel approach to reduce the determination time of the local deformation. In their work, a pre-set *Zero-Mean Normalized Cross-Correlation* (ZNCC) coefficient was used to identify a matched point. Once a computed point was found to be greater than the threshold value, the respective point was considered as the seed point for 4 neighbouring points. Again, these newly determined 4 seed points were used for another 8 neighbouring points. By using this approach, more and more points in the captured images could be analysed concurrently and the computation time was found to be 6–7 times shorter than that of the IC-GN algorithm.

In fact, with the continuous improvement of 2D-DIC algorithms with respect to their computational efficiency, as well as the development of high-speed computers, the computation time of the DIC technique has been drastically reduced. Thus, the authors believe that the 2D-DIC technique will become a popular tool for the real-time surface deformation and strain measurement.

3.2. Improvement of 2D-DIC algorithms regarding their measurement accuracy

In order to improve the accuracy of the *Digital Image Correlation* (DIC) algorithm, Lu and Cary [20] applied the second-order approximation to determine the first and second-order displacement gradients. Besides, they also implemented the third-order polynomial interpolation known as Bi-cubic Spline to reconstruct the grey intensity values in each location of the images. This was carried out as an improvement of the previous DIC algorithms that used only the first-order approximation for determining the deformation parameters. In their work, the authors adopted the second-order Taylor series approximation of the displacement field; the second-order shape functions are expressed in the form of:

$$\zeta_2 = u + \frac{\partial u}{\partial x} \Delta x + \frac{\partial u}{\partial y} \Delta y + \frac{1}{2} \frac{\partial^2 u}{\partial x^2} \Delta x^2 + \frac{1}{2} \frac{\partial^2 u}{\partial y^2} \Delta y^2 + \frac{\partial^2 u}{\partial x \partial y} \Delta x \Delta y, \quad (7)$$

$$\eta_2 = v + \frac{\partial v}{\partial x} \Delta x + \frac{\partial v}{\partial y} \Delta y + \frac{1}{2} \frac{\partial^2 v}{\partial x^2} \Delta x^2 + \frac{1}{2} \frac{\partial^2 v}{\partial y^2} \Delta y^2 + \frac{\partial^2 v}{\partial x \partial y} \Delta x \Delta y, \quad (8)$$

where: ζ_2 and η_2 are the total displacements of the subset; u and v are the translations; $\frac{\partial u}{\partial x}$, $\frac{\partial v}{\partial y}$, $\frac{\partial u}{\partial y}$ and $\frac{\partial v}{\partial x}$ are the first-order displacement gradients; $\frac{\partial^2 u}{\partial x^2}$, $\frac{\partial^2 v}{\partial x^2}$, $\frac{\partial^2 u}{\partial y^2}$, $\frac{\partial^2 v}{\partial y^2}$, $\frac{\partial^2 u}{\partial x \partial y}$ and $\frac{\partial^2 v}{\partial x \partial y}$ are the second-order displacement gradients; Δx and Δy are the distances between point

P and point Q in x and y -directions, respectively, as referred to Fig. 4. By using the twelve deformation parameters, the deformation range was increased and the large deformation measurement was accurately obtained.

In 2002, Cheng *et al.* [21] applied the Bi-cubic Spline (B-Spline) deformation function to represent the two-dimensional continuous intensity field throughout the image area. According to Cheng *et al.*, the conventional subset-based correlation process normally causes a small error as the calculation points in the reference and the deformed images are still in the discrete form. By using the proposed method, an arbitrary decision on the subset size was eliminated and the continuity of displacement was guaranteed. In 2007, Meng *et al.* [41] introduced an iteration and finite element smoothing technique to improve the accuracy of the displacement field results. Basically, the algorithm was implemented to smooth the full-field displacement results that were determined by the Newton-Raphson iteration scheme. They stated that this process was necessary because the obtained strain data did not perfectly represent the actual strain values as a high level of noise occurred in the reference and the deformed images.

In 2010, Cofaru *et al.* [42] improved the accuracy of DIC technique in measuring the displacement and the strain fields of a structural component subjected to low and high spatial frequency variations. In their work, they proposed an adaptive spatial regularization in which the details of neighbouring movements were analysed and processed, so that the movements of selected subsets were accurately estimated. The proposed algorithm starts with minimizing

the correlation coefficient for all subsets in the entire image, followed by introducing the determined translation values into the Newton-Raphson iteration scheme. These iteration processes are terminated once convergence is obtained and updating of the displacement vector of the respective subset is stopped. The obtained deformation parameters are used for the neighbouring subsets in order to increase the accuracy of displacements and strains. In 2012, Cofaru *et al.* [43] introduced an image adaptive subset algorithm which uses irregular subset sizes in the image matching process. It was required that the adaptive subset size was big enough to cover one speckle. By a series of numerical simulations and experiments the proposed approach proved to be capable for increasing the full-field displacements accuracy, especially for the irregularly shaped samples.

In 2013, Pan [44] applied the Gaussian low-pass filter [45] to pre-smooth both reference and deformed images as the measurement accuracy of DIC technique depends heavily on the bias error concealed inside the noisy images. Basically, the Gaussian pre-filtering has been widely used to reduce the image noise by removing the high-frequency components of an image prior to the correlation process. By doing this, even a simple bi-cubic interpolation scheme can be performed more accurately and hence reduce the bias error of the conventional Newton-Raphson method. In 2014, Zhou *et al.* [46] suggested an adaptive subset offset approach in order to reduce the systematic error in incremental digital image correlation. Since the interpolation of intensity values in the sub-pixel positions normally causes the systematic error in the determined results, the additional interpolation of intensity values for the reference image is totally unnecessary, especially when using the automatic reference image updating scheme for a large deformation or a serious change of illumination. In the adaptive subset offset approach, a subset from the updated reference image is translated onto the integer position and hence the interpolation of sub-pixel intensity values for the reference image can be avoided.

Recently, Yuan *et al.* [47] developed a self-adaptive sampling algorithm to improve the accuracy of deformation measurement using the DIC technique. In the conventional subset-based DIC analysis, the fine sampling grid used during the correlation process eventually leads to a longer determination time. In addition, the fine sampling grid is simply not necessary to improve the measurement accuracy of a structural member subjected to a simple loading condition. The self-adaptive sampling algorithm basically starts with a sparse initial sampling grid and this sampling grid is refined only around the local deformation of a sample. Based on the results, the proposed algorithm has successfully increased the accuracy and robustness of the conventional equidistant sampling DIC technique. Also, to increase the measurement accuracy of DIC technique for large-scale objects, Mazzoleni *et al.* [48] applied the Gaussian pre-filtering to the images with numerically-designed speckle patterns adhered to the sample surface. In their work, the numerically-designed speckle patterns with well-defined and regular shapes, size and spacing were created by spraying paint through a pre-cut stencil. Next, the Gaussian filter was used to blur the low-resolution image which was generated by 10 times down-sizing the original high-resolution image. By doing this, the bias error and the uncertainty of DIC technique were reduced during determination of displacements and strain fields. Based on these research papers, the accuracy of surface deformation and strain measurement using the DIC technique has been improved significantly over the years. With the current rate of improvement, the measurement accuracy of DIC technique will become as good as that of the interferometric technique.

4. Accuracy analysis of Two-Dimensional Digital Image Correlation

For the last three decades, the *Two-Dimensional Digital Image Correlation* (2D-DIC) algorithms have been developed, modified and improved in order to increase the accuracy of deformation measurement and also to significantly reduce its computation time. Due to

digitization of the captured images, assumption of the subset's deformed shape and the systematic errors from the experimental set-up, various potential errors occur in the DIC technique. In 1988, Sutton *et al.* [49] performed the first modelling works to identify the factors which influence the accuracy of DIC technique. In their work, a selected subset was deformed by known values and the model was examined numerically. Based on their findings, the digitization process of images, the ratio of the signal and sampling frequencies, and the interpolation scheme used to reconstruct the intensity values in non-pixel locations were found to be the main sources of errors.

Schreier *et al.* [50, 51] examined such systematic errors as the improper intensity interpolation and the inappropriate subset's shape functions – in 2000 and 2002, respectively. Based on their research, a higher-order interpolation scheme was highly recommended to decrease the intensity interpolation error that occurred during reconstruction of intensity values in the non-pixel locations. On the other hand, the time required for the image correlation was substantially longer than that obtained with using the lower-order interpolation scheme. Meanwhile, for the subset's shape functions, Schreier *et al.* also stated that the higher-order shape functions were able to reduce the systematic error successfully. In the same year, Wang *et al.* [52] studied the measurement uncertainty of an imaging system used in their experiments. By using one image to represent a deformation state, the measurement uncertainties ranging from 0.25 to 0.30 pixels were reported in measurements of the displacement parameters, as well as from 0.017 to 0.032 in measurements of the displacement gradients. Based on their findings, the measurement uncertainties can be improved by using the multiple image approach in which 15 images are captured within a specified period of time to average the deformation state. For the multiple image approach, the measurement uncertainties ranging from 0.06 to 0.14 pixels were reported in measurements of the displacement parameters, as well as from 0.0039 to 0.0085 in measurements of the displacement gradients.

In 2003, Zhang *et al.* [53] examined the influences of the subset size, speckle size and speckle pattern quality on the accuracy of deformation measurement using the DIC. In their report, the optimum subset sizes ranged from 31×31 pixels to 51×51 pixels if the high accuracy and computational speed were given the highest priority. Besides, the optimum speckle size was found to be dependent on the subset size. For example, the optimal speckle size ranged from 3 to 8 pixels when the subset size was 51×51 pixels. In 2006, Lecompte *et al.* [54] explored the influence of the mean speckle size on the accuracy of DIC technique with respect to the subset size. In their work, the speckles' patterns were numerically deformed in order to avoid the experimental errors. By the finite element simulations, the relationship between the mean speckle size and the subset size that produced the most accurate results was successfully revealed. In 2007, Sun and Pang [55] further examined the influence of the subset size on the accuracy of measured quantity. They stated that the larger the subset size, the smaller the standard deviations and higher level of accuracy. Nevertheless, it is important to emphasize that the choice of the subset size depends on the speckle pattern quality as well as the field of application. For example, during the image matching processes some speckle patterns require a subset size large enough to contain sufficient features that enable distinguishing itself from the deformed subset. In comparison with the sharp contrast speckle pattern, a small subset contains features that are sufficient to provide accurate results.

In 2008, Haddadi and Belhabib [56] studied the errors associated with the lighting, optical lens, camera sensor, out-of-plane displacement, speckle pattern, grid pitch, subset size and correlation algorithm. They concluded that the light source must provide a uniform lighting condition over the entire region of interest. In addition, special clumps and a tele-centric lens were suggested in order to reduce the out-of-plane displacement. Lastly, an optimized pattern such as the printed type on the specimen surface was proposed to get more reproducible results. In the same year, Sutton *et al.* [57] examined the effect of out-of-plane movements, such as

translation and rotation, on the two-dimensional DIC measurements. Again, they stated that the tele-centric lens can effectively minimize the out-of-plane movements. In 2010, Jerabek *et al.* [58] further analysed the effects of various parameters related to the DIC experimental set-up, such as the light intensity, camera shutter time, speckle pattern, machine and vibration. In their work, they revealed that a fine speckle pattern and light intensity gave accurate results when it was in the condition of overexposure. Besides, they reported that vibration had no meaningful effect on decreasing the strain measurement accuracy.

In 2012, Liu *et al.* [59] examined the accuracy of constant intensity model, linear intensity change model and non-linear intensity change model that were used in the DIC technique. These models are used to represent the relationship of the pixel's grey intensity values in the reference and deformed images. Based on their findings, the constant intensity model appeared to be suitable only for images captured in strictly controlled conditions in which the light exposure was assumed to be constant throughout the testing. Even though the linear intensity change model can rectify this issue, the non-linear intensity change model is highly recommended for serious changes of illumination. In 2013, Crammond *et al.* [60] studied the influence of the speckle's size and density in a subset on the accuracy of deformation measurement using the DIC technique. In the measurement with the use of high magnification images, the errors – as a consequence of a sparse speckle pattern – were found to be high. By using an airbrush to generate the fine speckles, the measurement errors were significantly reduced as the number of speckles in a subset increased. On the other hand, bigger-sized speckles were suggested as they provide more differentiated shape features.

In the same year, Hoult *et al.* [61] studied different sources of errors: lighting, camera quality and out-of-plane movements in measurements using the DIC technique. In their work, they have concluded that the source of biggest measurement errors was the out-of-plane movement. Pan *et al.* [62] and Dufour *et al.* [63] studied the systematic error due to lens distortion in the two-dimensional DIC. Most of the time, the image distortion inevitably existed due to the irregularity of lens. As a result, the displacements determined from the captured images were different from the real displacements of the object. Based on their works, they concluded that the displacement and strain errors due to the lens irregularity were randomly scattered across the entire captured image. Nonetheless, the errors were found to be small at the image centre.

In 2014, Zappa *et al.* [64, 65] quantified the uncertainty of DIC technique in dynamic applications. In their work, the uncertainty due to the movement effect was determined by analysing the simulated reference images, as well as the images which were captured during the vibration tests. By the validation exercises, the uncertainties in displacement and strain were found to be 0.2 pixels and 0.008, respectively, when subjected to a 7-pixel movement. Also, for a 3.5-pixel movement, the uncertainties in displacement and strain were reduced to 0.02 pixels and 0.0008, respectively. Even though the DIC technique has been improved progressively for the past 33 years, the surface deformation and strain measurement accuracy of 2D-DIC still depends heavily on the construction of the experimental set-up, the quality of the imaging system, the quality of speckle pattern and the image matching algorithms. As a result, special attention must be given to these factors if a high-level accuracy of measurement is to be achieved.

5. Applications of Digital Image Correlation in Two-Dimensional Measurement

The *Digital Image Correlation* (DIC) technique was first applied in experimental mechanics to perform the *Two-Dimensional Deformation Measurement* [22]. In the last three decades, the DIC technique was used to study the deformation behaviour of various materials. For example, the full-field displacement analysis of composite cylinder, measurement of the surface profile of a plastic specimen under loading and examining specimens with heterogeneous deformation

pattern were performed using the DIC technique [66, 70]. Besides, various applications have been reported as the DIC technique was widely employed in many mechanical testing procedures. Nevertheless, the most common applications of the two-dimensional DIC were found in the field of fracture mechanics, *e.g.* the study of plastic deformation around a notch or an open hole and the study of fracture surfaces of a specimen subjected to tensile loading [71–74]. Later on, the researchers have further extended the crack propagation study onto the measurement of *Stress Intensity Factors* (SIFs) using the DIC technique [75–78]. Technically, the SIFs were determined based on the displacement parameters around the crack tip. In determination of SIFs, the DIC technique requires no assumptions regarding the boundary conditions, as well as the loading conditions of the specimen. For this reason, this approach was widely accepted since the determination of stress intensity factors was much easier and cheaper in comparison with the interferometry methods. Starting in 2002, the DIC technique was first applied in the fatigue analysis and the literature is focused on four particular areas, *e.g.* development of a real-time strain value feedback system [79, 80], development of a monitoring system for detecting an early fatigue damage [81, 82], improvement of fatigue crack growth rate estimation [83–87] and characterization of strain life properties of thin sheet metals [88, 89].

Over the years, the researchers also applied the DIC technique to identify the mechanical properties of materials through the deformation measurement [90], *e.g.* the strain measurements of paper using the DIC technique – by Sutton and Chao [91] and determination of mechanical properties of wood and concrete – by Choi *et al.* [92] and Huang *et al.* [93], respectively. However, according to Sanchez-Arevalo *et al.* [94], the interferometry method provides a more accurate result than the DIC technique. Nonetheless, the DIC technique gives better visual information about the surface of deformed material. In 2010, Tung *et al.* [95] performed a tensile test on a steel specimen and it appeared that the strain values determined by the strain gauge and the DIC technique were close to each other. In their report, the modulus of elasticity of a steel specimen determined by the DIC technique was equal to 201 GPa, whereas its benchmark value was 206 GPa. Several researchers further employed the concept and studied mechanical properties of materials with the finite element-based integrated DIC technique [96–101]. According to Leclerc *et al.* [102], the DIC technique was applied to update material properties in simulation of finite elements, so that the difference between the real and simulated displacements was minimized. Besides, the deformation parameters determined by the DIC technique can be used to verify the accuracy of finite element model.

Since the DIC technique is an inherently non-contact strain measurement method, it offers a valuable solution for the high-temperature deformation measurements [103]. In 1996, Lyons *et al.* [104] applied the DIC technique to measure the full-field surface deformations of a specimen inside a furnace. In their experimental work, they proved that the DIC technique was able to determine the displacements and strains of an Inconel 718 bar at the temperatures of up to 650 °C. Nevertheless, special precautions must be considered as the radiation from a hot surface contributes to the decorrelation issue. Subsequently, Grant *et al.* [105] applied a wavelength filtration technique, together with the blue illumination to determine the Young's modulus and the coefficient of thermal expansion of a nickel-based super-alloy at the temperatures of up to 1400 °C. Also, Pan *et al.* [106] used transient aerodynamic heating simulation devices and the developed reliability-guided DIC algorithm [32] to determine the thermal deformation at temperatures ranging from the room temperature to 550°C. By doing this, the usage of a high-temperature furnace was avoided and therefore the errors associated with changing the refractive index along the optical path were eliminated.

The DIC technique was employed in Ophthalmology study in 1987. According to Wu *et al.* [107], the mechanical properties of retina subjected to tensile loading were successfully determined. In 1995, Hjortdal and Jensen [108] adopted the DIC technique to measure the

multi-directional corneal strain, thickness and radius of curvature in human eyeballs. Also, measurement of strain distributions within the articular cartilage and measurement of diabetic retinopathy with the DIC technique were reported in 2002 and 2009, respectively [52, 109]. Besides, the DIC technique was applied to monitor the structural integrity of man-made structures. According to Lee and Shinozuka [110], the dynamic displacement measurement of a bridge using the DIC technique was very cost-effective and easy to use in comparison with using the conventional structural displacement sensors, such as *Linear Variable Differential Transformers* (LVDTs) and dial gauges. Furthermore, measurement of a high-speed dynamic event such as an explosion of thin metal frame [111], as well as the shape measurement [112] were successfully documented using the DIC technique.

With the rapidly developing field of micro-manufacturing, many engineering parts are becoming smaller and smaller. Hence, the deformation characteristics of micro-scale components must be tested and the DIC technique usefulness for this purpose was reported. In 1997, Sun *et al.* [113] stated that the DIC technique was capable to measure the in-plane surface deformations at the magnification of up to 2000 pixels per millimetre. The DIC technique was further extended onto the study of *Scanning Tunnelling Microscope* (STM) images [28, 114, 115] and *Scanning Electron Microscopy* (SEM) images [116]. According to Jin *et al.* [117], the mechanical properties of the electrodeposited nickel-based specimens at the micro-scale level were effectively determined with a combination of SEM imaging and DIC technique. In addition, Ya'akovovitz *et al.* [118] proved that the nanoscale displacement of as much as 100 nm could be accurately determined using the DIC technique. Based on this trend, together with the technological progress in imaging systems, the 2D-DIC technique will soon become an effective tool for the nanoscale deformation measurement.

6. Conclusion

Due to the limitations of other deformation measurement methods, as well as the technological progress in optical metrology, the *Two-Dimensional Digital Image Correlation* (2D-DIC) technique has been introduced in 1980's for surface deformation and strain measurements. Since then, algorithms used in the 2D-DIC have been modified and gradually improved in order to increase their computational efficiency and measurement accuracy. Recently, the 2D-DIC technique has become a popular tool for the displacement measurement in experimental solid mechanics. In comparison with the interferometry methods, the 2D-DIC technique does not require a stringent experimental set-up and a complex optical system. Therefore, the 2D-DIC technique is relatively easy to use in laboratories. Also, in the outdoor applications the 2D-DIC technique proved to be very cost-effective in monitoring the structural integrity of man-made structures. This is because any changes of a structure can be identified easily and efficiently by analysing the images captured in two different states. Moreover, numerous successful applications of the technique in different areas of engineering can be found in the literature, which presents another proof of its usefulness. Nevertheless, special attention must be given to using suitable experimental set-ups, imaging systems, speckle patterns and image matching algorithms since the accuracy and consistency of measurements depends heavily on these factors. In this review, the 2D-DIC technique was introduced and its fundamental concepts and operating principles were discussed. Besides, the development and accuracy analysis of 2D-DIC technique in the past three decades was chronologically presented. By referring the presented extensive review of the literature, the authors believe that the applications of 2D-DIC technique will keep expanding and the technique will become a popular tool for the surface deformation and strain measurements. Lastly, due to the fact that the 2D-DIC technique can provide better visual information about the deformed surfaces,

the deformation behaviour of materials can be further studied by observation and quantification of the obtained surface data.

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