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EXTENDED WARRANTY OF MEDICAL EQUIPMENT SUBJECT TO IMPERFECT REPAIRS: AN APPROACH BASED ON GENERALIZED RENEWAL PROCESS AND STACKELBERG GAME

ROZSZERZONA GWARANCJA NA SPRZĘT MEDYCZNY PODLEGAJĄCY NIEPEŁNYM NAPRAWOM: PODEJŚCIE OPARTE NA UOGÓLNIONYM PROCESIE ODNOWY I MODELU STACKELBERGA

Due to its advanced technology, maintenance services of healthcare equipment have been commonly executed by the original equipment manufacturer (OEM), which can be characterized as a monopolist. In this context, hospitals require high availability of their equipment at a reasonable servicing cost, whereas OEM aims to maximize its profit by selling extended warranty (EW) services for multiple consumers. The issue of drawing a maintenance contract between OEM and hospitals has already been treated by adopting a Stackelberg's game. However, the "as good as new" and "as bad as old" assumptions are usually considered, which are rather difficult to observe in practice, especially for healthcare institutions and their technology-intensive equipment. Thus, we here adopt generalized renewal processes (GRP) for modelling imperfect repairs, and we develop a discrete event simulation method for finding the best strategies of each player: OEM sets the prices for EW and on-demand maintenance that optimize its profit, while hospitals choose which option they should hire. We also present an application example with real data gathered from an angiography device, which is used for mapping blood vessels and diagnosing heart diseases.

Keywords: *medical equipment; extended warranty; Stackelberg game; generalized renewal process.*

Ze względu na zaawansowanie technologiczne sprzętu medycznego, jego obsługą serwisową zazwyczaj zajmuje się producent sprzętu oryginalnego (OEM), co czyni go monopolistą w tym zakresie. Podczas gdy szpitalom zależy na wysokiej gotowości sprzętu przy rozsądnych kosztach obsługi, OEM dąży do maksymalizacji zysku poprzez sprzedaż rozszerzonej gwarancji na usługi serwisowe wielu klientom. Istnieją już badania, w których kwestię zawierania umowy o świadczenie usług serwisowych między OEM a szpitalami analizowano z zastosowaniem modelu Stackelberga. Jednak zwykle badania te zakładają, że stan po naprawie może być albo "jak fabrycznie nowy" albo "jak przed uszkodzeniem", co rzadko spotyka się w praktyce, zwłaszcza w przypadku placówek służby zdrowia i ich zaawansowanego technologicznie sprzętu. W związku z tym, w przedstawionej pracy, przyjęto uogólniony proces odnowy (GRP) do modelowania niepełnych napraw oraz opracowano metodę symulacji zdarzeń dyskretnych w celu znalezienia najlepszych strategii dla każdego gracza: OEM ustala ceny rozszerzonej gwarancji oraz konserwacji na żądanie, tak by zoptymalizować swój zysk; szpital natomiast ustala, którą opcję powinien wybrać. W pracy przedstawiono również przykład zastosowania omawianego podejścia z wykorzystaniem rzeczywistych danych zebranych z angiografu, który służy do obrazowania naczyń krwionośnych i diagnozowania chorób serca.

Słowa kluczowe: sprzęt medyczny; rozszerzona gwarancja; gra Stackelberga; uogólniony proces odnowy.

1. Introduction

Medical equipment plays an important role in modern healthcare institutions because they present the following purposes: diagnosis, disease prevention, monitoring and patient treatment. During the last decades, as technology has advanced, the maintenance of such equipment has become too complex to be done in-house. Therefore, this activity has been commonly assigned to the Original Equipment Manufacturer (OEM).

In this context, the importance of studying more deeply the warranty issue is reinforced by the role maintenance management plays in guaranteeing the quality of healthcare services. In diverse situations, human lives depend on the correct operation of medical devices. For instance, it is estimated that an amount of US$ 24.83 billion will be annually spent in the medical equipment maintenance market by 2022 [20]. However, maintenance outsourcing for medical equipment is yet to be fully explored by current research (Cruz & Rincon [5]).

Warranty policies define responsibilities for both parties: the OEM and hospital managers. Indeed, the OEM is responsible for repairing the devices' eventual failures related to problems of equipment design, manufacturing and/or quality. The customers in turn should make proper use of equipment, i.e., they must comply to the specifications defined by the OEM (Rahman & Chattopadhyay [32]).

Given that, a new trend has been intensified by manufacturers, selling an additional, optional coverage, which begins after the expiration of the base warranty, called Extended Warranty (EW) (Murthy

& Djamaludin [26]). Thus, the customer decides whether to pay an extra value at the purchase epoch (Murthy & Jack [27]), whereas the OEM will correctively maintain equipment for a given period even after the ordinary warranty expires.

It is important to emphasize that EW ends up creating a conflict of interests between the owner of the equipment and the OEM. Specifically, the customer needs a high availability of its equipment at a reasonable servicing cost, whereas the manufacturer aims to maximize their profit with the addition of post-selling services. Consequently, EW affects both buyers' and manufacturer's outcomes (Ye & Murthy [41]), once the actions of one interferes in the results of the other. Due to this situation, Game Theory provides an appropriate approach to solve this problem (Forgó et al. [10]).

Among different games that can be used to model the interaction between agents, the leader-follower Stackelberg Game (SG) is a good option for drawing maintenance service contracts of medical equipment. In this context, the OEM is commonly the only party able to perform maintenance, since it has the technical knowledge, expertise, technology and spare parts for the repair execution (Rinsaka & Sandoh [33]). The hospital in turn needs its medical device available in suitable condition to provide a good service. From these two perspectives, an uneven power relationship is noted, which can be modeled via SG. The leader role is assigned to the OEM, which determines the terms of the EW, while the health institution acts as the follower, responding to actions taken by the OEM.

Quantitative studies about warranty, maintenance outsourcing and maintenance contracts are present by Kim et al. [18], Bouguerra et al. [3], Husniah et al. [14], Huang et al. [13], Moura et al. [23] and Darghouth et al. [6]. However, such studies have simplifying assumptions with respect to the state of the system after a corrective maintenance (CM) intervention. In fact, those papers considered that the system returns to either an "as good as new" condition (perfect repair) or an "as bad as old" condition (minimal repair); these two situations are modeled respectively according to a Renewal Process (RP) and a Non-Homogeneous Poisson Process (NHPP); Ross [34] describes RP and NHPP in details. The use of these assumptions may yield inadequate managerial decisions, which can result in significant losses in company profits because of incorrect definition of warranty policies.

In practical terms, maintenance actions typically return the equipment to an intermediate condition between the perfect and minimal repairs, which is called imperfect repair (Kijima & Sumita [17]; Wang & Pham [37]). Kijima & Sumita [17] proposed two methods to tackle imperfect repairs: Kijima type I and Kijima type II, which gave rise to the Generalized Renewal Process (GRP) and introduced the concept of "virtual age". Furthermore, these situations generalize RP and NHPP; other approaches may be seen in Pham & Wang [31].

In this paper, we aim to join SG and GRP to model imperfect repairs – a more realistic and general assumption. This approach considers the interaction between OEM and multiple customers. The proposed model will be characterized as follows. First, we considered the OEM offers to the hospital managers two maintenance options for the period after the ordinary warranty expires: (i) an extended warranty or (ii) on-call service. EW states that for a fixed price P, the OEM should repair all failures without any additional cost over the period of the contract; if a failed unit does not get repaired before a set time τ, a penalty, which increases over time, will be incurred. For the on-call service, failures will be repaired at a fixed cost C_s each, with no penalty incurred in case of delays. Given a number of hospitals that buy the equipment, this model estimates P^* and C_s^*, which are the maximal prices hospitals accept to pay for each maintenance option. Finally, we find the optimal number of customers, i.e., number of hospitals that maximize OEM's profit.

To that end, we develop a Discrete Event Simulation (DES) based method to reproduce the GRP-queue system and obtain the perfor-

mance indicators of interest. Simulation models allow for analysis of systems with complex behavior, require fewer assumptions when compared with analytical models and are used as tools to perform experimentations with systems (Marsaro & Cavalcante [21]). Due to present model's complexity, an analytical solution is unfeasible. In this sense, simulation is useful for describing equipment behavior (Ding & Kamaruddin [7]). Thus, a simulation approach is employed for obtaining a solution for the present model.

The remainder of this paper unfolds as follows. In Section 2, the theoretical background is provided, containing the adaption of the SG for the context of maintenance contracts, as well as the characteristics of GRP, emphasizing the method proposed by Yañez et al. [39]. Section 3 presents the proposed model, the players' optimal strategies and the equilibrium of the game. In Section 4, a numerical example is presented, using real data from an angiograph, which is a device used for blood vessels mapping and diagnosis of organ diseases. Finally, Section 5 concludes remarks.

2. Theoretical background

2.1. Generalized renewal process

RP and NHPP may be adopted to model perfect and minimal repairs respectively. According to (Lins & Droguett [19]), such methods have simplifying assumptions that may be unreal in many practical situations such as the healthcare technology intensive environment. To overcome the limitations of RP and NHPP, Kijima & Sumita [17] developed a probabilistic virtual age based model, known as the Generalized Renewal Process (GRP) that deals with all classes of maintenance actions. According to this model, q (rejuvenation parameter) may generally assume values between 0 and 1:

- $q = 0$ represents a perfect repair (as good as new);
- $q = 1$ corresponds to a minimal repair (as bad as old);
- $0 < q < 1$ indicates imperfect repair (better than old, worse than new).

Cases, where $q < 0$ and $q > 1$, are also possible corresponding to *better than new* and *worse than old* conditions respectively. Generally, GRP may be classified into two types (Kijima Type I and II), according to the method used to calculate the virtual age. These types can be seen in details in Moura et al. [22] and other virtual age-based representations could be found in Guo et al. [12], Tanwar et al. [35], Ferreira et al. [9], Oliveira et al. [29] and Wang & Yang [38].

This paper uses Kijima type I so that equipment virtual age follows Eq. (1), according to which maintenance actions only act on the degradation incurred during x_i, which is the time between $(i-1)^{th}$ repair and the ith failure:

$$v_i = v_{i-1} + qx_i = q\sum_{j=1}^{i} x_j \qquad (1)$$

The Cumulative Distribution Function (CDF) for the time between the $(i-1)^{th}$ and i^{th} failures can be determined from the CDF of the time until a failure conditioned on the virtual age v_{i-1} as seen in Eq. (2):

$$F(x_i | v_{i-1}) = P(X \leq x_i | X > v_{i-1}) = \frac{F(v_{i-1} + x_i) - F(v_{i-1})}{1 - F(v_{i-1})} \qquad (2)$$

For our analysis, we consider the time to failure follows a conditioned Weibull distribution because of its flexibility to fit various deg-

radation stages. Then, Eq. (2) can be rewritten as Eq. (3). Note that, for $i = 1$, we have the Weibull distribution itself because $v_0 = 0$. When there are reasonably sufficient failure data available, Maximum Likelihood Estimators (MLE) can be used to estimate GRP parameters α, β, and q. To that end, the procedure described in Yañez et al. [39] can be followed.

$$F\left(x_i | v_{i-1}\right) = 1 - exp\left[\left(\frac{v_{i-1}}{\alpha}\right)^{\beta} - \left(\frac{v_{i-1} - x_i}{\alpha}\right)^{\beta}\right] \qquad (3)$$

In the proposed model, we consider a case with multiple clients and m service channels, where maintenance teams may serve a total of M clients. If the number of failed units is greater than the service capability, a queue is formed. The waiting time in queue is decisive to the interests of both hospitals and OEM, since the former aims high availability of equipment and the latter wants to serve a higher number of customers. Therefore, a queuing formulation is employed to incorporate system behavior due to the interaction between service level and number of clients to be served.

Then, GRP governs the equipment failure process and the queue discipline follows a FCFS (first come first served) logic. We also consider times to repair follow an exponential distribution. Thus, using the conventional notation, the queue can be described as GRP/Markovian/m/∞/M/FCFS. For this situation, analytical solutions are not available. Therefore, we proposed a DES-based algorithm is adopted to obtain the GRP-queue system measures; the DES formulation is described in Section 3.8.

2.2. Stackelberg game

SG is a non-cooperative sequential game developed by Heinrich von Stackelberg. It was originally proposed to evaluate the equilibrium of a duopoly, where competing companies decide the optimal quantity to be produced (Gibbons [11]) in a leader-follower interaction.

At the best of authors' knowledge, Murthy & Yeung [28] were the first authors to introduce SG as a tool to model maintenance service contracts. Murthy & Asgharizadeh [25] expanded the problem by creating a game between a customer and a manufacturer by assuming perfect repairs. Ashgarizadeh & Murthy [2] and Murthy & Asgharizadeh [24] incorporated multiple customers and service channels. Esmaeili et al. [8] considered a three-level service contract between a manufacturer, a customer and an independent third agent. Moura et al. [23] used priority queues to analyze the interaction among OEM and two priority classes of hospitals. All aforementioned models intend to maximize the clients' expected utility, considering parameters like risk aversion, revenue generated by the system, maintenance costs and times to repair. In such papers, the leader (OEM) provides the maintenance options for the follower (hospitals) and obtains the highest payoff, since it charges the prices that maximize profits.

Generally, in the context of complex medical equipment, the OEM has a well-trained staff, spare parts and dominates the equipment technology. Thus, OEM behaves as a leader, acts first (by defining services and respective prices) and is the only maintenance service provider. Hospitals need to guarantee minimum levels of availability for their equipment. However, they do not have expertise in the maintenance of complex equipment. Therefore, hospitals can be considered as followers, since they react to the OEM's action when choosing a service type to hire.

The SG's solution is obtained through backwards induction (Osborne and Rubinstein [30]) corresponds to a sub-game perfect Nash equilibrium of a two-stage game, with perfect information and players with different profiles (Amir [1]). Finally, it is noteworthy to say that the papers cited earlier in this Section, which adapted SG to the field

of maintenance outsourcing, make simplifying assumptions about the repair structure employed. At the best of authors' knowledge, none of them adopted the imperfect repair assumption, a more realistic hypothesis, especially for complex systems. Thus, this paper aims to incorporate the GRP to model this situation and attempts to make the model more suitable to the medical environment.

3. Model description

3.1. Game formulation

This paper aims to determine the optimal strategies for the problem of EW for the medical context. Thus, we employ a SG formulation to model the interaction between the OEM and each customer, and the game's equilibrium will be reached by finding the reservation prices, i.e., maximum prices that customers accept to pay.

This decision problem extends the model developed by Ashgarizadeh & Murthy [2] by considering imperfect repairs. Additionally, the problem is stochastic due to the uncertainty inherent in the presence of random variables in the model. Finally, decision makers (healthcare institutions and OEM) have their own objective function that will define the respective payoffs. Both OEM and healthcare institutions are aware about their alternatives, acting rationally, and choosing strategies that maximize their respective payoffs (Osborne & Rubinstein [30]).

3.2. Notation list

A_k :   Decision variable of the hospital;

C_b :   Purchase price of the equipment;

C_r :   OEM's mean cost to repair a failed unit;

C_s :   Price charged by the OEM per repair;

C_s^{max} :   Hospital's reservation price for the on-call service;

M :   Number of customers;

M^* :   Optimal number of customers;

n_{rep} :   Number of Monte Carlo replications;

$N_{j,1}$, $N_{j,2}$:   Number of failures occurred over the intervals W_1 and W_2, respectively, for the jth device (j^{th} device is held by the jth customer (hospital));

N_j :   Total number of failures during W for the jth device; $N_j = N_{j,1} + N_{j,2}$;

P :   Price of the extended warranty;

P^{max} :   Hospital's reservation price for the extended warranty;

R :   Revenue per hour;

T_0 :   Time of the purchase of the equipment;

T_a :   Time of next failure among available customers;

T_d :   Time of next completion of a maintenance intervention;

T_j^{dt} :   Total downtime for j^{th} system.

$T_{j,1}^{op}$, $T_{j,2}^{op}$:   Period that the jth system is on operational state during W_1 and W_2 respectively;

T_j^{op} :   Total period that the jth system is on operational state;

$T_{j,1}^{ov}$, $T_{j,2}^{ov}$: Overtime for the jth device during W_1 and W_2 respectively;

T_j^{ov} :   Total overtime for the jth system;

U :   Expected utility of the customer;

W_1 :   Base (ordinary) warranty period;

W_2 :   Extended warranty period;

$W = (W_1 + W_2)$: Model analysis period;

Y_{ij} :   Time between failure and completion of repairing the jth equipment after the ith failure.

τ_1 , τ_2 : Maximum time that the manufacturer has to return a failed device to an operational state without incurring penalty, when the failure occurs during coverage period W_1 and W_2 , respectively;

θ_1 : Penalty per hour for the base warranty period;

θ_2 : Penalty per hour for the EW period;

Π_{OEM} : OEM's expected profit;

δ: Hospital's risk-aversion parameter;

Π_H : Hospital's expected profit.

3.3. Problem description

OEM sells a technology-intensive medical device to multiple customers (hospitals) for a cost of C_b per unit. Each device, when in operational state, generates a revenue of R monetary units per time. Along with the purchase of the device, OEM provides a base warranty, during which OEM is responsible for all repairs required with no charge for the client. If a failed equipment is not returned to operational state within a period τ_1 after a failure occurs, OEM is charged with a penalty proportional to the overtime in repairing the equipment, which is the period from τ_1 to the time when the equipment returns to operation. Therefore, the penalty is $\theta_1(Y_{ij} - \tau_1)$ when $Y_{ij} > \tau_1$, and zero, otherwise; θ_1 is the penalty per time during overtime; Y_{ij} is the time between the occurrence of the ith failure of equipment j and the completion of its respective repair. This penalty exists because medical equipment is vital for patients' treatment and for the hospitals' profit, and then unavailability affects their payoff and reputation.

After the expiration of the base warranty, each hospital may choose a type of repair service: i) EW or ii) on-call services. These options are described as follows:

i) A_1 : EW – begins after the base warranty expires and has duration W_2 . The customer pays a fixed price P and the OEM repairs all failed units over W_2 at no additional cost. If a failed device is not returned to operational state within a period τ_2 after a failure occurs, the OEM is charged a penalty. Analogously to the base warranty, the penalty is $\theta_2(Y_{ij} - \tau_2)$, when $Y_{ij} > \tau_2$;

ii) A_2 : on-call services – the OEM executes each repair at a cost of C_s per intervention; no penalty is here incurred.

We also assume that option A_0 means the hospital chooses not to buy the equipment.

3.4. Hospital's decision problem

Considering the options presented by the OEM, the hospital manager decides whether to opt for either EW (A_1), or on-call services (A_2) or not to purchase the equipment (A_0), where the latter occurs if its expected utility is negative over the period W. Each strategy has consequences to the payoffs. Indeed, each customer's return, Π ,

depends on the option A_k, and thus the hospitals profit can be seen in Eqs. (4), (5) and (6):

$$\Pi_{H(A_0)} = 0 \tag{4}$$

$$\Pi_{H(A_1)} = RT_j^{op} + \theta_1 T_{j,1}^{ov} + \theta_2 T_{j,2}^{ov} - C_b - P \tag{5}$$

$$\Pi_{H(A_2)} = RT_j^{op} + \theta_1 T_{j,1}^{ov} - C_b - N_{j,2} C_s \tag{6}$$

where:

$$T_j^{op} = \sum_{i=1}^{N_j} X_{ij} + \widetilde{X}_j$$

$$T_{j,1}^{op} = \left[\sum_{i=1}^{N_{j,1}} \max\left\{0, (Y_{ij} - \tau_1)\right\} \right]$$

$$T_{j,2}^{op} = \left[\sum_{i=1}^{N_{j,2}} \max\left\{0, (Y_{ij} - \tau_2)\right\} \right]$$

For hospital (equipment) j, ($1 \le j \le M$), N_j is the total number of failures over the mission time W ; X_{ij} ($0 \le i \le N_j$) is the time between the ith repair and the $(i+1)^{th}$ failure; $\tilde{X}_j$ is the time between the last failure and W ; $N_{j,1}$ and $N_{j,2}$ are the number of failures occurred over W_1 and W_2 respectively; Y_{ij} ($0 \le i \le N_j$) is the total time to finish repairing the jth equipment since the occurrence of the ith failure, i.e., Y_{ij} includes the waiting time in queue and repair time; T_j^{op} is the total operational time during W ; $T_{j,1}^{op}$ and $T_{j,2}^{op}$ are the respective operational times during W_1 and W_2 ; T_j^{ov} is the total overtime during W ; $T_{j,1}^{ov}$ and $T_{j,2}^{ov}$ are the total overtimes during W_1 and W_2 respectively.

We consider the hospital's risk is modeled according to a utility function U , which indicates how the customer chooses among distinct options; thus, the preferred options are represented by higher utilities. The utility function considered in this model has been used in Murthy & Asgharizadeh [25] and is shown in Eq. (7), where Π represents the associated wealth:

$$U(\Pi) = \left(\frac{1 - e^{-\delta\Pi}}{\delta} \right) \tag{7}$$

Thus, the choice A_k is strongly affected by equipment availability, pricing structure and the hospital's degree of risk aversion (δ). We assume all customers are homogeneous with respect to their risk aversion and all equipment units are identical regarding their reliability.

3.5. OEM's decision problem

The OEM is considered risk neutral and its expected profit is related to the customer's optimal choice. Consequently, the OEM's payoff can be denoted as $\Pi_{OEM}(P, C_s, M, A_k)$, where (P, C_s, M) are OEM's decision variables and $k = 0,1,2$. In this way, the manufacturer's profits for each of the customer's possible actions A_k are shown respectively in Eqs. (8), (9) and (10). Then, OEM may choose the combination P , C_s , M that maximizes its expected profit, taking into account the customer's optimal strategy A^* :

$$\Pi_{OEM}(P, C_s, M, A_0) = 0 \tag{8}$$

$$\Pi_{OEM}\left(P,C_s,M,A_1\right) = \sum_{j=1}^{M} P - C_r N_j - \theta_1 T_{j,1}^{ov} - \theta_2 T_{j,2}^{ov} \qquad (9)$$

$$\Pi_{OEM}\left(P,C_s,M,A_2\right) = \sum_{j=1}^{M} N_{j,2}\left(C_s - C_r\right) - N_{j,1}C_r - \theta_1 T_{j,1}^{ov} \quad (10)$$

3.6. Assumptions

In order to make the model manageable, we consider some assumptions:

I   Equipment is repairable and subject to imperfect repair. The probabilistic failure modelling is handled by GRP;

II   The times between failures are random variables. The time to first failure follows a Weibull distribution, as seen in Yañez et al. [39];

III   The times to repair are exponentially distributed with parameter μ;

IV   The OEM has m parallel service channels, i.e., in total they are capable of processing m units simultaneously (one unit per service channel);

V   The equipment's failures are critical. Moreover, the OEM carries out just corrective maintenance interventions;

VI   The manufacturer and the customer have complete information about the model's parameters, which implies that the leader is aware about the customer's risk parameter and the hospital knows the equipment reliability;

VII   If there are more failed units than the number of servers, a queue following a FCFS is generated. This formulation describes a queuing system with finite population M.

3.7. Players' strategies

In order to find the optimal solution for the players and the game equilibrium, we determine how the hospital's and OEM's optimal strategies are defined, as well as understand their relation, and the degree of influence between them. Then, these strategies are shown as follows.

3.7.1. Hospital's optimal strategy

The customer's expected utility U is derived from two random variables $\left(X_{ij}, Y_{ij}\right)$, the customer's decision A_k, and the pricing structure $\left(P^{max}; C_s^{max}\right)$ imposed by the OEM. Given the assumptions of Section 3.6, the expected utilities for each decision are given in Eqs. (11), (12) and (13) obtained by using Eqs. (4), (5) and (6). For a pair $\left(P, C_s\right)$ determined by the manufacturer, the customers analyze their expected utilities and choose the option that returns the highest payoff.

$$E\left[U\left(A_0,P,M,C_s\right)\right] = 0 \qquad (11)$$

$$E\left[U\left(A_1,P,M,C_s\right)\right] = \frac{1}{\delta}\left\{1 - \exp\left[\delta\left(P+C_b\right)\right]E\left[\exp\left(-\delta\left(RT_j^{op} + \theta_1 T_{j,1}^{ov} + \theta_2 T_{j,2}^{ov}\right)\right)\right]\right\} \qquad (12)$$

$$E\left[U\left(A_2,P,M,C_s\right)\right] = \frac{1}{\delta}\left\{1 - \exp\left[\delta C_b\right]E\left[\exp\left(-\delta\left(RT_j^{op} + \theta_1 T_{j,1}^{ov} - N_{j,2}C_s\right)\right)\right]\right\} \qquad (13)$$

The reservation prices depend on equipment expected number of failures $E\left[N_j\right]$, and the number of units sold M. Then, they are fairly influenced by the repair assumption. For instance, by considering imperfect repair ($0 \leq q \leq 1$), it is expected that devices with the same shape $\beta > 1$ and scale $\alpha > 0$ fail less frequently than in the context presented by Moura et al. [23], who considered minimal repairs. Thus, the change in repair hypothesis can modify the expected payoffs, and consequently a change of strategies. The mathematical approach used to reproduce the imperfect repair is the GRP, a which makes use of DES and allows for modeling imperfect, perfect and minimal repair assumptions, adapting to a broader range of scenarios. Another point to emphasize corresponds to the reservation prices (P^{max}, C_s^{max}), which are defined by Varian [36] as the highest prices a consumer is willing to pay. These prices affect the decision for what strategy A_k is chosen. Thus, determine the hospital's reservation prices is essential to find the OEM's expected profit, once the pricing structure has strong influence on the healthcare's institutions decision. Indeed, given the hospital's reservation prices and the pricing structure imposed by the OEM, the EW model can be seen on its extensive form, as a sequential game tree in Figure 1a, which shows all possible decisions for the players.
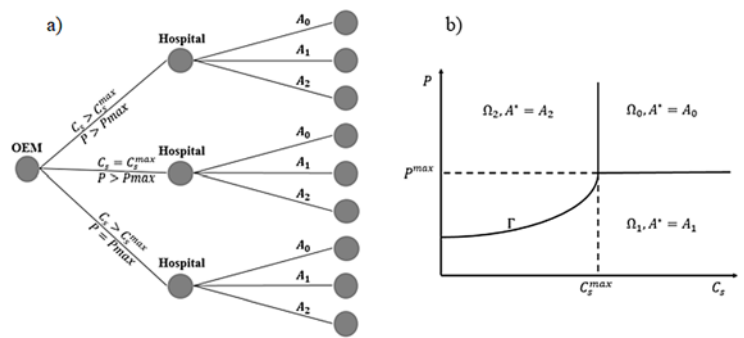

Fig. 1. a) The game tree – P^{max} and C_s^{max} represent the customer's maximum willingness to pay for the EW and on-call maintenance interventions respectively; b) Customer's optimal options adapted from Murthy & Asgharizadeh [25]

In the case the prices charged by the OEM are higher than the reservation prices ($P > P^{max}$; $C_s > C_s^{max}$), then the equipment is not purchased (A_0). Otherwise, if the EW price is superior than P^{max}, while C_s^{max} is not reached ($P > P^{max}$; $C_s \leq C_s^{max}$), the hospital manager should choose A_2. Analogously, when $P \leq P^{max}$; $C_s > C_s^{max}$, A_1 should be chosen. Figure 1b presents the $P - C_s$ plan, and shows that the customer's optimal choice is characterized by three regions Ω_0, Ω_1 and Ω_2. In Ω_0, $A_k^* = A_0$; in Ω_1, $A_k^* = A_1$; and in Ω_2, $A_k^* = A_2$. The curve Γ is obtained by equalizing the expected utilities for options A_1 and A_2, and represents indifference between the maintenance options since the same payoff is returned to the hospital.

3.7.2. OEM's optimal strategy

Since the OEM is considered to be risk neutral, its optimal strategy corresponds to the pricing structure that maximizes its expected profit (P^{max}, C_s^{max}). To choose its optimal strategy, OEM compares the expected profit for each type of service provided, varying the number of customers M, and then the optimal number of customers is determined.

Considering assumption VI in Section 3.6 (complete information), we conclude the OEM knows the hospital's reservation prices. Thus, OEM builds a structure that captures all the consumer surplus, which implies in the maximization of the producer profit.

Given that, we use Eqs. (12) and (13) to obtain the reservation prices. Indeed, we equalize Eq. (12) to zero, and then isolate P in order to determine P^{max}P^max. Thus, the reservation price of the EW can be given in (14):

$$P^{max} = -C_b - \frac{1}{\delta} \ln E\left[\exp\left(-\delta\left(RT_j^{op} + \theta_1 T_{j,1}^{ov} + \theta_2 T_{j,2}^{ov}\right)\right)\right] \quad (14)$$

Now, we calculate C_s^{max} by using Eq. (13). However, since it's impossible to isolate C_s, we use a numerical method to find its expected value. Eq. (15) shows C_s^{max} equilibrium equation:

$$\delta C_b + \ln E\left[\exp\left(-\delta\left(RT_j^{op} + \theta_1 T_{j,1}^{ov} - N_{j,2} C_s^{max}\right)\right)\right] = 0 \quad (15)$$

Notice that in Eqs. (14) and (15), P^{max} and C_s^{max} can be obtained provided the values of T_j^{op}, $T_{j,1}^{ov}$, $T_{j,2}^{ov}$ and $N_{j,2}$, which are random variables. Therefore, we developed a DES-based algorithm, which is explained in more detail in the following Section, to obtain these values.

3.8. GRP-queue model simulation

As seen in Section 2.1, the presented formulation describes a queuing system with finite population (M), where times until failures follow a GRP. According to assumption III in Section 3.6, times to repair are exponentially distributed. If the number of failed units is greater than the number of servers (m), a queue is formed, and follows a FCFS rule.

In order to find the optimal reservation prices P^{max} and C_s^{max}, it is necessary to simulate the alternating failure-repair process considering a GRP/Markovian/ m / ∞/M//FCFS queuing system. We adopted a DES-based approach to represent the behavior of the system of interest, allowing us for modeling and solving problems that would otherwise be considered intractable or too complex (Zio [42]).

The DES algorithm we developed for queue simulation is shown in Figure 2. This proposed algorithm is similar to what was presented by Moura et al. [23], with two main changes: (i) our simulation model covers both base and extended periods, while the aforementioned paper only simulates the extended period; (ii) our model is formulated for a single customer class, while the aforementioned work allows for two priority classes, which are not implemented here. Our formulation is explained as follows.

At $t = 0$, all M devices are considered new, the time to first failure for each equipment follows a Weibull distribution, queue is empty, and the first failure immediately begins its respective repair. Next, we track if the future events are failures or repairs based on the min (t_a, t_d), where t_a is the next time of arrival (occurrence of a failure) and t_d is the time of departure (next repair completion). Note that t_d is initially set to infinity in step 1.2, since there is no equipment being repaired; as a consequence, it will never be smaller than t_a. If a failure occurs when all service crews are busy, i.e., there are at least m failed units, the just failed device waits in queue. If there is at least one available service crew when a failure occurs, this failed device will immediately begin its repair.

This procedure will continue over the total period (W). We also consider that if a failure occurs during W_1, but its respective repair is completed during W_2 with overtime, the penalty charged is θ_1. During simulation, all events of interest (times of failures and repairs, number of failures for each device, downtime and overtime of each device) are logged so that they can be used to obtain information about system availability, number of failures (N_j), downtime of equipment (T_j^{dt}) and overtime (T_j^{ov}). These are the main outputs of the simulation (step 3), used later to obtain the prices of EWs and on-call services.

After the queue is simulated, customer's optimal strategy is defined. To that end, we estimate the reservation prices for each option offered by the OEM, which are given by Eqs. (14) and (15), using data from many replications (n_{rep}) of the simulation. The complete process used to find the optimal number of customers and their respective reservation prices is given in Figure 3, and is more detailed in Figure 4. Note that the prices that the customers are willing to pay for each option depends on their risk aversion δ.

Initially, the population size (number of devices) is $M = 1$, and then the DES is repeated for the defined number of replications, resulting in estimates for P^{max} and C_s^{max}. Since the OEM has limited service capability (the number of service crews), M cannot be increased indefinitely because high values of M result in longer waiting times in queue, thus reducing operational time and increasing the occurrence of penalties. Therefore, M is increased and this process is repeated until the OEM's profits decrease for both repair service strategies (so that optimal number of hospitals served is guaranteed to be found); this is possible since, increasing M beyond its optimal value results in considerable increases in queue waiting times and OEM penalties (Ashgarizadeh & Murthy [2]). Finally, the optimal number of hospitals and the respective service prices are found by choosing the number of devices that result in the highest values for OEM's profit.

Inputs: $\alpha, \beta, q, \mu, m, \tau_1, \tau_2, W_1, W_2$
1. **Initialization**
   1.1. Generate first failure time for each equipment and set t_a equal to the earliest failure time
   1.2. For convenience, set $t_d = \infty$
   1.3. Set initial time: $t = 0$
2. **Simulation**
   2.1. Is the next event a failure ($t_a \leq t_d$) or a completion of repair ($t_d < t_a$)?
      2.1.1. Failure of device j (only if $t_a < W$)
         ▪ Update current time $t = t_a$
         ▪ Store failure information
            ● Increment number of failures for device j for current warranty period ($N_{j,w}$)
            ● Increment number of failed devices
            ● Store failure time
         ▪ If there is at least one available service crew
            ● Allocate failed device to a service crew, and mark the crew as busy
            ● Generate repair duration r and set $y = r$
            ● $t_d = t + y$
            ● Increase downtime by y for device j (T_j^{dt})
            ● Store repair duration and departure time
            ● If $y > \tau_i$, register overtime of $y - \tau_i$ for device j for current warranty period i ($T_{j,w}^{ov}$)
         ▪ Set t_a equal to the earliest failure time among the remaining operational equipment
      2.1.2. Completion of repair on device j
         ▪ Update current time $t = t_d$
         ▪ Store repair information
            ● Decrement number of failed devices
            ● Generate a failure time t_a' for this device
            ● If $t_a' < t_a$, then set $t_a = t_a'$
         ▪ Are there any remaining failed devices queued?
            ● Yes
               ● Let u = time spent in queue
               ● Generate repair duration r and set $y = u + r$
               ● $t_d = t + y$
               ● Increase downtime by y for device j (T_j^{dt})
               ● Store repair duration and departure time
               ● If $y > \tau_i$, register overtime of $y - \tau_i$ for device j for current warranty period i ($T_{j,w}^{ov}$)
            ● No
               ● Set service crew as free
               ● Set $t_d = \infty$
   2.2. Is $t_a > W$ and the queue empty?
      2.2.1. Yes: go to step 3.
      2.2.2. No: repeat step 2.
3. **Output generation**: for each device, during each warranty period, return these measures:
   3.1. Number of failures ($N_{j,1}, N_{j,2}$)
   3.2. Downtime (T_j^{dt})
   3.3. Overtime ($T_{j,1}^{ov}, T_{j,2}^{ov}$)

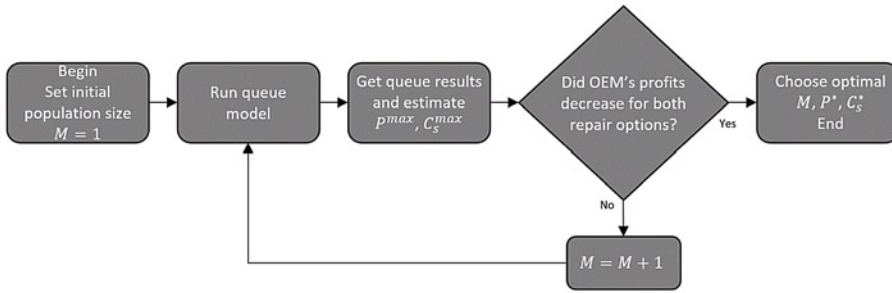*Fig. 2. DES algorithm used to simulate the queue system*

Fig. 3. Diagram of the optimization process

Inputs: $\alpha, \beta, \mu, q, \theta_1, \theta_2, \delta, \tau_1, \tau_2, R, C_b, C_r, W_1, W_2, n_{rep}$
1. **Initialization**: set initial population size $M = 1$
2. **Model execution**
   2.1. Run queue simulation for the current population M for the required n_{rep}
   2.2. Get queue measures from simulation
        2.2.1. Estimate C_s^{max} and P^{max}, using Eqs. (14) and (15)
        2.2.2. Choose best strategy
        2.2.3. Store OEM's profit for each strategy
   2.3. Did OEM's profits decrease for both repair service options?
        2.3.1. No: increment population size $M = M + 1$ and repeat step 2.
        2.3.2. Yes: go to step 3.
3. **Results definition**
   3.1. Choose population size and strategy which resulted in highest OEM profit (M^*)
   3.2. Define C_s and P accordingly

Fig. 4. Monte Carlo based algorithm for the optimization process

4. Application example

4.1. Model's parameters estimation

An application example is here presented by using a failure database of an angiography device, which is technology-intensive, and supports the treatment and diagnosis of cardiovascular diseases. The angiogram, used for visualization of arteries based on x-rays, begins with introduction of an iodine contrast material injection into blood vessels though a catheter. X-ray angiogram provides anatomical information about blood vessels (Çimen et al. [4]). By watching the flow of the contrast fluid, the doctor can identify obstructions and narrowing, proceeding with treatment.

Angiography failures can result in incorrect diagnosis and inappropriate patient treatment, having negative consequences in the patient's health and hospital's reputation. Thus, angiographies are fundamental for hospitals profitability, and their unavailability

represents a great loss of revenue, resulting in a negative economic impact.

Table 1 shows 38 times between critical failures and their respective times to repair. Considering this device is subject to imperfect repairs, we need to use the simulation-based solution proposed in the previous Section to reproduce the GRP-queue system. To that end, we first obtain the MLEs for the GRP parameters by using the procedure described in Yañez et al. [39].

We also obtained the MLE's for parameters α and β when the assumptions of perfect $(q = 0)$ and minimal $(q = 1)$ repairs are adopted. By restricting q to fixed values, it is expected that MLE's result in inferior likelihood, or at most as good as that of obtained by imperfect repair assumption, since parameter search space is restricted. The MLE's for a, β and q for each repair hypothesis are given in Table 2, which also shows the mean squared error (MSE) for simulation data with each repair assumption in relation to observed data. Note that the lowest MSE is obtained when we considered imperfect repairs, which attests that imperfect repair is the most suitable assumption for this case.

Next, we estimate the expected number of failures $E\left[N_j\right]$ by using the procedure described by Yañez et al. [39], and we compared the results against the observed failure data in Table 1. Figure 5 shows this comparison under assumptions of imperfect repairs.
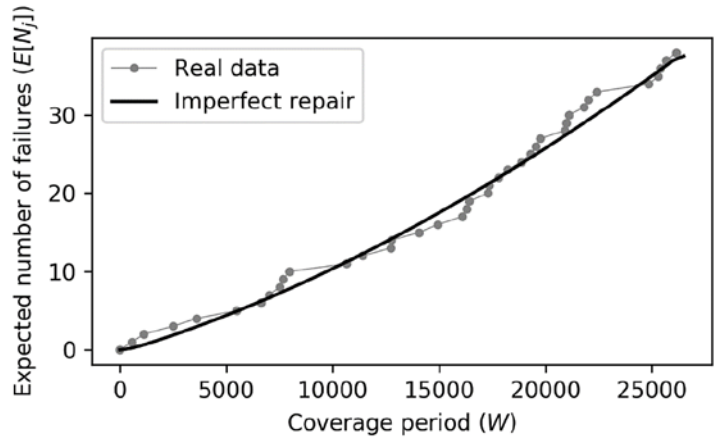

Fig. 5. Comparison between real and simulated times to failure under different repair assumptions

Table 1. Failure dataset of an Angiography device

ith failure	X_i (hours)	Y_i (hours)	ith failure	X_i (hours)	Y_i (hours)	ith failure	X_i (hours)	Y_i (hours)
1	576	4.25	14	24	1.5	27	192	1.45
2	552	2.25	15	1,320	1.5	28	1,152	1.25
3	1,368	4.75	16	864	1.5	29	72	4.75
4	1,104	6	17	1,152	1	30	120	2
5	1,872	3.25	18	216	2.5	31	720	4
6	1,152	2.25	19	120	4	32	192	1.25
7	384	2.25	20	888	4	33	408	2.5
8	504	1.5	21	48	1	34	2,424	5
9	144	2.25	22	432	7.25	35	456	0.25
10	312	3.5	23	432	2	36	96	0.5
11	2,688	4.75	24	648	2.25	37	264	3.5
12	744	3.5	25	432	1	38	480	2.5
13	1,320	1.25	26	264	1.5	–	–	–

For the application example, we will use the parameters shown in Table 3. The inputs given above were used to feed the simulation algorithm described in Figure 2. In order to find the results, the procedures given in Figure 3 and Figure 4 are executed using $n_{rep} = 1,000,000$ replications and varying the number of equipment (M), and then P^{max} and C_s^{max} are estimated for each M.

By using those parameters and GRP MLE's, the optimal number of equipment, and hospital's reservation prices obtained were respectively: $M^* = 47$, $P^{max} = \$ 183,991$ and $C_s^{max} = \$ 8,550$. As it can be

Table 2. Parameter estimates

Repair hypothesis	$\hat{\alpha}$	$\hat{\beta}$	$\hat{q}$	MSE
Minimal Repair	1,622.55	1.309	1	1.62
Imperfect Repair	1,351.83	1.658	0.097	1.54
Perfect Repair	717.83	1.120	0	9.18

Table 3. Parameters for the application example

Angiograph sale price (C_b)	$ 1,476.49 (10^3)$
Revenue per time (R)	$ 0.094 (10^3)$ h^{-1}
Cost of the repair (C_r)	$ 2.5 (10^3)$
Mean repair rate (μ)	0.2 h^{-1}
Penalty per time (base warranty) (θ_1)	$1 (10^3)$ h^{-1}
Penalty per time (under extended warranty) (θ_2)	$3 (10^3)$ h^{-1}
Hospital's risk aversion (δ)	0.1
Period of the game (W)	2 years = 17,520 h
Base warranty period (W_1)	1st year = 8,760 h
Extended warranty period (W_2)	2nd year = 8,760 h
Maximal time to repair the equipment under EW ($\tau_1 = \tau_2 = \tau$)	12 h

seen, the price of hiring an EW is near the range between 5% and 12% of the purchase price C_b, which is consistent to what happens in the clinical environment (Murthy & Djamaludin [26]). The OEM's expected profit for the options (A_1 , A_2) are respectively $E\left[\Pi_{OEM}\left(A_1\right)\right]$ = $ 2,979,171 and $E\left[\Pi_{OEM}\left(A_2\right)\right]$ = $ 2,155,601. Then, the Nash Equilibrium occurs when the OEM sets $P = P^{max}$ and $C_s > C_s^{max}$, returning an expected profit of $ 2,979,171. Finally, the optimal choice for the client is A_1 and the expected utility for the customer is zero, as the OEM extracts all the consumer's surplus.

4.2. Sensitivity analysis

For better understanding on how sensitive the model is regarding parameters and repair assumptions, we present a sensitivity analysis. Note that in the tables presented throughout this analysis, base results are highlighted in grey.

4.2.1. Effect of *q* variation

First, in order to identify possible output changes, we disregard the MLE value found

for q in the previous Section, and vary the rejuvenation parameter q as shown in Table 4. As seen in Section 2.1, the rejuvenation parameter measures the quality of repair. When q approaches zero, the quality of repair increases, returning the failed unit almost to "as good as new". Thus, the wear of equipment is reduced, and it is reasonable to expect a lower failure rate. As q increases in turn the quality of repair decreases, and equipment suffers higher wear over time. Queue length, overtime ($T_j^{dt} W_i$) and amount of penalty incurred rise under option A_1, making the OEM serves fewer clients, which reduces their profit. For $q \geq 0.16$, the increased number of failures and amount of penalties are so high so that result in a change of strategy.

4.2.2. Variation on the model parameters

Generally, consumers with high risk-aversion are less willing to pay for the services, reducing OEM's profit. In this situation, the OEM tends to sell equipment to fewer customers. Table 5 shows how the players' optimal strategies change for different risk-aversion parameter values. For $\delta \leq 0.08$, strategy A_2 is chosen since customers become more tolerant to risk, thus not choosing EW anymore. For this case, OEM can perform maintenance for a greater number of hospitals, thus increasing waiting time in queue and downtime. However, profit is still increased, since customers are not as risk-averse as in the other tested cases, accepting to pay more for services. For $\delta > 0.08$, the hospital's optimal strategy is to hire the EW (A_1).

The variations on results due to changes in the device characteristic life α are given in Table 6. Higher values of α result in longer times to failures, which in turn decreases the expected number of failures $E\left[N_j\right]$. Consequently, devices have increased availability and generated revenue, also increasing the OEM's profits. Yet, the number of sold units M increases along with α, and due to the occurrence of fewer failures, customers pay considerably more for on-

Table 5. Optimal solution changes due to δ variations

δ	M	P^{max} $	C_s^{max} $	A^*	$E[\pi]$ $	$E\left[N_j\right]$	$E\left[T_j^{dt}\right]$	$E\left[T_j^{ov}\right]$
05	90	247,750	10,143	A_2	4,655,854	21.34	254.98	94.56
0.08	82	215,350	8,969	A_2	3,349,710	21.39	226.87	73.95
0.09	48	185,928	8,796	A_1	3,047,552	21.51	154.68	29.21
0.10	47	183,991	8,550	A_1	2,979,171	21.51	153.24	28.47
0.15	45	178,207	7,543	A_1	2,746,489	21.51	150.48	27.09
0.20	44	174,883	7,003	A_1	2,612,681	21.51	149.12	26.42

Table 4. Optimal solution changes due to q variations

q	M	P^{max} $	C_s^{max} $	A^*	$E[\pi]$ $	$E\left[N_j\right]$	$E\left[T_j^{dt}\right]$	$E\left[T_j^{ov}\right]$
0.0	108	185,591	15,785	A_1	9,281,009	14.09	124.11	31.64
0.1	47	183,991	8,550	A_1	2,979,171	21.51	153.24	28.47
0.15	34	183,407	7,222	A_1	1,816,625	25.08	168.29	28.19
0.16	57	203,265	6,887	A_2	1,679,855	25.60	232.35	62.65
0.2	49	203,694	6,233	A_2	1,219,007	28.11	245.85	63.04
0.3	29	194,319	5,189	A_2	500,567	34.12	246.42	46.99

call repairs, since the total value they are willing to pay for repair services is now spread across fewer payments. P^{max} also increases with α increment, however at a lowest rate, because while devices fail less frequently, the OEM serves more devices as α increases, which causes more time spent in queues. For $\alpha \leq 1,140$, strategy A_2 is selected, due to increased failure frequency and, consequently, higher amount of penalties to the OEM.

Changes in optimal strategies due to variations in the shape parameter β can be seen in Table 7. For higher values of β, equipment wears

Table 6. *Optimal solution changes due to α variations*

α (h)	M	P^{max} $	C_s^{max} $	A^*	$E[\pi]$ $	$E[N_j]$	$E[T_j^{dt}]$	$E[T_j^{ov}]$
1,100	49	201,301	6,648	A_2	1,244,617	28.29	241.34	59.40
1,140	53	201,513	6,951	A_2	1,485,647	26.95	235.11	59.75
1,150	31	183,076	7,101	A_1	1,531,656	26.75	176.50	28.61
1,200	35	183,493	7,463	A_1	1,857,136	25.25	170.36	28.77
1,300	43	183,963	8,168	A_1	2,574,340	22.66	158.96	28.75
1,351.83	47	183,991	8,550	A_1	2,979,171	21.51	153.24	28.47
1,400	51	184,093	8,854	A_1	3,373,132	20.53	148.59	28.37
1,500	60	184,386	9,545	A_1	4,244,186	18.73	140.40	28.40

Table 7. *Optimal solution changes due to β variations*

β	M	P^{max} $	C_s^{max} $	A^*	$E[\pi]$ $	$E[N_j]$	$E[T_j^{dt}]$	$E[T_j^{ov}]$
1.5	58	184,520	9,247	A_1	3,945,706	19.60	146.41	29.37
1.6	51	184,329	8,791	A_1	3,296,711	20.82	151.21	29.01
1.658	47	183,991	8,550	A_1	2,979,171	21.51	153.24	28.47
1.7	45	184,073	8,369	A_1	2,770,587	22.00	155.66	28.60
1.74	43	184,033	8,188	A_1	2,588,444	22.46	157.59	28.54
1.75	72	204,856	8,019	A_2	2,569,419	22.46	221.40	66.84
1.8	69	205,154	7,808	A_2	2,454,541	23.03	224.62	66.48

Table 8. *Optimal solution changes due to μ variations*

μ (h⁻¹)	M	P^{max} $	C_s^{max} $	A^*	$E[\pi]$ $	$E[N_j]$	$E[T_j^{dt}]$	$E[T_j^{ov}]$
0.15	46	209,088	8,484	A_2	1,350,270	21.38	232.33	77.10
0.175	60	204,565	8,450	A_2	2,024,136	21.40	217.29	66.91
0.19	42	185.088	8.536	A_1	2540.077	21.50	157.04	30.39
0.2	47	183,991	8,550	A_1	2,979,171	21.51	153.24	28.47
0.225	60	181,767	8,508	A_1	4,149,651	21.52	145.25	24.65
0.25	73	179,841	8,525	A_1	5,408,147	21.53	138.11	21.44

out faster, resulting in higher number of failures $E[N_j]$, which in turn causes greater unavailability. Thus, the probability of penalty being incurred also increases, and the OEM chooses to serve fewer customers. In fact, as $\beta \geq \beta \geq 1.75$, EW is no longer advantageous, and A_2 becomes the optimal strategy; notice that the significant increase in M for $\beta \geq 1.75$ occurs because the OEM pays no penalty at all for strategy A_2, which allows it for serving more customers. The increase in P^{max} when $\beta \geq 1.75$ is due to the number of customers being served, which increases the likelihood of overtime, and the customers' willingness to pay for services. Notice in Eq. (14) that P^{max} increases along with the expected overtime in repairing the device $E[T_j^{ov}]$, which is higher as more customers are served, resulting in longer waiting times in queue.

Table 8 presents how optimal strategies change due to variation in the service rate μ. With higher values of μ, the service crew can repair a greater number of failed devices per time unit, decreasing the queue size. Therefore, the OEM can deal with a greater number of

customers, increasing its own profit. On the other hand, when μ decreases, the service crew can repair fewer items per time unit. Notice that for μ≤0.175, the service crew is too slow to repair the equipment by τ, resulting in a large amount of penalty under the option A_1. Then, the OEM's optimal strategy changes to set $P > P^{max}$ and $C_s = C_s^{max}$, which induces the customer to choose A_2, i.e., on-call services.

Table 9 shows how the optimal solution changes due to the maximum times to repair τ, which influences how long equipment waits (in queue or being repaired) without penalizing the OEM. The higher the value of τ, less penalty is incurred, because the frequency of those delays in returning the equipment to operational state will be lower. In other words, the OEM's payoff increases as τ increases. When the acceptable tolerance τ is smaller, overtime and penalty increase under EW option. Then, for values of $\tau \leq 10$, it is best for the manufacturer to sell on-call repairs.

Table 10 presents the behavior of optimal solution due to revenue R variations. Notice that with the increase of monetary incomes to the hospital, they are willing to pay considerably more for each

Table 9. Optimal solution changes due to τ variations

τ (h)	M	P^{max} $	C_s^{max} $	A^*	$E[\pi]$ $	$E[N_j]$	$E[T_j^{dt}]$	$E[T_j^{ov}]$
8	66	218,036	8,729	A_2	2,252,960	21.45	185.89	74.04
10	74	211,672	8,553	A_2	2,535,263	21.42	204.35	72.04
11	45	187,130	8,590	A_1	2,721,277	21.51	150.48	31.25
12	47	183,991	8,550	A_1	2,979,171	21.51	153.24	28.47
14	51	178,860	8,461	A_1	3,461,705	21.50	159.11	24.06
16	55	174,856	8,369	A_1	3,900,469	21.49	165.46	20.81

Table 10. Optimal solution changes due to R variations

R ($ 10^3/h)	M	P^{max} $	C_s^{max} $	A^*	$E[\pi]$ $	$E[N_j]$	$E[T_j^{dt}]$	$E[T_j^{ov}]$
0.093	43	164,882	7,846	A_1	2,191,878	21.52	147.80	25.77
0.0935	45	174,420	8,197	A_1	2,576,855	21.51	150.48	27.09
0.094	47	183,991	8,550	A_1	2,979,171	21.51	153.24	28.47
0.0945	49	193,620	8,885	A_1	3,397,473	21.50	156.13	29.95
0.095	51	203,276	9,206	A_1	3,830,324	21.50	159.11	31.51

option. Since hospitals generate more revenue with operation of their devices, they become willing to pay significantly more for maintenance services. Notice, however, that hospital's decision did not change with variations in R.

Finally, Table 11 and Table 12 show the changes in optimal solution due to variations on penalty parameters θ_1 and θ_2 respectively. Even though θ and τ both influence the total amount of penalties, their effects are distinct. High values for θ_1 increases the importance of repairing the equipment by τ, since delays will be severely penalized.

For smaller values of θ_1, the OEM is able to serve a greater number of hospitals due to the decrease in penalties. However, when the number of hospitals increases, the penalty during W_2 also increases (since the reduction in penalty rate is only for W_1, in this case), which may cause strategy A_2 to be selected. This happens for $\theta_1 \leq 0.7$. In the case of θ_2, when it rises too much, strategy A_2 is chosen, since penalties during W_2 increase considerably. This occurs for $\theta_2 \geq 3.3$.

Table 11. Optimal solution changes due to θ_1 variations

θ_1($ 10^3/h)	M	P^{max} $	C_s^{max} $	A^*	$E[\pi]$ $	$E[N_j]$	$E[T_j^{dt}]$	$E[T_j^{ov}]$
0.5	87	208,257	8,130	A_2	3,354,136	21.36	243.67	86.11
0.7	85	208,439	8,227	A_2	3,079,027	21.37	236.67	80.98
0.75	48	183,288	8,471	A_1	3,034,574	21.51	154.68	29.21
1	47	183,991	8,550	A_1	2,979,171	21.51	153.24	28.47
1.25	46	184,484	8,553	A_1	2,914,144	21.51	151.84	27.77
1.5	46	185,253	8,593	A_1	2,844,846	21.51	151.85	27.77

Table 12. Optimal solution changes due to θ_2 variations

θ_2 ($ 10^3/h)	M	P^{max} $	C_s^{max} $	A^*	$E[\pi]$ $	$E[N_j]$	$E[T_j^{dt}]$	$E[T_j^{ov}]$
1	74	187,862	8,434	A_1	5,604,879	21.42	204.35	58.55
2	58	186,192	8,464	A_1	3,972,368	21.48	170.56	37.78
3	47	183,991	8,550	A_1	2,979,171	21.51	153.24	28.47
3.2	46	183,978	8,516	A_1	2,823,874	21.51	151.84	27.77
3.3	74	201,541	8,444	A_2	2,766,223	21.42	204.36	58.40
4	74	203,169	8,436	A_2	2,758,993	21.42	204.36	58.56
5	74	204,831	8,436	A_2	2,758,993	21.42	204.36	58.56

5. Concluding remarks

In this paper, a decision model for an Extended Warranty involving hospitals and OEM was proposed. For modelling this situation and determining the players' optimal strategies, a SG formulation was employed, with the OEM being the leader and the hospital the follower. This situation is commonly found in the market of technology-intensive equipment, which is characterized by a greater bargaining power for the manufacturer, which is the only part capable of performing maintenance interventions adequately.

In order to approximate the problem to a more realistic context, we considered the equipment is subject to imperfect repairs, and to model this issue, two approaches were joined: GRP and queueing theory. Additionally, an application example was presented with real failure data of an angiograph to determine the optimal strategies for each player and demonstrate applicability of the model. Furthermore, we perform a series of sensitivity analyses by showing how model results and players' strategies behave under different scenarios.

Some limitations of the presented approach may also be pointed out. In real-world situations, customers do not present homogeneous risk behavior, consequently, different customers often choose different strategies. Also, different agents commonly have access to different levels of information (asymmetric information), so that it is difficult to predict the actions of other players. Based on these limitations, and also intending to extend the present model, the following features could be implemented:
- Consideration of information asymmetry by employing a principal-agent formulation (Jiang et al. [15], Jin et al. [16]).
- Analysis of consumer usage rate, along with the definition of a two-dimensional warranty policy (Yang et al. [40]).
- A dynamic SG with a greater time horizon to analyze the possibilities for renewal of the extended warranty, analyzing the behavior of the players during longer periods.
- Incorporation of a heterogeneous market, with different profiles of customers, represented by different risk-aversion parameters.

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