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POSITION ESTIMATION USING UNSCENTED KALMAN FILTER

ABSTRACT Position estimation in integrated navigation systems often calls for operations on nonlinear system models. Dynamics nonlinearity of an object, which position we want to estimate requires using special filters. The Extended Kalman Filter based on linearization of nonlinear functions is generally accepted solution. The paper presents the Unscented Kalman Filter based on Unscented Transform. Filter performance with comparison to extended Kalman filter is discussed on the theoretical base and simulation results showing accuracy increase are presented.

INTRODUCTION

The integrated navigation systems use data from different navigation sensors. The most common method of data integration is to combine the Dead Reckoning (DR) and external navigation system (e.g. GNSS). DR sensors are simply designed but they are more immune to active and passive disturbance. The main fault of DR systems is poor long-term accuracy – errors increase as time is going. This problem is not present in navigation systems – especially in satellite ones. Their accuracy is better than DR systems but they are more unreliable however less immune to disturbance. Major disadvantage of navigation systems is possibility to lose information as a result of disturbance or signal fading.

There are two basic methods of navigation data integration: compensation and filtering. The compensation method distinguishes one navigation sensor at which output data are compensated by data from another sensor. In the filtering method data from several sensors are filtered in order to get rid of disturbance and noise and than are used to estimate position or another navigation elements. The integration process is based on a variety of Kalman Filter working as an error estimators or navigation algorithms. These filters operate in a discrete time.

Kalman Filter allows to estimate error or state of an object in k- th step on the basis of measurements in k-1th step. Kalman Filters use information about dynamics of the object (system). Knowledge about system dynamics and its correct modeling is the main issue in Kalman Filter implementation. Depending on system dynamics few kinds of Kalman Filter are used. For systems with linear dynamics it is legitimate to apply standard Kalman Filter. In systems with nonlinear dynamics linearization of the dynamics model is necessary and in such case Extended Kalman Filter (EKF) is used [3, 8]. Linearization is carried out by means of partial derivatives of nonlinear state functions or their Taylor series expansion. An alternative for EKF is the UKF (Unscented Kalman Filter). UKF is a recursive estimating filter and its properties meet well requirements of strongly nonlinear systems. UKF does not linearize the model but manipulate on statistical parameters of nonlinear transformed state and measurement vector. UKF is based on Unscented Transform (UT) [6]. UT converts the state vector into a set of weighted Sigma Points. These points are than used in algorithms of UKF. The UKF algorithm is a set of equations which are necessary to do prediction, innovation and correction steps.

UNSCENTED TRANSFORM

Unscented Transform is designed for calculating statistics of n-dimensional random variable exposed to nonlinear transform with assumption that it is easier to estimate a distribution than a nonlinear function [2]. To compute mean and variance of n-dimensional random variable what is a result of its nonlinear transform we have to determine a set of 2n+1 weighted Sigma Points $S_i = \{W_i, \mathbf{X}_i\}$. Assume that random variable $\mathbf{x} \in \Re^n$ has Gaussian distribution $N(\bar{\mathbf{x}}, \mathbf{P}_x)$ and its components undergo a nonlinear function $\mathbf{y} = f(\mathbf{x})$. The aim is to have as accurate approximation of distribution of variable $\mathbf{y}$ as possible. Approximation of mean and covariance of this variable is as follows:

$$\bar{\mathbf{y}} \approx f(\bar{\mathbf{x}}), \tag{1}$$

$$\mathbf{P}_{yy} \approx \mathbf{G}\mathbf{P}_x\mathbf{G}^{\mathrm{T}}, \tag{2}$$

where $\mathbf{G}$ is the Jacobian of the transformation through the f function under assumption $x = \bar{\mathbf{x}}$

$$\mathbf{G} = \frac{df(\mathbf{x})}{d\mathbf{x}}\Big|_{\mathbf{x}=\bar{\mathbf{x}}} . \tag{3}$$

Sigma Points are computed according to following formulas:

$$\mathbf{X}_0 = \bar{\mathbf{x}}, \tag{4}$$

$$\mathbf{X}_i = \bar{\mathbf{x}} + \left(\sqrt{n+\lambda}\sqrt{\mathbf{P}_\mathbf{x}}\right)_i \qquad i = 1, \dots, n, \tag{5}$$

$$\mathbf{X}_i = \bar{\mathbf{x}} - \left(\sqrt{n+\lambda}\sqrt{\mathbf{P}_\mathbf{x}}\right)_i \qquad i = n+1, \dots, 2n. \tag{6}$$

Weights of these points are given by:

$$W_0^{(m)} = \lambda / (n+\lambda), \tag{7}$$

$$W_0^{(c)} = \lambda / (n+\lambda) + \left(1 - \alpha^2 + \beta\right), \tag{8}$$

$$W_i^{(m)} = 1/[2(n+\lambda)], \qquad i = 1, \dots, 2n, \tag{9}$$

$$W_i^{(c)} = 1/[2(n+\lambda)], \qquad i = 1, \dots, 2n. \tag{10}$$

The λ parameter is defined as follows:

$$\lambda = \alpha^2 (n+\kappa) - n. \tag{11}$$

Constants *α, β, κ* are parameters of this transform fulfilling following assumptions:

$$0 \le \alpha \le 1, \ \kappa \ge 0, \ \beta \ge 0. \tag{12}$$

The κ parameter decides about distances between Sigma Points and the mean $\bar{\mathbf{x}}$. Its optimum value in majority of applications is zero [1, 2, 7]. The α parameter is responsible for influence from high order nonlinearities of function *f*. The β parameter controls weight of zero Sigma Point and often is set up to 2. Expression $\left(\sqrt{n+\lambda}\sqrt{\mathbf{P}_\mathbf{x}}\right)_i$ is the *i*th column of the matrix square root of $(n+\lambda)\mathbf{P}_x$.

Sigma Points undergo the nonlinear transformation of function *f*.

$$\mathbf{Y}_i = f(\mathbf{X}_i) \quad i = 0, \dots, 2n \tag{13}$$

The mean and covariance of variable $\mathbf{y}$ are determined from following expressions:

$$\bar{\mathbf{y}} \approx \sum_{i=0}^{2n} W_i^{(m)} \mathbf{Y}_i, \tag{14}$$

$$\mathbf{P}_y \approx \sum_{i=0}^{2n} W_i^{(c)} (\mathbf{Y}_i - \bar{\mathbf{y}})(\mathbf{Y}_i - \bar{\mathbf{y}})^\mathrm{T}. \tag{15}$$

This method allows to calculate distribution parameters of output random variable $\mathbf{x}$.

UKF ALGORITHM

The transform shown above is the basis of Unscented Kalman Filter. UKF operates on the same state model as EKF [4]:

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1},\ \mathbf{w}_{k-1}), \qquad \mathbf{w}_{k-1} \sim N(0,\ \mathbf{Q}_{k-1});$$

$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k,\ \mathbf{v}_k), \qquad \mathbf{v}_k \sim N(0,\ \mathbf{R}_k). \tag{16}$$

In this filter state vector is incremented by disturbance vectors w and **v** with their corresponding covariance **Q** and **R**. In UKF algorithm are used following denotations:

- $\mathbf{x}_k = \mathbf{x}(t_k)$ is the system state (n-dimensional vector) at step t_k. System state is the smallest set of quantities completely characterizing results of influences on this system $\mathbf{x} = [x_1, x_2, \ldots, x_n]^{\mathrm{T}}$,
- $\mathbf{y}_k$ is p-dimensional measurement vector. The measurement may be given from system sensors (position, velocity etc.),
- **F** is $n\times n$ state matrix describing system dynamic. It describes how system state changes between steps t_{k-1} and t_k,
- **Q** describes increase of uncertainty in discrete dynamic model from stem t_{k-1} to t_k. i.e. it is the $n\times n$ process noise covariance matrix,
- **H** is $p\times n$ measurement matrix,
- **R** is $p\times p$ measurement errors covariance matrix,
- $\mathbf{P}_k$ is state vector covariance matrix.

UKF algorithm includes following steps:

- initialization – calculating the mean and priori covariance of the state vector:

$$\bar{\mathbf{x}}_0^a = E[\mathbf{x}_0] = (\bar{\mathbf{x}}_0^{\mathrm{T}}\ \mathbf{0}\ \mathbf{0})^{\mathrm{T}}, \tag{17}$$

$$\mathbf{P}_0^a = E\left[(\mathbf{x}_0 - \bar{\mathbf{x}}_0^a)(\mathbf{x}_0 - \bar{\mathbf{x}}_0^a)^{\mathrm{T}}\right] = \begin{bmatrix} \mathbf{P}_0 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R} \end{bmatrix}. \tag{18}$$

- Sigma Points – they are calculated according to the method described in Unscented Transform:

$$\mathbf{X}_{k-1} = \left[\ \bar{\mathbf{x}}_{k-1}^a,\ \bar{\mathbf{x}}_{k-1}^a + \left(\sqrt{n+\lambda}\sqrt{\mathbf{P}_{\mathbf{x}}}\right)_i,\ \bar{\mathbf{x}}_{k-1}^a - \left(\sqrt{n+\lambda}\sqrt{\mathbf{P}_{\mathbf{x}}}\right)_i\ \right] \tag{19}$$

- prediction – estimating the state vector distribution on the basis of Sigma Points:

$$\mathbf{X}_{k|k-1} = \mathbf{f}\left(\mathbf{X}_{k-1}^{x},\ \mathbf{X}_{k-1}^{w}\right), \tag{20}$$

$$\overline{\mathbf{x}}_{k}^{-} = \sum_{i=0}^{2n} W_{i}^{(m)} \mathbf{X}_{i,k|k-1}\ , \tag{21}$$

$$\mathbf{P}_{k}^{-} = \sum_{i=0}^{2n} W_{i}^{(c)} \left(\mathbf{X}_{i,k|k-1} - \overline{\mathbf{x}}_{k}^{-}\right)\left(\mathbf{X}_{i,k|k-1} - \overline{\mathbf{x}}_{k}^{-}\right)^{\mathrm{T}}\ . \tag{22}$$

- correction – estimating the mean and covariance of the state vector and Kalman gain:

$$\mathbf{Y}_{k|k-1} = \mathbf{h}\left(\mathbf{X}_{k|k-1}^{x}, \mathbf{X}_{k-1}^{v}\right), \tag{23}$$

$$\overline{\mathbf{y}}_{k}^{-} = \sum_{i=0}^{2n} W_{i}^{(m)} \mathbf{Y}_{i,k|k-1}\ , \tag{24}$$

$$\mathbf{P}_{yk,yk} = \sum_{i=0}^{2n} W_{i}^{(c)} \left(\mathbf{Y}_{i,k|k-1} - \overline{\mathbf{y}}_{k}^{-}\right)\left(\mathbf{Y}_{i,k|k-1} - \overline{\mathbf{y}}_{k}^{-}\right)^{\mathrm{T}}, \tag{25}$$

$$\mathbf{P}_{xk,yk} = \sum_{i=0}^{2n} W_{i}^{(c)} \left(\mathbf{X}_{i,k|k-1} - \overline{\mathbf{x}}_{k}^{-}\right)\left(\mathbf{Y}_{i,k|k-1} - \overline{\mathbf{y}}_{k}^{-}\right)^{\mathrm{T}}, \tag{26}$$

$$\mathbf{K}_{k} = \mathbf{P}_{xk,yk} \mathbf{P}_{yk,yk}^{-1}\ , \tag{27}$$

$$\overline{\mathbf{x}}_{k} = \overline{\mathbf{x}}_{k}^{-} + \mathbf{K}_{k}\left(\mathbf{y}_{k} - \overline{\mathbf{y}}_{k}^{-}\right), \tag{28}$$

$$\mathbf{P}_{k} = \mathbf{P}_{k}^{-} - \mathbf{K}_{k} \mathbf{P}_{yk,yk} \mathbf{K}_{k}^{\mathrm{T}}\ . \tag{29}$$

UKF requires computing matrix square root. This may be done with the aid of Cholesky factorization to $n^3/6$ order [5], but covariance matrixes may be recursive formulated and then square root may be computed with factors n^2 order. This is the most overall form of the Unscented Kalman Filter.

UKF ALGORITHM WITH ZERO MEAN ADDITIVE NOISE

For special (but often existing) case in which process and measurement noise are simply additive calculations may be significantly reduced. In this case system state does not have to be augmented according to increase of the number of noises taken into consideration. This reduces Sigma Points dimension and number of them. Covariances of these noises are then included with the aid of simply additive procedure [6-7].

Equations of the UKF with additive noise have following form:

$$\bar{\mathbf{x}}_0 = E[\mathbf{x}_0], \tag{30}$$

$$\mathbf{P}_0 = E[(\mathbf{x}_0 - \bar{\mathbf{x}}_0)(\mathbf{x}_0 - \bar{\mathbf{x}}_0)^{\mathrm{T}}]. \tag{31}$$

- Sigma Points:

$$\mathbf{X}_{k-1} = \left[\ \bar{\mathbf{x}}_{k-1},\ \bar{\mathbf{x}}_{k-1} + \left(\sqrt{n+\lambda}\sqrt{\mathbf{P}_{\mathbf{x}}}\right)_i,\ \ \bar{\mathbf{x}}_{k-1} - \left(\sqrt{n+\lambda}\sqrt{\mathbf{P}_{\mathbf{x}}}\right)_i\ \right] \tag{32}$$

- Time updating equations:

$$\mathbf{X}^*_{k|k-1} = \mathbf{F}\left(\mathbf{X}_{k-1}, \mathbf{u}_{k-1}\right), \tag{33}$$

$$\bar{\mathbf{x}}_k^- = \sum_{i=0}^{2n} W_i^{(m)} \mathbf{X}^*_{i,k|k-1}, \tag{34}$$

$$\mathbf{P}_k^- = \sum_{i=0}^{2n} W_i^{(c)} \left(\mathbf{X}^*_{i,k|k-1} - \bar{\mathbf{x}}_k^-\right)\left(\mathbf{X}^*_{i,k|k-1} - \bar{\mathbf{x}}_k^-\right)^{\mathrm{T}} + \mathbf{Q}. \tag{35}$$

- Renewed calculating Sigma Points:

$$\mathbf{X}_{k|k-1} = \left[\ \mathbf{X}^*_{k|k-1},\ \mathbf{X}^*_{0,k|k-1} + \left(\sqrt{n+\lambda}\sqrt{\mathbf{Q}}\right)_i,\ \ \mathbf{X}^*_{0,k|k-1} - \left(\sqrt{n+\lambda}\sqrt{\mathbf{Q}}\right)_i\ \right] \tag{36}$$

- Correction – estimating the mean and covariance of state vector and Kalman Gain:

$$\mathbf{Y}_{k|k-1} = \mathbf{H}\left(\mathbf{X}_{k|k-1}\right), \tag{37}$$

$$\bar{\mathbf{y}}_k^- = \sum_{i=0}^{2n} W_i^{(m)} \mathbf{Y}_{i,k|k-1}, \tag{38}$$

$$\mathbf{P}_{\bar{y}k,\bar{y}k} = \sum_{i=0}^{2n} W_i^{(c)} \left(\mathbf{Y}_{i,k|k-1} - \bar{\mathbf{y}}_k^-\right)\left(\mathbf{Y}_{i,k|k-1} - \bar{\mathbf{y}}_k^-\right)^{\mathrm{T}} + \mathbf{R}, \tag{39}$$

$$\mathbf{P}_{xk,yk} = \sum_{i=0}^{2n} W_i^{(c)} \left(\mathbf{X}_{i,k|k-1} - \bar{\mathbf{x}}_k^-\right)\left(\mathbf{Y}_{i,k|k-1} - \bar{\mathbf{y}}_k^-\right)^{\mathrm{T}}, \tag{40}$$

$$\mathbf{K}_k = \mathbf{P}_{xk,yk} \mathbf{P}_{yk,yk}^{-1}, \tag{41}$$

$$\bar{\mathbf{x}}_k = \bar{\mathbf{x}}_k^- + \mathbf{K}_k\left(\mathbf{y}_k - \bar{\mathbf{y}}_k^-\right), \tag{42}$$

$$\mathbf{P}_k = \mathbf{P}_k^- - \mathbf{K}_k \mathbf{P}_{yk,yk} \mathbf{K}_k^{\mathrm{T}}. \tag{43}$$

RESEARCH OF EKF AND UKF

The purpose of these research is to compare the performance and accuracy of position estimation by the use of UKF and EKF. This comparison is done on the base of nonlinear system in which the state matrix causes nonlinearity. Position estimation of an object moving around a radio beacon working in a short range navigation system is applied as a model of nonlinear system. The radio beacon takes distance measurements to the object and its azimuth. These measurements are applied to the filter input. Its task is to estimate the object position in Cartesian co-ordinate system. Figure 1 shows the model in graphical form.

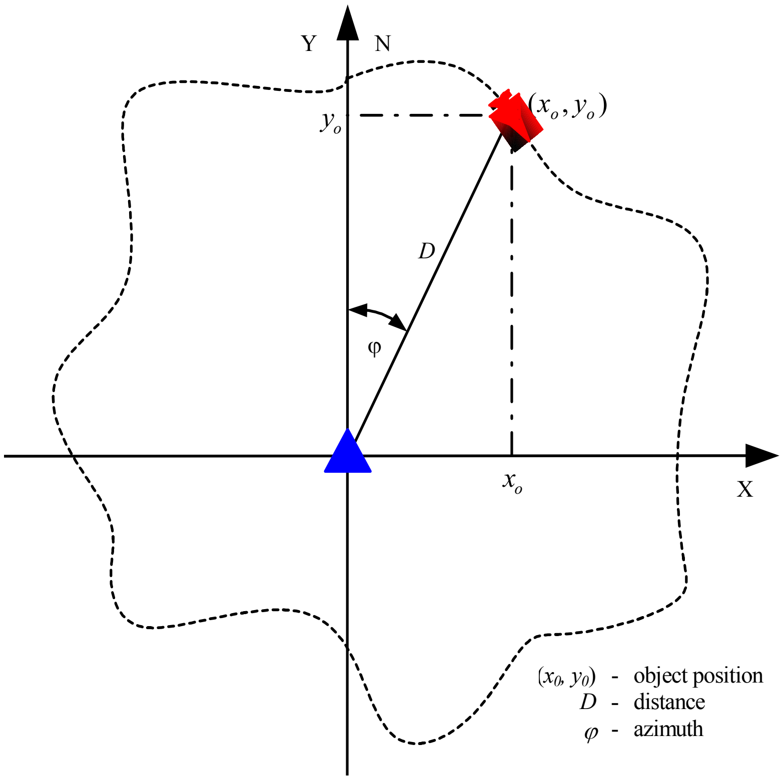

Fig. 1. Graphical form of nonlinear system

Nonlinearity of the state matrix is caused by necessity of transform polar coordinates (distance and azimuth) into rectangular (x, y) one. The system and filters were designed in MATLAB and based on time-discrete model. Block diagram of the system for simulation is shown in the Fig. 2.

Signals of distance and azimuth are exposed to additive noise. Information on dynamics model and measurements without noise (to obtain ideal position and route) reach both filters. Measurements are the basis of object position estimation and traveled route drawing.

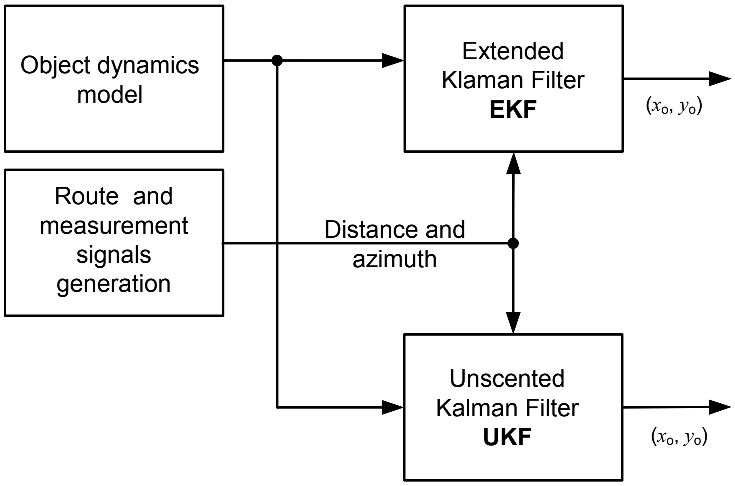

Fig. 2. Block diagram of the system for simulation

State vector in this system has the following form

$$\mathbf{x} = [x_o, D, y_o, \varphi]^{\mathrm{T}} \tag{44}$$

where: x_o, y_o - rectangular coordinates of the object,
D - distance to the radio beacon,
φ - azimuth.

Measurement matrix

$$\mathbf{H} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{45}$$

In the object analyzed above there is nonlinear relationship between polar coordinate system in which measurements are done and rectangular system in which object position is estimated. This relationship is described with the help of state vector.

$$\begin{bmatrix} x_o \\ D \\ y_o \\ \varphi \end{bmatrix} = \begin{bmatrix} D\sin\varphi \\ D \\ D\cos\varphi \\ \varphi \end{bmatrix} \tag{46}$$

Nonlinearity of state matrix comes from this relationship. State matrix has another form for UKF and EKF. In UKF this is typical state matrix and in EKF the state matrix essential to EKF algorithm is determined by partial derivatives from this nonlinear function with respect to elements of state vector.

Nonlinear function in EKF is in the same form as in equation 46 but distance and azimuth are with measurement noise which is function of elements of measurement noise variance matrix **R**:

$$\mathbf{R} = \begin{bmatrix} R_D & 0 \\ 0 & R_\varphi \end{bmatrix}, \tag{47}$$

$$D' = D + f(R_D)\,, \tag{48}$$

$$\varphi' = \varphi + f(R_\varphi)\,. \tag{49}$$

Nonlinear function of state vector in EKF has the following form:

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} D'\sin\varphi' \\ D' \\ D'\cos\varphi' \\ \varphi' \end{bmatrix}, \tag{50}$$

where: R_D - distance measurement variance,

R_φ - azimuth measurement variance,

D' - distance with measurement noise,

φ' - azimuth with measurement noise.

Matrix $\mathbf{F}_{EKF}$ in EKF results from geometric analysis of object movement, determining derivatives of individual rows of matrix in equation 50 with respect to elements of state vector and has following form:

$$\mathbf{F}_{EKF} = \begin{bmatrix} 0 & \sin\varphi' & 0 & D'\cos\varphi' \\ 0 & 1 & 0 & 0 \\ 0 & \cos\varphi' & 0 & -D'\sin\varphi' \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{51}$$

The state matrix $\mathbf{F}_{UKF}$ in UKF is described as follows:

$$\mathbf{F}_{UKF} = \begin{bmatrix} 0 & \sin\varphi' & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \cos\varphi' & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{52}$$

To ensure equal conditions for examination process, noise matrix **Q** and vector state covariance matrix **P** in considered system have the same form:

$$\mathbf{Q} = \begin{bmatrix} Q_{11} & 0 & 0 & 0 \\ 0 & Q_{22} & 0 & 0 \\ 0 & 0 & Q_{33} & 0 \\ 0 & 0 & 0 & Q_{44} \end{bmatrix}, \tag{53}$$

$$\mathbf{P} = \begin{bmatrix} P_{11} & 0 & 0 & 0 \\ 0 & P_{22} & 0 & 0 \\ 0 & 0 & P_{33} & 0 \\ 0 & 0 & 0 & P_{44} \end{bmatrix}. \tag{54}$$

Similarly to matrixes **P** and **Q** initial values of state vector for both filers are the same and include only zero-worth elements:

$$\mathbf{x} = [0,\ 0,\ 0,\ 0]^{\mathrm{T}}. \tag{55}$$

To simulate object maneuvers around the radio beacon the following signals were generated:

$$D' = D + C\cos(Z\varphi) + f(R_D), \tag{56}$$

$$\varphi' = \varphi_T + f(R_\varphi). \tag{57}$$

where: D - constant basic distance,
C, Z - route shape factor,
φ_T - azimuth varying with constant step $T = 2°$ from 0° to 360°.

EKF and UKF algorithms were realized in MATLAB. Parameters of Unscented Transform were assumed as optimal: α = 0,5, β = 2, κ = 0 [2, 4] and following values of route shape factors: C = 1, Z = 6, basic distance D = 8 km, azimuth step $T = 2°$.

Initial state vector covariance matrix has following form:

$$\mathbf{P} = \begin{bmatrix} 1000\,[\mathrm{m}^2] & 0 & 0 & 0 \\ 0 & 1000\,[\mathrm{m}^2] & 0 & 0 \\ 0 & 0 & 1000\,[\mathrm{m}^2] & 0 \\ 0 & 0 & 0 & 1000\,[\mathrm{deg}^2] \end{bmatrix} \tag{58}$$

Figure 3 shows a fragment of route traveled by the object formed on the basis of positions estimated by filters, an ideal route and the route as a result of measurements.

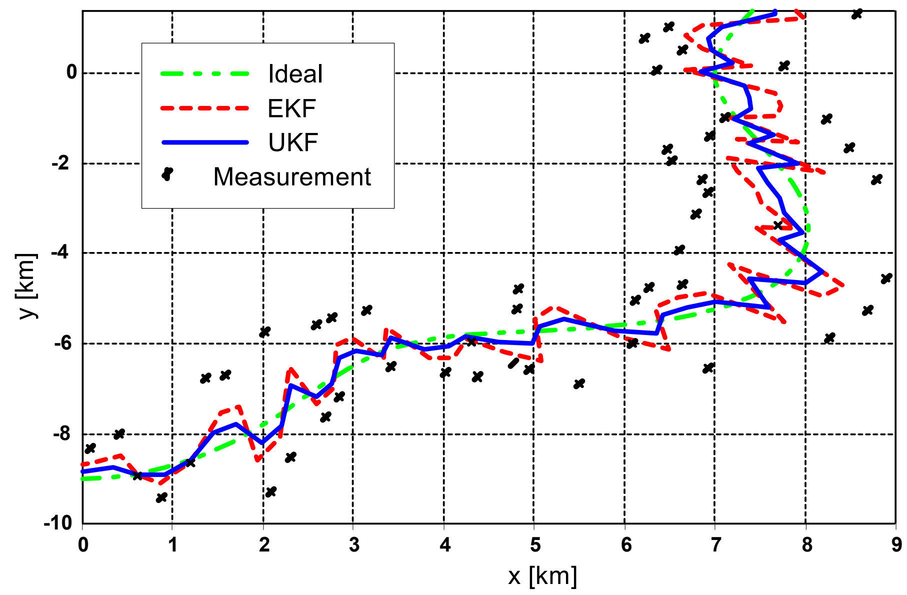

Fig. 3. Fragment of route traveled by the object

Figures 4 and 5 present values of position errors in rectangular coordinates. These errors show difference between the ideal position and the position estimated by individual filters.

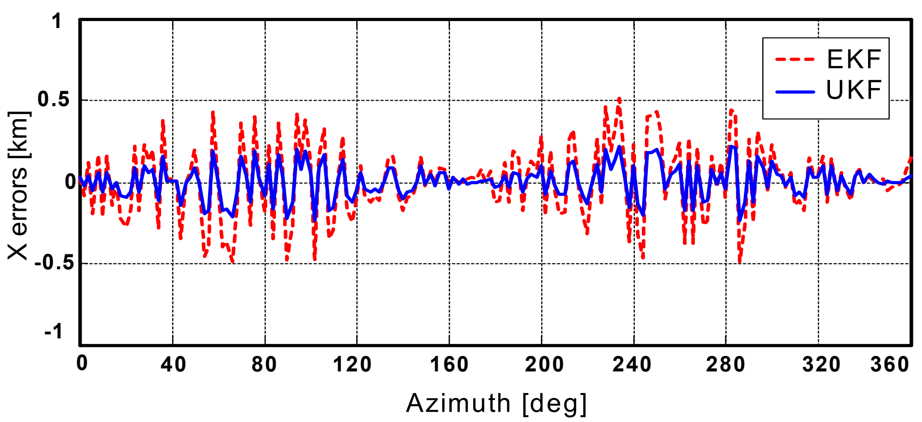

Fig. 4. Position error (X coordinate) in rectangular coordinates

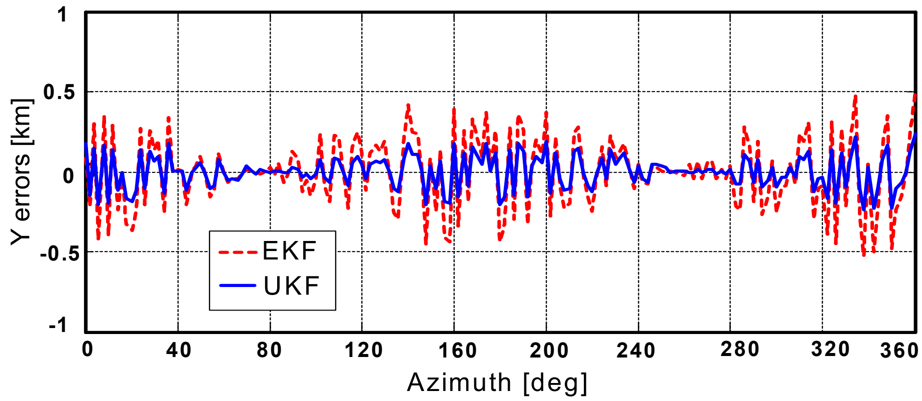

Fig. 5. Position error (Y coordinate) in rectangular coordinates

Measurements in this system are done in polar coordinates and they are estimated too. Errors of these coordinates are shown in figures 6 and 7.

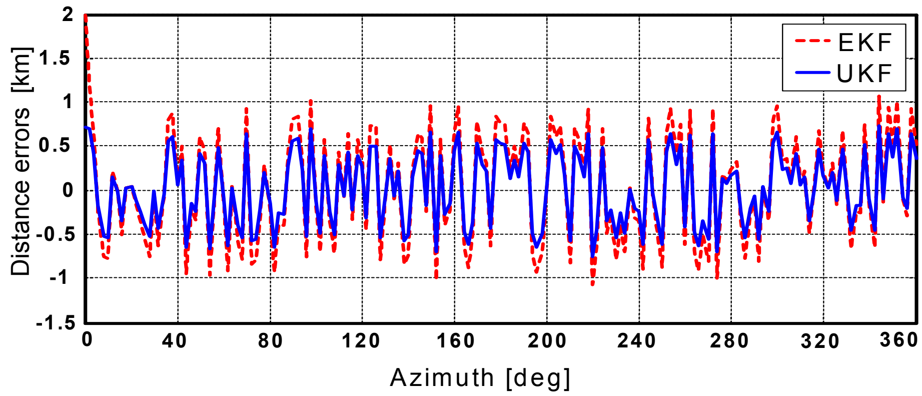

Fig. 6. Distance error in polar coordinates

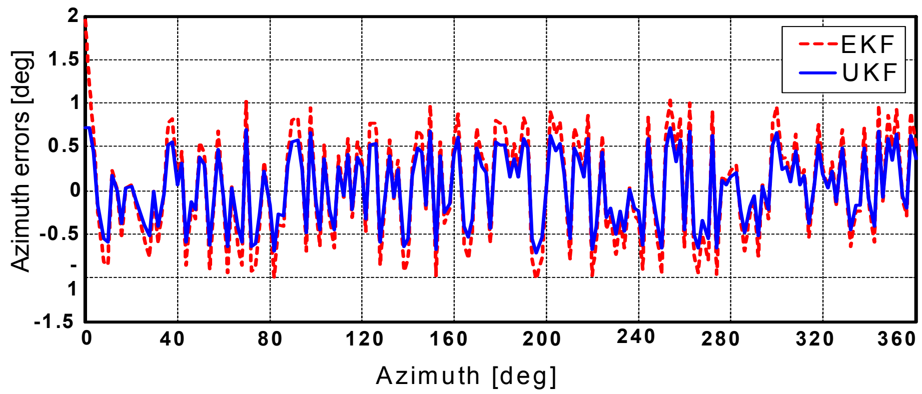

Fig. 7. Azimuth error in polar coordinates

CONCLUSIONS

The integrated navigation systems use variety of Kalman Filter working as an error estimators or navigation algorithms. These filters work in time-discrete mode. Kalman Filters utilize information about dynamics of the object (system). Knowledge about dynamics and its correct modeling is the main issue in implementation of the Kalman Filters. The systems with linear dynamics use basic Kalman Filter. Systems with nonlinear dynamics require linearization of the system model and use of Extended Kalman Filter (EKF) [3].

Unscented Kalman Filter (UKF) is an alternative for EKF [6]. UKF is a recursive estimating filter, which properties meet all requirements of strongly nonlinear systems. UKF does not make linearization of the model but manipulate on statistical parameters of nonlinear transformed state and measurement vector. UKF is based on Unscented Transform (UT) [6]. UT converts the state vector into a set of weighted Sigma Points. These points are than used in algorithms of UKF. The UKF algorithm is a set of equations which are necessary to do prediction, innovation and correction steps.

Simulation results of position estimation using EKF and UKF have shown that UKF used as data processing algorithm gives better accuracy of estimation in system with nonlinear dynamics than EKF. Nonlinearity in system used in simulation is caused by transformation of systems coordinates. Such situation takes place very often in navigation. This shows that UKF is more suitable to systems with strong nonlinearities than EKF. Better accuracy of position estimation using UKF calls for large number of computations (especially evaluation of matrix square root), what makes it more demanding for computation units of integrated navigation systems. UKF may also be used to estimate errors in integrated navigation system based on the compensation mode.

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