

Application of Markov decision process as a mathematical model of operation and maintenance process

Zastosowanie decyzyjnego procesu Markowa jako matematycznego modelu procesu eksploatacji

Bogdan Landowski

University of Technology and Life Sciences, Machine Maintenance Department
Uniwersytet Technologiczno-Przyrodniczy, Wydział Inżynierii Mechanicznej
85–791 Bydgoszcz, ul. Prof. S. Kaliskiego 7, e-mail: lbogdan@mail.utp.edu.pl

Key words: urban transport system, operation and maintenance process, Markov decision process

Abstract

This paper presents an example of applying Markov decision process to model and analyse the bus operation and maintenance process within an urban transport system and to forecast the influence of the operation and maintenance strategies realised for the technical objects on the transport system behaviour. Setting the values of the indices describing the process under the analysis is performed on the basis of a computerised simulation of the Markov decision process, being a mathematical model of the technical objects operation and maintenance process. In order to simulate the operation and maintenance process (Markov decision process) and to evaluate the influence of the decisions being made on the course and effectiveness of the process being realised within the study object a simulation algorithm has been elaborated and a computer calculation program has been written. The model presented herein has been created in such a way to assure that it shall be possible to use it in as extensive as possible class of the problems regarding the operation and maintenance of the technical objects.

Słowa kluczowe: komunikacja miejska, proces eksploatacji, decyzyjny proces Markowa

Abstrakt

W artykule przedstawiono przykład zastosowania decyzyjnego procesu Markowa do modelowania i analizy procesu eksploatacji autobusów komunikacji miejskiej oraz prognozowania wpływu realizowanych strategii eksploatacji obiektów technicznych na zachowanie się systemu transportowego. Wyznaczanie wartości wskaźników charakteryzujących analizowany proces realizowane jest na podstawie komputerowej symulacji decyzyjnego procesu Markowa, będącego matematycznym modelem procesu eksploatacji obiektów technicznych. W celu symulacji procesu eksploatacji (decyzyjnego procesu Markowa) i oceny wpływu podejmowanych decyzji na przebieg i efektywność procesu realizowanego w obiekcie badań opracowano algorytm symulacji i napisano komputerowy program obliczeniowy. Przedstawiony w pracy model został skonstruowany w taki sposób, by zapewnić możliwość jego wykorzystania w jak najszerszej klasie problemów związanych z eksploatacją obiektów technicznych.

Introduction

The object of the investigations is generally defined operation and maintenance system of the technical objects. The controlled processes being the components of the operation and maintenance process are realized in this system.

As an example of the investigation object, serving to illustrate all the considerations, a real system

of an urban bus operation and maintenance system within a large (approx. 400 thousand of residents) agglomeration has been chosen.

The subject of the investigations is a determined set of the operation and maintenance states of the means of transport and processes of their maintenance as well as the sets of possible decisions made in each of the w states affecting the flow of the operation and maintenance process, including

relations occurring between the elements of the said sets and between them and effectiveness of the operation and maintenance process.

Supporting a decision maker in the decision making process regarding realization of the operation and maintenance process of the means of transport may be realized by forecasting the way of the operation and maintenance is going to behave and by evaluating the influence of the chosen decision variants on the flow of the operation and maintenance process [1]. The analysis of the results from investigations of the operation and maintenance process models may be helpful here. The investigations of such a type include determination of the value of the selected measures of the technical and economic efficiency of the process being realized for the assessed values of the model parameters corresponding to the analysed decision variants. A change of the model input parameters value may reflect a change of the way the internal and external factors affect the system behaviour [2, 3]. The analysis of the results of the investigations realized in such a manner makes it possible to evaluate operation of the system under investigation and of the process being realized in it as well as to determine the decision indices enabling reasonable control of the system.

As examples of the criterion function, the following parameters may be considered: values of the transport system reliability features, values of the incomes discounted in a long range of time or an average value of the incomes for the determined ranges of time [4].

This paper presents an example of using the Markov decision process to model and analyse the urban bus transport operation and maintenance process. Determination of the values of the indices describing the process being analysed is performed on the basis of a computerized simulation of the Markov decision process, being a mathematical model of the technical objects operation and maintenance process.

In order to illustrate the investigations a calculation example and the chosen results of the simulation experiments have been presented herein.

Mathematical model of the operation and maintenance process

It has been assumed that the stochastic process $\{X_t, t \in T\}$ with finite number of the states from the space $S = \{1, 2 \dots n\}$, where the set T means non-negative real semiaxis $t \geq 0$ is a mathematical model of the bus operation and maintenance process. The changes of the process state occur at the

moments $t = t_1, t_2 \dots t_n$. The states of the analysed stochastic process correspond to the identified bus operation and maintenance states.

A finite set $A_i = \{1, 2 \dots \overline{A}_i\}$ of decisions (actions, alternatives), which may be applied in the state $i \in S$ corresponds to each state $i \in S$. The power of the set A_i is denoted by \overline{A}_i .

The space of possible decisions $A = \bigcup_{i \in S} A_i$.

The decisions are defined as determination of the proceeding way, for instance in relation to a technical object, at the moment of entering a particular state. In a real operation and maintenance system there may be various methods of servicing (repairs, inspections) or using the object (e.g. various routes, over which a bus is used), having an influence on the suffered costs and generated incomes as well as on the frequency and types of damages to the object [5].

Let $p_i \geq 0$ denotes a probability that the process will be in the state $i \in S$ at the moment $t = 0$. Naturally $\sum_{i \in S} p_i = 1$. The vector $p = [p_i]$, $i \in S$ is a stochastic vector of the initial process distribution $\{X_t, t \in T\}$.

If at the moment t_k , $k \in \mathbb{N}$ the process is in the state i and the decision $a \in A_i$ was made when entering this state then p_{ij}^a , where $i, j \in S$ represent the probability that the process will be in the state j at the moment t_{k+1} (the next state of the process will be the state j).

It is assumed that $\sum_{j \in S} p_{ij}^a = 1$ and $p_{ij}^a \geq 0$ for all $i, j \in S$ and $a \in A_i$.

To simplify the considerations it has been assumed in the calculation example that:

$$p_{ij}^a = p_{ij}$$

Let r_i^a denote an income, when $r_i^a > 0$ (a loss, when $r_i^a < 0$) per a time unit generated by the system, when the process $\{X_t, t \in T\}$ is in the state $i \in S$ at the moment t and the decision $a \in A_i$ was made when entering the state i . It is assumed that the income r_i^a is limited for all $i \in S$ and $a \in A_i$.

The random variable denoting duration of the state $i \in S$ of the process, when $j \in S$ will be the next state and the decision $a \in A_i$ with the distribution determined by the distribution function $F_{ij}^a(t)$ was made when entering the state i has been identified with the symbol T_{ij}^a .

To simplify further investigations it has been assumed that $F_{ij}^a(t) = F_i^a(t) = F_{ia}(t)$, $i, j \in S$, $a \in A_i$. The function $F_{ia}(t)$ is a distribution function of the distribution of the state $i \in S$ duration provided that the decision a was made when entering this state.

The random variable denoting the duration of the state $i \in S$, with the distribution determined by the distribution function $F_{ia}(t)$ has been identified with the symbol T_{ia} .

The stochastic process $\{X_t, t \in T\}$ determined this way and is a special case of the Markov decision process with incomes.

Decision model of an operation and maintenance process

The investigations presented hereunder assume that from the point of view of the investigation aim, it is possible to identify n separable subsets of the homogenous objects in the set of the technical objects being operated and maintained in a transport system.

Various maintenance events requiring from the decision makers of the operation and maintenance system to make current decisions regarding the way of operating and maintaining the vehicles, the results of which affect the flow of the operation and maintenance process, including its economic effect occur when operating and maintaining the buses in an urban transport system [5]. The bus operation and maintenance states are characterized by the distributions of these states durations and by unitary incomes (costs) generated by the system, when a vehicle stays in a specific state.

The simplified model of operation and maintenance process of an urban bus transport system presented hereunder illustrates possibility to apply Markov decision process and a computerized simulation to analyse and control the operation and maintenance process of the technical objects.

Three bus operation and maintenance states have been analysed. The state S_1 – using a technical object. A technical object including its operator carries out the transport tasks assigned to it. The state S_2 – servicing a technical object performed by so called technical service (TS) units. An unserviceable technical object staying in the environment of the operation and maintenance system is affected by the actions performed by mobile technical service units aimed at bringing back its serviceability. The state S_3 – servicing a technical object within the operation and maintenance system. An unserviceable technical object is affected by the actions aimed at bringing back its serviceability within a subsystem of assuring serviceability of the ana-

lysed vehicle operation and maintenance system or in another organizational unit.

Such a state occurs in the investigated system at the moment when such a damage occurred that cannot be removed by the technical service units outside the service station or the vehicle was directed to the service station for other reasons.

The space of the states S of the process consists of three states $S = \{i : i = 1, 2, 3\}$ in the investigated model. The states i of the analysed stochastic process correspond to the identified bus operation and maintenance states S_i .

On the basis of identification of a real urban bus transport operation and maintenance system the possible transitions between the identified bus operation and maintenance states have been determined. The changes of the operation and maintenance states are described with the stochastic process $\{X_t, t \in T\}$. The transition matrix $P = [p_{ij}]$, $i, j \in S$ of the Markov chain inserted into the process $\{X_t, t \in T\}$ has been assessed on the basis of the investigations realized in a real system of the urban bus operation and maintenance system (Table 1).

Table 1. Probability values p_{ij} , $i, j \in S$ of the process states changes $\{X_t, t \in T\}$

Tabela 1. Wartości prawdopodobieństw p_{ij} , $i, j \in S$ zmian stanów procesu $\{X_t, t \in T\}$

State <i>i</i>	<i>j</i>		
	1	2	3
1	0	0.81	0.19
2	0.9	0	0.1
3	1	0	0

When entering the state S_1 a decision may be made about the kind of the transport task (determined by the kind of so called communication route within the urban transport systems). Realization of the transport tasks over a respective route may have an influence on the frequency and kind of the damages to the vehicles realizing the transport tasks. It is related to the condition of the roads, number of passengers, route length, number of bus stops etc. The income per a time unit related to being in this state is determined on the basis of so called buskilometre and average velocity of the travel over the respective route. The buskilometre price is settled on tender basis. Therefore, it is of significant importance for the transport companies to determine the technical and economic aspects of the task realization over the individual communication routes.

In the state S_2 a decision may be made to send a technical service unit of a special type (the tech-

nical service units used in the analysed operation and maintenance system differ from one another by their type and technical equipment which is crucial for the scope of the realized repairs).

In the state S_3 a decision may be made about performing the service within a subsystem of assuring serviceability of the analysed vehicle operation and maintenance system or to have one of the external companies realize such a service. The following factors are related to the type of the unit realizing the service process: scopes of the realized actions, diagnostic apparatuses in use, or service duration, number of people needed to realize the service, etc. which in an obvious way affect the costs of the services being realized.

In order to simplify the consideration, it has been assumed hereunder that a decision regarding selection of the respective communication route may be made only in the first state. Four communication routes have been analysed. The decision k made when entering the state i has been denoted by a_{ik} , ($i, k \in N$). The decision space may be presented as $A = \{a_{11}, a_{12} \dots a_{14}\}$.

Chosen results of the simulation experiment

In order to simulate the operation and maintenance process (Markov decision process) and to evaluate the influence of the taken decisions on the flow and effectiveness of the process being realized within the investigation object a simulation algorithm has been elaborated and a computer calculation program has been developed.

The subject of the analysis in the considered example was transport realization effectiveness over the respective communication routes.

The simulation experiment has been carried out for all possible deterministic strategies for the considered example, assuming that there are 200 maintained and operated vehicles in the system and the duration of the simulation is equal to 31 days. The deterministic strategy is understood as a strategy to make the same decision in the respective state.

The chosen simulation results presented hereunder refer to four deterministic strategies and they are denoted respectively St-1, St-2 ... St-4. The strategy denoted with the code St- k , $k = 1, 2 \dots 4$ means that the decision k (a_{1k}) was made when entering the state 1.

The data needed for the simulation regarding the probability of transition between the states and the times of staying in the states (for the strategy denoted with the code St-1) have been preliminary assessed on the basis of the fragmentary investiga-

tion results. The remaining data needed for the simulation are hypothetical ones and serve to illustrate the considerations.

Table 2. Chosen simulation results – incomes generated within 24 hours by a system due to operation and maintenance of a single vehicle depending on the adopted strategy

Tabela 2. Wybrane wyniki symulacji – dochody uzyskane w ciągu jednej doby przez system w wyniku eksploatacji jednego pojazdu w zależności od przyjętych strategii

Strategy code	Average value	Standard deviation
St-1	260.75	35.25
St-2	325.25	31.95
St-3	363.05	31.67
St-4	396.02	32.55

The chosen simulation experiment results are presented in the tables 2 and 3. The results presented in the table 2 include average values of the incomes generated by the system due to realization of the respective strategies by a single vehicle calculated per one day of realization of the transport tasks. However, the table 3 includes number of entries to the respective states (jointly for all the vehicles) depending on the adopted strategies.

The figure 1 shows the flow of the momentary probability values $P_i(t) = P\{X_t = i\}$, $i \in S$, that means such probabilities that at the moment t the process $\{X_t, t \in T\}$ is in the state i .

Table 3. Chosen simulation results – number of entries to the respective states depending on the adopted strategy

Tabela 3. Wybrane wyniki symulacji – liczba wejść do poszczególnych stanów w zależności od przyjętych strategii

State	Strategy code			
	St-1	St-2	St-3	St-4
1	1643	4895	5518	6106
2	1388	4030	4595	5036
3	516	1370	1477	1663

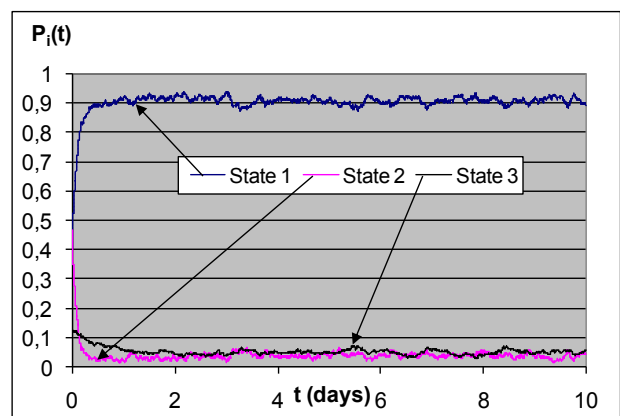


Fig. 1. Flow of the momentary probability values $P_i(t)$
Rys. 1. Przebieg chwilowych wartości prawdopodobieństw $P_i(t)$

Conclusion

The aim of the investigations was, among other things, to present possibility of applying chosen stochastic processes (that means Markov decision processes) for mathematical modelling the system and vehicle operation and maintenance process.

The investigated example of the model of the urban bus transport operation and maintenance process is significantly simplified (due to the nature of the elaboration). However, the presented way of building and analysing such models shows that it is possible to apply them to provide preliminary prognosis for a system state, and to evaluate the co-operation results of the primary (using) and the auxiliary (servicing) processes and supporting the decision makers in the process of making decisions regarding the control of the operation and maintenance process and the system in which it is being realized.

The elaborated simulation program has been designed in such a way to assure possibility of using it in as extensive as possible class of problems related to the operation and maintenance of the technical objects. Realization of the simulation experiments enables determination of the parameter values (including momentary values) describing the vehicle operation and maintenance process that cannot be determined by means of an analytic method (for the most complex models).

It seems that the analysis of the investigation results of the presented model types, for different values of their parameters (decision variants) assessed on the basis of the results coming from the operation and maintenance investigations may facilitate making reasonable decisions regarding the control of the vehicle operation and maintenance process. Such an analysis enables, among other things, to evaluate preliminarily both the technical and organizational aspects of the vehicle operation and maintenance and the economic ones of the realized transports, which may be the basis for affecting a real operation and maintenance system in order to secure its rational operation.

Because of the nature of the affair the mathematical models of the operation and maintenance processes, realized within complex systems, constitute a simplification of the real processes [6]. The consequence of the above is a necessity to formulate carefully the practical conclusions resulting from investigating these models. However, it does

not limit the purposefulness of creating and analysing models of this type. The analysis of the results obtained by investigating these models, for the model parameter values, determined on the basis of the results from the operation and maintenance investigations realized in a real transport system, makes it possible to formulate conclusions and evaluations both in terms of quality and (in limited extent) quantity.

The presented method to model and analyse the operation and maintenance process, due to the complex degree of the description generality and the system-based approach to the problem may be used to analyse an operation and maintenance process being realized in other operation and maintenance systems than an urban bus transport system.

References

1. LANDOWSKI B.: Method of determination values of the chosen decision variables to control rationally the operation and maintenance process in the transport system [doctoral thesis]. Academy of Technology and Agriculture, Bydgoszcz 1999.
2. LANDOWSKI B.: Operation and maintenance model of a certain class of the technical objects. Scientific Journals, 2000, 229, Mechanics 48.
3. WOROPAY M., LANDOWSKI B., NEUBAUER A.: Simulation of the method to optimise the operation and maintenance process of the buses in an urban transport system. International Scientific Conference "Transport of the 21st Century", section 5, Warsaw 2004.
4. WOROPAY M., LANDOWSKI B., NEUBAUER A.: Using semi-Markov decision processes to model and simulate a bus operation and maintenance process. Scientific Publication of the Polish Scientific Association of Automotive Engineering, Archive of Automotive Engineering, 2004, 7, 1.
5. LANDOWSKI B., WOROPAY M., NEUBAUER A.: Controlling reliability in the transport systems. Library of Maintenance Problems. Maintenance Technology Institute, Bydgoszcz–Radom 2004.
6. LANDOWSKI B., WOROPAY M., PERCZYŃSKI D.: Method of supporting the decision makers in the control process of a transport system operation. Systems Research Institute of the Polish Academy of Sciences, Warsaw 2004.
7. WOROPAY M., GRABSKI F., LANDOWSKI B.: Semi-Markov model of the vehicle operation and maintenance processes within an urban transport system. Scientific Publication of the Polish Scientific Association of Automotive Engineering, Archive of Automotive Engineering, 2004, 7, 3.

Recenzent:

*dr hab. inż. Andrzej Adamkiewicz, prof. AM
Akademia Morska w Szczecinie*