PROBLEMY MECHATRONIKI
UZBROJENIE, LOTNICTWO, INŻYNIERIA BEZPIECZEŃSTWA

ISSN 2081-5891 **8, 1 (27), 2017, 101-114**

PROBLEMS OF MECHATRONICS
ARMAMENT, AVIATION, SAFETY ENGINEERING

Effect of the Man-Weapon System on the Trajectory of a Projectile Fired from a Machine Pistol

Zbigniew DZIOPA[1*], Krzysztof ZDEB[2]

[1] *Kielce University of Technology,*
Faculty of Mechatronics and Mechanical Engineering
7 1000-lecia P.P. Avenue, 25-314 Kielce, Poland
[2] *Forensic Laboratory of the Provincial Police Command in Kraków*
109 Mogilska Street, 31-571 Kraków, Poland
[*] *Corresponding author's email address: zdziopa@tu.kielce.pl*

Received by the editorial staff on 13 June 2015.
The reviewed and verified version was received on 20 December 2016.

DOI 10.5604/01.3001.0009.8997

Abstract. The process of burst-firing two projectiles from a Skorpion vz. 61 7.65 mm calibre machine pistol was recorded during a test at a firing range operated by the Polish Police. This experimental test involved Browning 7.65 x 17 mm SR ammunition rounds. The shots were fired by a Police Special Forces operative. The shots were recorded with a Phantom v.9.1 slow-motion video camera, complete with the essential accessories. TEMA, a software suite dedicated to video recording analyses, was used to determine the initial kinematic parameters of flight for each of the two projectiles. By applying the theorem of momentum and spin change, a theoretical projectile flight model was developed with a respective simulation runtime in SciLab. The projectile flight results were verified with the experimental test results to validate the developed model.

This work has been compiled from the paper presented during the 11th International Armament Conference on Scientific Aspects of Armament and Safety Technology, Ryn, Poland, 19 to 22 September 2016

Having delivered a reliable projectile flight model, an analysis was carried out to study the effectiveness of the shots fired by the Police Special Forces operative towards a target 30 metres away. This paper presents a determined process of burst firing of two projectiles. It is also a representative example of repetitive shooting. The ultimate task of this research is to develop guidelines for the design of machine pistol dynamic properties that reduce the scatter of projectiles shot in series.
Keywords: mechanics, machine pistols, experimental research, projectile flight theoretical model, firing accuracy.

1. INTRODUCTION

The operator of a firearm has a critical impact on the process of firing projectiles from a machine pistol (MP). It is impossible to achieve full unification of the human body. A human being is a complex biological system whose performance depends on its actual psychophysical condition, acquired experience, predisposition, and knowledge. To minimise the negative impact of inexperienced shooters, the research involved an experienced Police Special Forces (PSF) operative, for whom firearms use is a critical part of profession, see Fig. 1.

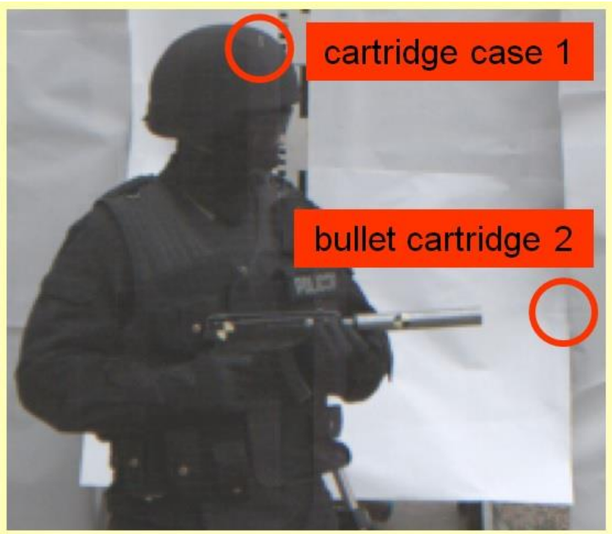

Fig. 1. PSF operative firing a burst from the Skorpion vz. 61 7.65 mm MP

In total, the PSF operative shot several dozen projectiles in various shooting stances. The firing of projectiles from the Skorpion vz. 61 7.65 mm calibre MP was recorded at a certified outdoor police firing range with a good target visibility and in still air [1]. The experimental firing tests were carried out with Browning 7.65 x 17 mm SR ammunition rounds.

The shooting process was recorded with a Phantom v.9.1 slow-motion digital camera, and the recorded video image was analysed with the TEMA software suite.

This paper presents a determined process of burst firing of two projectiles. Given the adopted mode of analysis, the paper lists a representative example of the projectile firing process. Based on the experimental data acquired from live-fire shooting, a theoretical model of the projectile flight was developed and validated. This stage of research is presented in another paper of the authors hereof. This paper focuses on the determination of the effect of the man-firearm kinetic system on the projectile flight [2]. The projectile scatter is an axiom dictated by practical experience in operation of firearms (not only by PSF operatives). The experimental tests conducted as described herein confirmed this. The theoretical projectile flight trajectory model was verified for a machine pistol in a state of static balance. In real-life conditions, the values of initial kinematic flight parameters of each projectile largely depend on the man-firearm kinematic system [3]. The dynamic characteristics of the man-firearm kinematic system largely determine the trajectory of each projectile. Consequently, it is the optimum concentration of projectiles at the target which is an estimator that permits evaluation of the accuracy of the man-firearm kinematic system [4]. This paper is a stage of an ultimate task of controlling the dynamic characteristics of automatic machine pistols as to achieve the spread of projectiles in fully automatic fire or burst fire.

2. EXPERIMENTAL TESTS

The experimental tests are detailed by the authors hereof in the paper titled "Empirical research of human-weapon system" (pending publishing). This paper only lists the information which is essential for completing the current research task. Fig. 2 shows the PSF operative in two distinct moments which are defined by the departure of projectiles 1 and 2 from the MP.

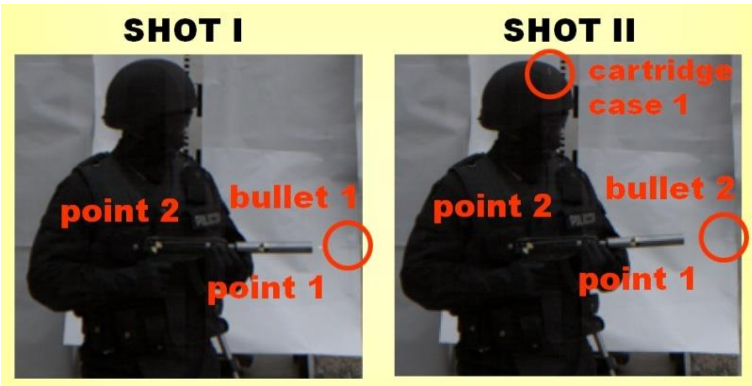

Fig. 2. PSF operative firing two shots from the Skorpion MP

The silencer and the breech casing had two markers applied, referred to as positions 1 and 2. The TEMA software was used to produce a series of variability of the kinematic values which defined the motion of both positions. To determine the kinematic values characteristic of the initial kinematic flight parameters of each projectile, relationships were applied that described the motion of position 2 and the rotary motion of the MP. Vector physical values were determined with the recorded series to describe the moment the projectile left the MP. The vector physical values included the effect of the man-firearm kinematic system on the line of departure position and the initial velocity of each projectile. The series of variability of the position 2 linear velocity vector components and the angle and angular velocity in the motion of MP tilt are shown in Figs. 3, 4, 5 and 6. The time points marked in the figures are as follows [5]:

a) First shot: I
   1) $t = 0.0448$ [s] – the breech block moves backwards,
   2) $t = 0.0462$ [s] – the cartridge 1 projectile leaves the barrel,
   3) $t = 0.0532$ [s] – the cartridge 1 case is ejected from the breech casing,
   4) $t = 0.0581$ [s] – the breech block reaches the back limit position,
   5) $t = 0.0749$ [s] – the breech block moves forward,
   6) $t = 0.0973$ [s] – the breech block reaches the front limit position;

b) Second shot: II
   7) $t = 0.1022$ [s] – the breech block moves backwards,
   8) $t = 0.1036$ [s] – the cartridge 2 projectile leaves the barrel,
   9) $t = 0.1134$ [s] – the cartridge 2 case is ejected from the breech casing,
   10) $t = 0.1162$ [s] – the breech block reaches the back limit position,
   11) $t = 0.1484$ [s] – the breech block moves forward,
   12) $t = 0.1743$ [s] – the breech block reaches the front limit position;

c) The shooter lowers the MP into the starting position
   13) t = 0.2604 [s] – the MP is in the starting position.

Given the focus of this paper, the most critical time points are 2, when projectile 1 leaves the MP barrel, and 8, when projectile 2 leaves the MP barrel.

The projectile linear velocity vector determined by the motion of the MP at the moment of projectile departure from the MP barrel was determined as follows:

$$\overrightarrow{v_p} = \left[v_{px}, v_{py}\right] \qquad (1)$$

$$v_{px} = v_{x2} - \mathrm{l_p}\dot{\vartheta}\sin\vartheta$$

$$v_{py} = v_{y2} + \mathrm{l_p}\dot{\vartheta}\cos\vartheta$$

with:

v_{x2}, v_{y2} – components of linear velocity vector;

$\vartheta, \dot{\vartheta}$ – angle and angle velocity during inclination of weapon;

l_p – distance measured between point 2 and muzzle.

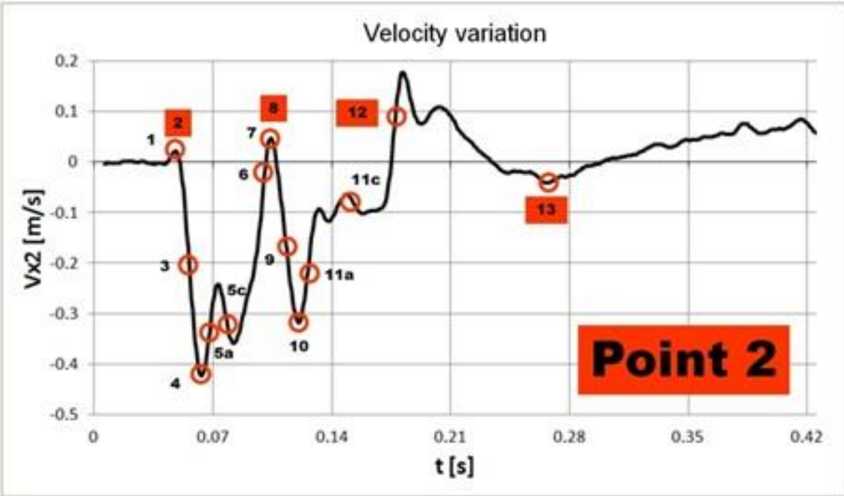

Fig. 3. Position 2 linear velocity towards the axis 0x

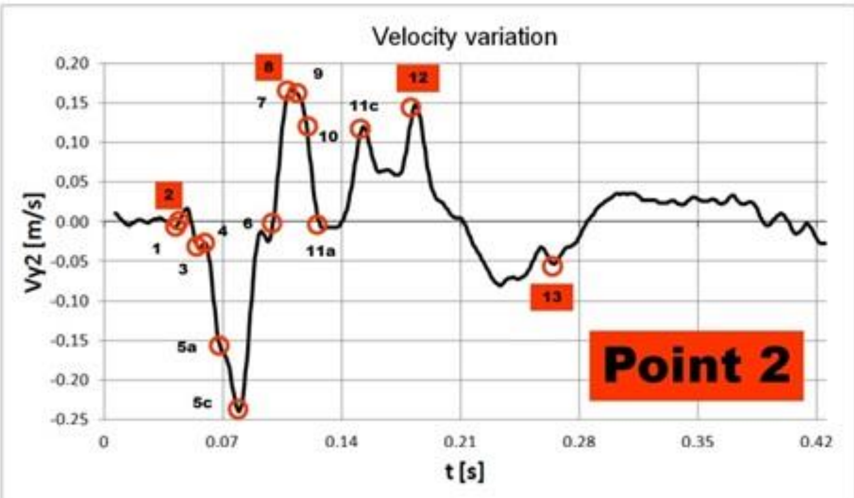

Fig. 4. Position 2 linear velocity towards the axis 0y

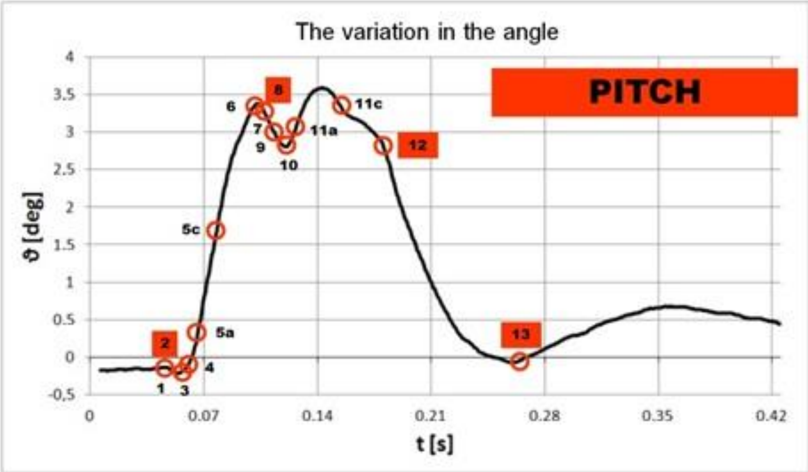

Fig. 5. Series of temporal variability of the MP tilt angle

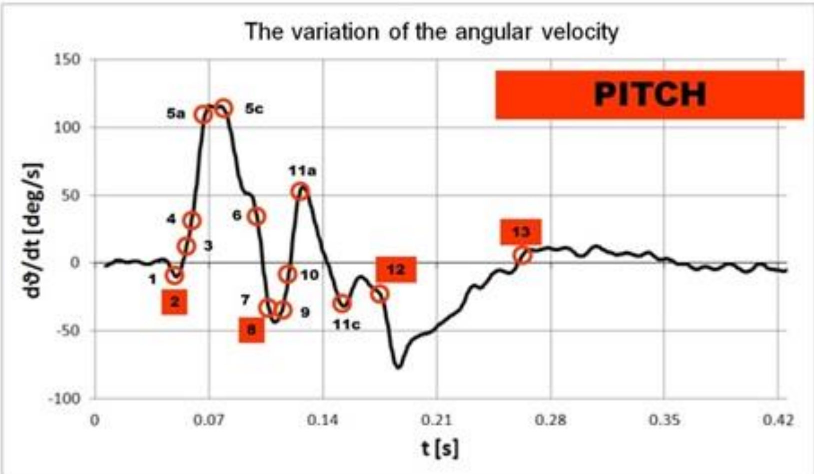

Fig. 6. Series of temporal variability of the angular velocity of the MP tilt motion

3. THEORETICAL MODEL

The developed theoretical model of the projectile flight trajectory had three translation equations, three equations of rotation about a fixed point, and twelve kinematic relationships [6-10].

Translation equations in the coordinate system $Sx_v y_v z_v$:

$$m\dot{v} = -mg\sin\gamma - P_{x_a}$$
$$mv\dot{\gamma} = -mg\cos\gamma + P_{y_a}^{\alpha} - P_{y_a}^{\beta\omega_x}$$
$$mv\dot{\chi}\cos\gamma = P_{z_a}^{\beta} + P_{z_a}^{\alpha\omega_x} \tag{2}$$

with:

$$P_{x_a} = c_{x_a}\frac{\rho v^2}{2}S$$
$$P_{y_a}^{\alpha} = c_{y_a}^{\alpha}\alpha\frac{\rho v^2}{2}S \qquad P_{z_a}^{\beta} = c_{z_a}^{\beta}\beta\frac{\rho v^2}{2}S$$
$$P_{y_a}^{\beta\omega_x} = c_{y_a}^{\beta\omega_x}\beta\omega_x\frac{\rho v}{2}SL \qquad P_{z_a}^{\alpha\omega_x} = c_{z_a}^{\alpha\omega_x}\alpha\omega_x\frac{\rho v}{2}SL$$

Equations of rotation about a fixed point in the coordinate system $Sxyz$:

$$I_{xx}\dot{\omega}_x + \left(I_{zz} - I_{yy}\right)\omega_y\omega_z = -M_x^{\omega_x}$$
$$I_{yy}\dot{\omega}_y + (I_{xx} - I_{zz})\omega_x\omega_z = -M_y^{\omega_y} + M_y^{\beta} + M_y^{\alpha\omega_x} \tag{3}$$
$$I_{zz}\dot{\omega}_z + \left(I_{yy} - I_{xx}\right)\omega_x\omega_y = -M_z^{\omega_z} + M_z^{\alpha} - M_z^{\beta\omega_x}$$

with:

$$M_x^{\omega_x} = m_x^{\omega_x}\omega_x\frac{\rho v}{2}SL^2 \qquad M_y^{\omega_y} = m_y^{\omega_y}\omega_y\frac{\rho v}{2}SL^2 \qquad M_z^{\omega_z} = m_z^{\omega_z}\omega_z\frac{\rho v}{2}SL^2$$
$$M_y^{\beta} = m_y^{\beta}\beta\frac{\rho v^2}{2}SL \qquad M_z^{\alpha} = m_z^{\alpha}\alpha\frac{\rho v^2}{2}SL$$
$$M_y^{\alpha\omega_x} = m_y^{\alpha\omega_x}\alpha\omega_x\frac{\rho v}{2}SL^2 \qquad M_z^{\beta\omega_x} = m_z^{\beta\omega_x}\beta\omega_x\frac{\rho v}{2}SL^2$$

Kinematic relationships with transformation $R_{\chi\gamma}$:

$$v_{x_g} = v\cos\gamma\cos\chi$$
$$v_{y_g} = v\sin\gamma \tag{4}$$
$$v_{x_g} = -v\cos\gamma\sin\chi$$

Kinematic relationships with transformation $R_{\psi\vartheta\varphi}$:

$$v_x = v_{x_g}a_{11} + v_{y_g}a_{12} + v_{z_g}a_{13}$$
$$v_y = v_{x_g}a_{21} + v_{y_g}a_{22} + v_{z_g}a_{23} \tag{5}$$
$$v_z = v_{x_g}a_{31} + v_{y_g}a_{32} + v_{z_g}a_{33}$$

with:

$a_{11} = \cos\vartheta\cos\psi$
$a_{12} = \sin\vartheta$
$a_{13} = -\cos\vartheta\sin\psi$
$a_{21} = \sin\varphi\sin\psi - \cos\varphi\sin\vartheta\cos\psi$
$a_{22} = \cos\varphi\cos\vartheta$
$a_{23} = \sin\varphi\cos\psi + \cos\varphi\sin\vartheta\sin\psi$
$a_{31} = \cos\varphi\sin\psi + \sin\varphi\sin\vartheta\cos\psi$
$a_{32} = -\sin\varphi\cos\vartheta$
$a_{33} = \cos\varphi\cos\psi - \sin\varphi\sin\vartheta\sin\psi$

Kinematic relationships with transformation $R_{\beta\alpha}$:

$$\begin{aligned} v_x &= v\cos\alpha\cos\beta \\ v_y &= -v\sin\alpha\cos\beta \\ v_z &= v\sin\beta \end{aligned} \tag{6}$$

Kinematic relationships with transformation $R_{\psi\vartheta\varphi}$:

$$\begin{aligned} \dot{\psi} &= \frac{\omega_y\cos\varphi - \omega_z\sin\varphi}{\cos\vartheta} \\ \dot{\vartheta} &= \omega_y\sin\varphi + \omega_z\cos\varphi \\ \dot{\varphi} &= \omega_x - (\omega_y\cos\varphi - \omega_z\sin\varphi)\mathrm{tg}\vartheta \end{aligned} \tag{7}$$

with:

- $Sx_vy_vz_v$ – coordinate system of the projectile flight trajectory,
- $Sxyz$ – projectile coordinate system,
- v – projectile linear velocity,
- γ, χ – respectively: trajectory tilt angle and trajectory sense angle,
- $\omega_x, \omega_y, \omega_z$ – projectile angular velocity vector $\vec{\omega}$ components,
- $v_{x_g}, v_{y_g}, v_{z_g}$ – projectile linear velocity vector $\vec{v}$ components in the normal terrestrial coordinate system,
- v_x, v_y, v_z – projectile linear velocity vector $\vec{v}$ components in the projectile coordinate system,
- α, β – respectively: attack angle and planning angle,
- ϑ, ψ, φ – respectively: projectile tilt angle, projectile yaw angle, and projectile bank angle,
- m – projectile weight,
- I_{xx}, I_{yy}, I_{zz} – projectile moments of inertia relative to the major central axes of inertia,
- g – gravitational acceleration,

$P_{x_a}, P_{y_a}^{\alpha}, P_{y_a}^{\beta\omega_x}, P_{z_a}^{\beta}, P_{z_a}^{\alpha\omega_x}$ – components of the major vector of the aerodynamic forces loading the projectile,
$M_x^{\omega_x}, M_y^{\omega_y}, M_y^{\beta}, M_y^{\alpha\omega_x}, M_z^{\omega_z}, M_z^{\alpha}, M_z^{\beta\omega_x}$ – components of the major moment of the aerodynamic forces loading the projectile.

The following problems were defined and solved to develop the physical flight model of the projectile fired from the Skorpion vz. 61 7.65 mm calibre MP:

1. Inertial components
   1.1. Projectile as a rigid body.
2. Cartesian orthogonal dextrogyrate coordinate systems
   2.1. Galilean system,
   2.2. Non-inertial coordinate systems,
   2.3. Isometric transformation of coordinate systems.
3. Space
   3.1. Three-dimensional Euclidian space,
   3.2. Homogeneous gravitational field,
   3.3. The Earth's atmosphere.

The developed physical projectile model consisted of a rigid body with a mass and moments of inertia I_{xx}, I_{yy}, I_{zz}. The formulation of the projectile flight model, physical phenomena were considered as imposed by the projectile motion in the Earth's gravitational field and atmosphere. The Magnus effect and the gyroscopic stability process were also modelled. The model shown has six degrees of freedom.

The theoretical projectile flight model was built by applying the theorem of momentum and spin change. The projectile flight results produced by analysing the results of the completed simulations were verified with the experimental test results. The procedure enabled validation of the developed theoretical model. This paper discusses the relationships and parameters which assured the continuity of dissertation [11-15].

4. NUMERICAL SIMULATION

The mathematical model of the projectile flight was used to develop a software runtime. The runtime was programmed in SciLab and enabled an in-depth analysis of the projectile flight in virtual space [16-18]. The following shows examples of the results of the numerical simulation of firing two projectiles in burst from the Skorpion vz. 61 7.65 mm calibre MP. The studied projectiles were components of the Browning 7.65 x 17 mm SR cartridges.

The charts plotted in Figs. 7 to 9 concern the firing of projectiles 1 and 2 at a target 30 m away.

Figure 7 shows a selection of kinematic values which characterized the translations of projectiles 1 and 2.

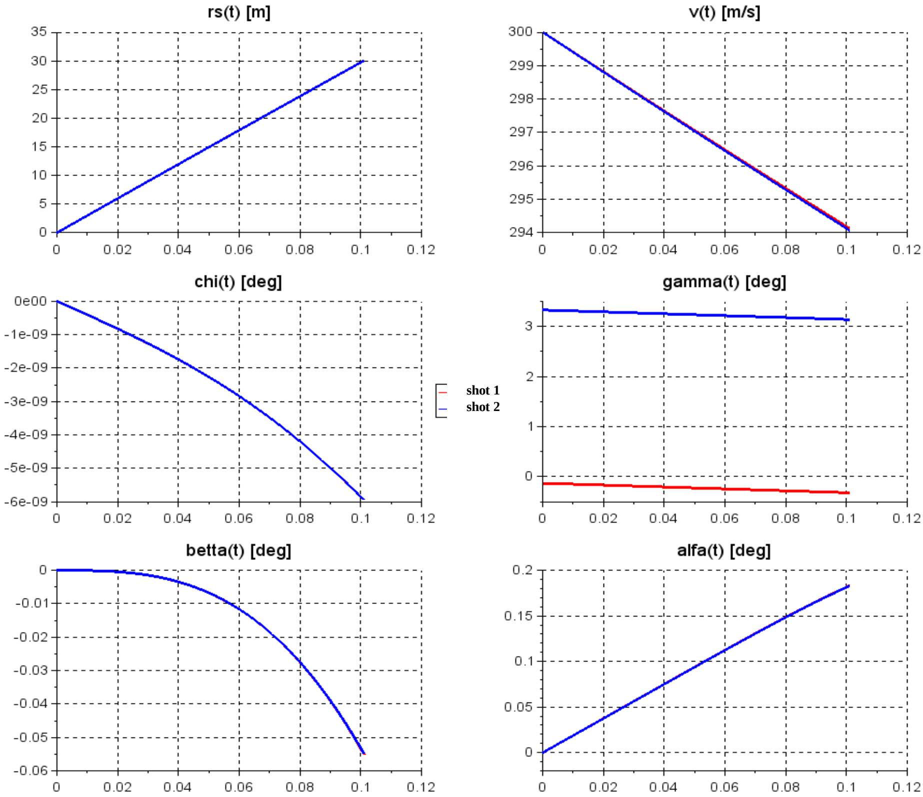

Fig. 7. Selection of kinematic values which characterized the translations of projectiles 1 and 2

The chart plots show the series of variability vs. time t in seconds for the following kinematic values:

- rs(t) – absolute value of the vector $\vec{r_S}$ which determines the projectile centre of mass S in the Earth's topocentric coordinate system $O_g x_g y_g z_g$. At the initial time t = 0[s], position S was aligned with position O_g.
- v(t) is v in the equations, i.e. the projectile linear velocity.
- gamma(t), chi(t) are γ, χ in the equations, i.e. the trajectory tilt angle and the trajectory sense angle, respectively.
- alpha(t), beta(t) are α, β in the equations, i.e. the attack angle and the planning angle.

Figure 8 shows a selection of kinematic values which characterized the rotation around a fixed point of projectiles 1 and 2.

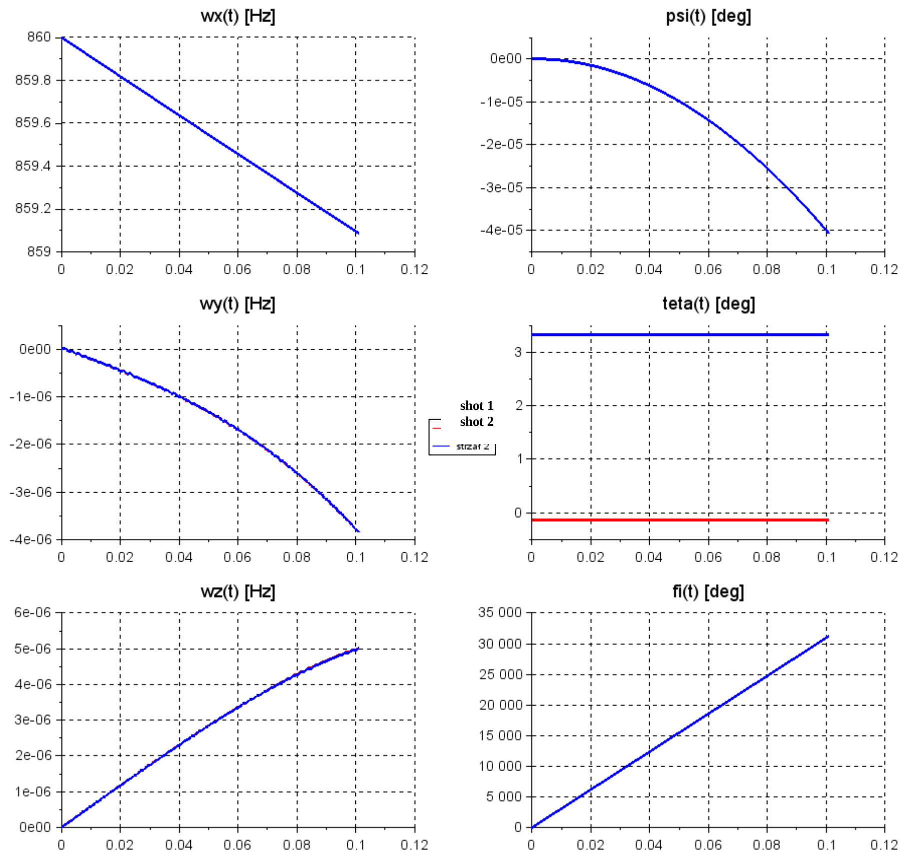

Fig. 8. Selection of kinematic values which characterized the rotation around a fixed point of projectiles 1 and 2

The chart plots show the series of variability vs. time t in seconds for the following kinematic values:

- wx(t), wy(t), wz(t) are $\omega_x, \omega_y, \omega_z$ in the equations, i.e. the projectile angular velocity vector components $\vec{\omega}$.
- theta(t), psi(t), phi(t) are ϑ, ψ, φ in the equations, i.e. the projectile tilt angle, the projectile yaw angle, and the projectile bank angle, respectively.

Figure 9 shows the hodograph of the vector $\vec{r_s}$ projected on two planes that were determined in the Earth's topocentric coordinate system $O_g x_g y_g z_g$ for the motion of projectiles 1 and 2.

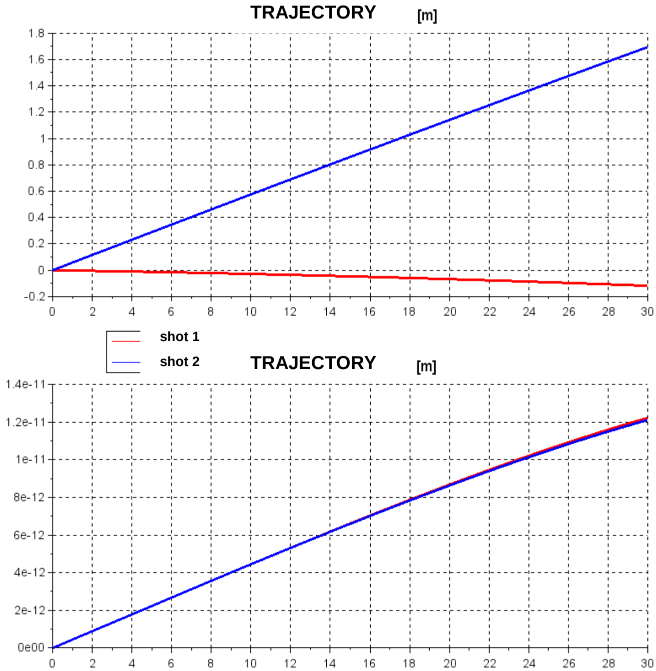

Fig. 9. Flight trajectories of projectiles 1 and 2 projected on two planes

The chart plots show the projectile flight trajectory determined on planes $O_g x_g y_g$ and $O_g x_g z_g$. The figure designations represent the following functions:
$yg(xg)$ – projectile flight trajectory in plane $O_g x_g y_g$ (top),
$zg(xg)$ – projectile flight trajectory in plane $O_g y_g z_g$ (bottom).

5. SUMMARY

The analysis of the experimental results confirmed the opinion of the users of the Skorpion vz. 61 7.65 mm calibre MP by which this firearm has a poor effect on the target with fully automatic fire. The paper shows a representative example of firing projectiles by a PSF operative, an individual with expert training and experience in operation of firearms. This example of firing two projectiles in burst shows that the difference between the hit locations on a target 30 m from the shooter was 1.8 m in the vertical plane. The cause for this staggering spread were the unfavourable initial flight conditions of both projectiles. The conditions were represented by the initial kinematic flight parameters, which depended on the dynamics of the man-firearm kinematic system. This paper is a stage of an ultimate task of adjusting the dynamic characteristics of automatic machine pistols.

The presented projectile flight model will help verify the spread of projectiles in full automatic firing in virtual space. This in turn will help optimize firearm parameters and verify the projectile spread without any need for prototype building or live-fire testing.

REFERENCES

[1] Białczak Bogusław.1988. *Podstawy budowy broni lufowej*. Course Book No. 155, Kielce.

[2] Ewertowski Janusz. 2007. „Analiza siły oddziaływania broni ramiennej na strzelca w czasie strzału”. *Biuletyn Wojskowej Akademii Technicznej* LVI (1) : 207-221.

[3] Suchocki Cyprian, Janusz Ewertowski. 2015. „Modeling and numerical simulation of semi-automatic pistol Dynamics”. *Journal of Theoretical and Applied Mechanics* 53 (1) : 81-91.

[4] Ejsmont Jerzy A. 2012. *Celność broni strzeleckiej*. *Praktyczny poradnik*. Warszawa: Wydawnictwa Komunikacji i Łączności.

[5] Dziopa Zbigniew, Krzysztof Zdeb. 2016. „Model teoretyczny układu człowiek-broń”. *Zeszyty Naukowe Akademii Marynarki Wojennej* LVII (2) : 53-64.

[6] Gacek Józef . 1997 & 1998. *Balistyka zewnętrzna*, Część I *Modelowanie zjawisk balistyki zewnętrznej i dynamiki lotu*, Część II *Analiza dynamicznych właściwości obiektów w locie*. Warszawa: Wydawnictwo WAT.

[7] Szapiro J. 1956 . *Balistyka zewnętrzna*. Warszawa: Wydawnictwo MON.

[8] Boiffier Jean-Luc. 1998. *The dynamics of flight – the equations*. John Wiley & Sons, Chichester-New York-Weinheim-Brisbane-Singapore-Toronto.

[9] Niczyporuk Jan, Stefan Wiśniewski. 1985. *Balistyka zewnętrzna*, Część I *Podstawy formułowania opisu matematycznego ruchu pocisków*. Warszawa: Wydawnictwo WAT.

[10] Sibilski Krzysztof. 2004. *Modelowanie i symulacja dynamiki ruchu obiektów latających*. Warszawa: Oficyna Wydawnicza MH.

[11] Dubiel Stanisław. 1984, 1985. *Dynamika lotu*, Część I *Aerodynamika*, Część II *Mechanika lotu*. Warszawa: Wydawnictwo WAT.

[12] Engel Zbigniew, Józef Giergiel. 1979, 1980. *Mechanika ogólna*, Część I *Statyka*, *Kinematyka*, Część II *Dynamika*. Course Books No. 718 and 719. Kraków: Wydawnictwo AGH.

[13] Gacek Józef, Jerzy Maryniak. 1987. „Modelowanie własności dynamicznych brył obrotowych miotanych z ruchomych obiektów”. *Biuletyn Wojskowej Akademii Technicznej*, 81-96.

[14] Gantmacher F.R. 1970. *Lectures in analytical mechanics*. Translated from the Russian by G. Yankovsky, Moscow: Mir Publishers.
[15] Osiecki Jan, Zbigniew Koruba. 2007. *Elementy mechaniki zaawansowanej*. Kielce: Wydawnictwo Politechniki Świętokrzyskiej.
[16] Giergiel Józef, Tadeusz Uhl. 1991. *Identyfikacja układów mechanicznych*, Warszawa: Wydawnictwo PWN.
[17] Górecki Henryk. 1993. *Optymalizacja systemów dynamicznych*. Warszawa: Wydawnictwo PWN.
[18] Mitkowski Wojciech. 1984. *Stabilizacja systemów dynamicznych*. Kraków: Wydawnictwo AGH.

Wpływ układu człowiek-broń na lot pocisku wystrzelonego z pistoletu maszynowego

Zbigniew DZIOPA[1], Krzysztof ZDEB[2]

[1] *Politechnika Świętokrzyska, Wydział Mechatroniki i Budowy Maszyn*
25-314 Kielce, al. 1000-lecia P.P.7
[2] *Laboratorium Kryminalistyczne Komendy Wojewódzkiej Policji w Krakowie*
31-571 Kraków, ul. Mogilska 109

Streszczenie. Na strzelnicy policyjnej przeprowadzono rejestrację procesu wystrzelenia z pistoletu maszynowego Skorpion wz. 61 kal.7.65 mm ogniem seryjnym dwóch pocisków. Do badań empirycznych użyto amunicji 7.65 x 17 mm SR Browning. Strzały oddane zostały przez policjanta ze służb specjalnych. Ich rejestrację przeprowadzono przy wykorzystaniu szybkiej kamery cyfrowej Phantom v.9.1 wraz z niezbędnym oprzyrządowaniem. Przy wykorzystaniu specjalistycznego programu TEMA służącego do analizy zarejestrowanego obrazu, określono początkowe kinematyczne parametry lotu każdego z pocisków. Wykorzystując twierdzenie o zmianie pędu i krętu opracowano model teoretyczny lotu pocisku oraz program symulacyjny w systemie Scilab. Zweryfikowano wyniki lotu na podstawie danych z badań empirycznych, co pozwoliło na walidację opracowanego modelu. Dysponując wiarygodnym modelem lotu pocisku, przeprowadzono analizę skuteczności strzelania wykonanego przez policjanta ze służb specjalnych do celu znajdującego się w odległości 30 m. W artykule przedstawiony jest proces zdeterminowany wystrzeliwania ogniem seryjnym dwóch pocisków. Stanowi on reprezentatywny przykład wielokrotnego procesu wystrzeliwania. Docelowym zadaniem podjętych rozważań jest opracowanie wytycznych zmierzających do takiego kształtowania właściwości dynamicznych pistoletów automatycznych, aby zmniejszyć rozrzut wystrzeliwanych ogniem seryjnym pocisków.
Słowa kluczowe: mechanika, pistolet maszynowy, badania empiryczne, model teoretyczny lotu pocisku, celność