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Computational mathematics in marine navigation

Matematyka obliczeniowa w nawigacji morskiej

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Abstract
This article is an overview aimed at a synthetic description of computational mathematics applications in marine navigation. The presented classification of algorithms is based on their application in specific fields of marine navigation. The literature referring to each group of algorithms is then discussed.

Słowa kluczowe: matematyka obliczeniowa, nawigacja morska, algorytmy obliczeniowe

Abstrakt
Artykuł ma charakter przeglądowy. Stanowi próbę syntetycznego ujęcia problematyki zastosowań matematyki obliczeniowej w nawigacji morskiej. Wprowadzono podział ze względu na zakres tematyczny głównych grup algorytmów odnoszących się do zastosowań matematyki obliczeniowej w nawigacji morskiej. Na tym tle dokonano przeglądu literatury obejmującej rozpatrywane zagadnienia.

Introduction

Mathematics can be perceived in a variety of ways. For some it embodies beauty, harmony and perfection. For others it is a key to the understanding of other sciences.

Mathematics delivers tools needed to reach scientific conclusions from assumptions made. Assumptions may refer to various areas of human thought, which makes mathematics intertwine with all of the sciences where the process of inference (reasoning) is in use. Therefore, in very general terms mathematics is an art of drawing conclusions from assumptions. If mathematical reasoning is correct, and so are assumptions made, then the conclusions (results) reached will certainly be correct. Any inaccuracy in reasoning will result in the fact this correctness is not guaranteed [1].

Computational mathematics

Mathematics can be simply divided into theoretical and applied mathematics. The former is developed with no close connections to specific applications (reality). Some mathematicians regard theoretical mathematics as a form of art. Although there were cases when some parts of theoretical mathematics found a practical application, generally it is developed for its beauty. Applied mathematics has a different purpose. Its role is more down to earth, but still very important: it delivers algorithms that enable solving specific problems. An algorithm is a finite ordered series of clearly defined operations necessary to execute a certain type of tasks. In most cases it comes down to the processing of some input data in a finite time and obtaining some output data. The interesting point about algorithms is that the user interested in solving a problem does not have to understand how the algorithm works. What the user needs is to know the form of data, be able to do basic operations and to interpret the results. Precisely defined, an algorithm is a finite set of multivalent vector functions in the following form:

$$\begin{aligned} &\mathrm{alg}^{(i)} : X_i \to X_{i+1} \quad \mathrm{alg}^{(i)}\left(\mathbf{x}^{(i)}\right) = \mathbf{x}^{(i+1)} \\ &\mathrm{alg}^{(k)} : X_k \to Y \quad \mathrm{alg}^{(k)}\left(\mathbf{x}^{(k)}\right) = \mathbf{y} \\ &1 \le i < k \end{aligned} \tag{1}$$

where: k – finite natural number of elementary operations (it need not be known at the start of the algorithm, so that some alg functions may be defined by iteration); X_i – space of all possible input data in an i-th elementary operation of the algorithm ($1 \le i \le k$); Y – space of all possible output data; $\mathbf{x}^{(i)} = [x_1^{(i)}, x_2^{(i)}, ..., x_n^{(i)}]^\mathbf{T}$ – vector of input data in an i-th elementary operation of the algorithm ($1 \le i \le k$); n_i – number of input data in an i-th elementary operation of the algorithm ($1 \le i \le k$); $\mathbf{y} = [y_1, y_2, ..., y_m]^\mathbf{T}$ – vector of output data; m – number of output data.

The oldest records of mathematics were in fact those of what we call applied mathematics. All calculations were made for some practical purposes. Initially mathematics was a set of calculating algorithms used for solving concrete computing problems. In the beginning, people were the only users of those algorithms. *Errare humanum est*, therefore it is not surprising that the final result (problem solution), despite an excellent algorithm, often happened to be burdened with an error. Applied mathematics flourished with the invention of computing machines that took over the role of people in operating algorithms and soon turned out to be far more effective in computing than human mind.

In the second half of the 20^{th} century the role of applied mathematics expanded rapidly. Computers led to algorithms much more complex than those the human can use with a pen and paper, algorithms that even an army of mathematicians would not be able to cope with. It was the computer that made it possible to implement complicated algorithms. It was then that computational mathematics was established, also referred to as computer mathematics or mathematics of computing.

Computing

What does the term *computing* mean? This verbal noun, derived directly from the word *computer* and originally standing for operations performed by computing machines, was first used by John von Neumann in 1945 in documentation of EDVAC – Electronic Discrete Variable Automatic Computer – to define automated computer systems. Till the early 1990s the term *computing* had been used to denote a common area of such disciplines as mathematics, computer science and engineering. Later computing science was incorporated into a broader concept: computer science [2]. Informatics became a European synonym of computer science (former computing science). According to one informatics dictionary [3], informatics is a field of science and technology that deals with information processing, which includes technologies of information processing and technologies of manufacturing systems for information processing. In other words, informatics deals with processes of designing, construction, evaluation, use and maintenance of systems for data storage, processing and transmission. These processes relate to hardware, software as well as human and organizational aspects. Most often information (data) processing involves the computer operating according to the algorithm implemented as a computer program. Such program reflects an algorithm recorded in a language the computer understands [4].

In various classifications informatics is often placed within mathematics and partly in engineering sciences. It follows that historically informatics became divided into theory (derived from mathematics) and practice (linked to engineering). These two aspects of informatics: computations and computing machines, mathematics and electronics, software and hardware – have been functioning jointly and it is hard to say which aspect is more important and decisive. This division can be seen in universities, in their educational and research activities. Theoretical informatics lectured at universities is connected with mathematics, while applied informatics taught or developed at universities of technology is connected with engineering applications (e.g. in mechanics, electronics etc.) [5].

Attempting at a historical description and classification of informatics, we have to indicate its main fields that have helped mankind in all kinds of computing activities. These fields include: calculations, machines and data, to which more precise categories can be attributed, respectively: computational methods, computing machines and data analysis. Computational mathematics is part of informatics that belongs to the field connected with computations. Distinguished as the main class of CCS – *Computing Classification System* [6], it was created and developed by the Association for Computing Machinery, unquestionable authority in the field. In Poland the system has been accepted by scientists and university authorities [7].

Computational mathematics is a branch of informatics that mainly deals with techniques and their applications in other sciences (physics, chemistry, biology, geography, medicine, economics, engineering sciences, agricultural sciences, or even the humanities or the science of law, which require support by means of mathematical tools and computational capabilities. Apart from supplying algorithms, computational mathematics has one more important experimental function. It enables modeling and simulation of systems and processes by means of computational experiments. It allows to make experiments in situations too complicated for a simplified theoretical analysis. Besides, it allows to test algorithms before their implementation in real conditions. Computational mathematics can also be regarded as a branch of existing and newly established interdisciplinary sciences that were established thanks to computational mathematics.

Naturally, the division of sciences is purely conventional. It is rather useless to discuss the boundaries between informatics and mathematics or another science. What is essential about a field of science is not the terminology, but a certain sense of identity, methodology, environment, language, and the consequent approach to solve a problem. It is hard to find a common language even within one scientific discipline. Should, therefore, computational mathematics not remain a branch of different sciences? There are arguments for and against it. Yes, it should, because the application of computational mathematics calls for the knowledge of the science. No, it should not, because people using computational mathematics have a common language, common methods and, frequently, common identity.

Computational algorithms in marine navigation

Navigation is an engineering discipline that belongs to sciences examining phenomena, processes and regularities taking place in the world of artifacts created through human activities known as technological production. In general, navigation deals with investigating spatial relations between various kinds of information about the Earth and its environment, developing methods of locating stationary objects as well as algorithms for controlling mobile objects (definition established in 2003 by the committee of Polish maritime universities). This definition refers to land, air, space and sea. The branch of navigation that refers to the latter is called marine navigation. Marine navigation as a science focuses on conditions of safe navigation of a ship through seas and oceans all over the world [8].

Over the past decades vessel traffic intensity has been on the rise, with vessels developing higher maximum speeds. As advantages of maritime transport are unquestionable, it is highly probable that this mode of transport will continue to expand. This inevitably increases marine risks, in other words, the level of navigational safety is decreasing (Fig. 1), particularly on waterways of heavy traffic and restricted depth – straits and channels, but also in the case of ships navigating in open sea areas.

Fig. 1. The tanker “Kashmir” on fire after a collision with the container ship “Sima Saman” off the coast of Dubai (10 February 2009) [9]
Rys. 1. Płonący zbiornikowiec „Kashmir” po zderzeniu z kontenerowcem „Sima Saman” u wybrzeży Dubaju (10 luty 2009 r.) [9]

To ensure a high safety level is not easy due to the complexity of navigational systems. The degree of complexity is affected by the following factors:

- difficulties in describing the sea system (models do not reflect faithfully all the system components);
- uncertainty of navigational systems (strong disturbances affecting objects in the marine environment caused by external forces: wind, current, waves);
- dynamics of changes and the associated necessity of constant system adaptation (e.g. change in parameters of ship dynamics due to changes of water depth);
- limited reliability and failures of machinery and equipment;
- measurement errors (e.g. GPS receivers);
- human factor, involving wrong navigator’s decisions,
- various aims of navigation, corresponding to various types of craft (carriage of goods, fishing, military, tourist, sailing, exploration, sports, search and rescue or salvage).

The above mentioned factors make the tasks and problems of marine navigation far from trivial. The constant pursuit of possibly high level of navigational safety results in continuous development of this branch of science.

The marine navigation is mainly geared to practical applications of its developments. Methods, i.e. algorithms facilitating safe ship conduct from a point of departure to its destination are what this scientific discipline copes with. The rapid advancements in computer technology recently witnessed have made these algorithms yet more sophisticated, which, in turn, solve navigational problems and tasks more effectively, thus contributing to the development of civilization. The latest parameters and capabilities of computers create a good basis for designing new and expanding the existing computational algorithms. Six thematic groups of computational mathematics applications in marine navigation can be distinguished:

- positioning algorithms (determining present movement parameters of own and target vessels);
- prediction algorithms (determining future movement parameters of own and target vessels);
- routing algorithms (determining an optimized voyage route which accounts for such criteria as: voyage time, track covered, fuel consumption, avoidance of dangers, e.g. adverse weather etc.);
- algorithms of navigational situation assessment (situation identification: safe or dangerous, forcing the navigator to take action to eliminate danger);
- algorithms for planning anti-collision manoeuvres (determining own ship's course and speed to assure safe passing of encountered vessels);
- automatic control algorithms (course autopilots, track autopilots).

The above classification does not include algorithms used in marine traffic engineering as this field is likely to become separated from marine navigation as an independent discipline dealing with qualitative and quantitative description of vessel traffic in a restricted area, that can be simply defined as a place of higher risk of collision (fairway, port entrance, anchorage, turning basin, port basin with quays, lock) [10, 11].

Overview of the literature: state of the research

A wide variety of publications on the application of computational mathematics in marine navigation, including the subjects such as safe and effective sea-going ship conduct, refers mainly to the groups of algorithms listed in the previous chapter.

Positioning algorithms have been dealt with by such researchers as: Bar-Shalom Y. [12], Gucma S. [13], Kazimierski W. [14], Kirubarajan T. [12], Li X. [12], Pietrzykowski Z. [15], Stateczny A. [14, 16, 17], Wąż M. [17]. Standard methods of ship position determination are described in the study [13]. Positioning problems have been solved by traditional statistical tools [12], as well as artificial intelligence methods, such as neural networks [14, 16, 17], or fuzzy logic [15, 16].

Algorithms of prediction have been of interest to Andrzejczak M. [18], Breda L. [19], Gucma L. [20], Narkiewicz J. [18], Passenier P. [19], Pietrzykowski Z. [21], Reich Ch. [21]. Ship movement prediction has mostly utilized methods based on hydrodynamic models or simple static models [18, 19]. However, some authors employed artificial intelligence, neural networks in particular [20, 21], to find solutions.

Scientists such as Abramowski T. [22], Chomski J. [23, 24], Drozd A. [25], Endo M. [26], Medyna P. [24], Wiśniewski B. [23, 24, 25, 27], Zwierzewicz Z. [22] have been dealing with routing algorithms. Fundamentals for the algorithmization of choosing the optimized ship's route are described in [27]. One of the tools applied in route planning was Pontriagin's maximum principle [22]. Problems of routing in the fuzzy environment have also been considered [25]. In [26] this problem was tackled by using expert systems. Besides, ship's routing was formulated as a multicriteria optimization problem, with evolutionary algorithms used to find solutions [23, 24].

The following authors have examined algorithms of navigational situation assessment: Furukawa Y. [28], Kearon J. [29], Kijima K. [28], Lenart A. [30], Pietrzykowski Z. [31, 32], Rutkowski G. [33], Wang F. [34], Wawruch R. [35], Wu Z. [34], Zhao J. [34]. The most common tool used in navigational situation analysis and assessment have been the criteria of closest point of approach and time to closest point of approach [29, 30]. In this approach the system of fuzzy inference has also been used [28]. Another criterion, the ship domain, has been mainly determined by analytical and statistical methods [33, 34, 35]. A more comprehensive approach, in which most of the factors affecting the correct analysis and assessment of a navigational situation, including the human factor, are taken into consideration, has resulted in the concept of ship fuzzy domain [31, 32].

Gawrychowski A. [36, 37], Gucma L. [38, 39], Lisowski J. [40, 41], Miloh T. [42], Pietrzykowski Z. [38, 39, 43], Seghir M. [41, 44], Sharma S. [42], Smierzchalski R. [36, 37, 45, 46, 47], Uchacz W. [43], Żak B. [48] have dealt with algorithms for planning anti-collision manoeuvres. One of the tools used in algorithms of optimized manoeuvres within the allowable strategies has been linear programming [40]. The problem was also formulated as a multicriteria optimization problem to be solved by non-linear programming [47, 48]. The works [45, 46] present the application of evolutionary algorithms for the optimization of anti-collision manoeuvre in ship encounter situations. Besides, methods of multistage control in the fuzzy environment [38, 39, 41, 43, 44] have been used in planning and performing anti-collision manoeuvres. The works [36, 37, 42] present the use of the theory of differential games in the process of collision avoidance.

Algorithms of automatic control of ship movement have been developed, among others, by these researchers: Amerongen J. [49], Astrom K. [50], Crisafulli S. [51], Fossen T. [52, 53, 54], Goodwin G. [51], Hwang C. [55], Kallstrom C. [50], Li Y. [54], Lisowski J. [56], McGookin E. [54], Morawski L. [57, 58], Murray-Smith D. [54], Paulsen M. [53], Piegat A. [59], Pomirski J. [57], Rak A. [58], Roberts G. [60], Sołdek J. [61], Sutton R. [60], Tiano A. [60], Tzeng C. [51], Zirilli A. [60], Zwierzewicz Z. [62]. This group of algorithms is most often discussed in the literature. The PID controller has been often used in ship movement automatic control [56, 61]. As the dynamic characteristics of the ship are not steady (depending on ship type and navigational conditions), settings of PID (PD) controller should also change. This requirement has initiated a number of concepts based on the notion of "space of states" and utilizing the theory of "stochastic control". Examples of adaptive autopilot designs can be found in [49, 50, 52, 57]. Ship movement automatic control algorithms have also been designed using non-linear control techniques [51, 53, 62]. Another solution overcoming the problem of non-linearity was found by means of computer technology and artificial intelligence methods: genetic algorithms [54], neural networks [58, 60] and fuzzy controllers [55, 59].

This author's aim in his research so far has been to prove that traditional mathematical tools characterized by scientific reasoning (scientific proofs) combined with methods of artificial intelligence, additionally supported by powerful computers of today – computational mathematics – lead to the creation of new methods, techniques and computational algorithms capable of solving contemporary problems, those of marine navigation in particular.

Positioning algorithms. In the era of satellite navigation, practically every sea-going vessel is fitted with a GPS receiver, indicating vessel's coordinates. However, relying exclusively on navigational information from a single autonomous receiver carries a risk of substantial errors or loss of position data. One shortcoming of these systems is that their operation can easily be disturbed. To reduce such risk to a minimum, navigational systems can be used where a ship's accurate position is obtained from data available from a number of sources. These data are processed: integrated and filtered in order to further minimize errors. One of the possible solutions is the algorithm of navigational data fusion [63, 64], which works using a multisensor Kalman filter. The author's computational algorithm enables combining data from various measuring devices into one signal. Such solutions are implemented in integrated navigation systems on board vessels equipped with a number of sensors measuring the same signals.

Prediction algorithms. To effectively solve a collision situation, the navigator must have information on future movement parameters of own and target ships. The accuracy of information presented to the navigator is of paramount importance for the correct assessment of the situation and decisions to be made. This is provided by systems of ship movement prediction. These systems enable the determination of basic movement parameters: position, course and speed, in a preset time interval. It turns out that the accuracy of prediction, i.e. its reliability, strongly depends on input data of the computational algorithm. Original computational algorithms related to this issue have been presented in the works [65, 66]. As prediction tools, one of these algorithms uses artificial neural networks taught by series of recorded data (increments of each movement parameter) of various lengths. The calculation of predicted values is based on weighted responses of all the networks. In the other algorithm, the method of linear prediction is used, where the present movement parameters are estimated by means of polynomial approximation.

Algorithms of navigational situation assessment. Navigator's actions on board depend on the existing conditions. When there are no risks (safe situation) they do daily routine, maintaining the course or turning the ship at waypoints. When they conclude the situation becomes dangerous, they are forced to take action to prevent loss (collision, damage, extended track). Such actions are mostly taken in encounter situations when other ships or fixed objects appear in vicinity. The works [67, 68] present an original algorithm of the probabilistic assessment of safety when two dynamic objects pass each other. This algorithm uses methods and tools of probability calculus. Supplementary to this algorithm is that of safe course determination [69], whose operation consists in seeking, through the probabilistic assessment of navigational situation, minimum course alteration, such that the preset safety level will be maintained (probability that the vessels will pass at a distance not less than that assumed as minimum).

Automatic control algorithms. The task of maintaining a steady course, i.e. keeping the ship on a preset course is aimed at the minimization of track, speed and fuel consumption as well as enhanced safety, as uncontrolled yawing may result in a collision. The ship course-keeping algorithm proposed in [70, 71] has been developed using a knowledge base, featuring a computer model of dynamics, built on the basis of appropriately selected set of signals recorded at object input and output. This allowed to overcome difficulties that appear in designing classical control algorithms when the model is complex and non-linear.

Fig. 2. Portable station of the prototype navigational decision support system installed on the training / research ship "Nawigator XXI"
Rys. 2. Przenośne stanowisko prototypu nawigacyjnego systemu wspomagania decyzji zainstalowane na statku „Nawigator XXI"

The author's algorithms herein described, connected with navigational data fusion and the prediction of own and other vessels' movement, have been successfully verified in real conditions, implemented in a prototype navigational decision support system. The system has been tested on board the training / research ship "Nawigator XXI" operated by the Maritime University of Szczecin (Fig. 2).

Summary

This article presents a synthetic description of computational mathematics applications in marine navigation and the state of the art in this field. The overview of relevant publications indicates a diversity of methods and tools in computational algorithms used in the marine navigation systems and equipment. It shows the complexity of these issues and the need to continue research on problems under consideration. It should also be noted that some of the works herein referred to deal with theoretical foundations only, not computational algorithms themselves nor their implementation, which should encourage researchers to develop work undertaken.

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