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Cross-Section Changes of Rates of Return on the Shares Traded on the Warsaw Stock Exchange

Introduction

The purpose of this paper is to demonstrate the possibilities of pricing of shares using, as an example, securities listed on the Warsaw Stock Exchange (WSE), in light of the ICAPM theory. With this in mind, an aggregated factor line model is constructed.[1] This model attempts to price shares listed on the Polish market in 1995–2005, the years preceding Poland's accession to the European Union.[2] Theoretical procedure for the description of rates of return combines Fama and French research [1992, 1993, 1995, 1996], Campbell's [1996] indications[3] and author's own considerations. This paper, however, differs from analysis methods hitherto used, in that the accepted model factors take into account both the known and unknown future parameters of various methods of investment. The selected explanatory variables are based on values of the FUN functional, defined in section 1 (see equation (2)). The FUN takes into account both share assessment and share pricing factors of listed companies.

The hypothetical investor could successively attain an above–the–average rate of return, on condition that the future state of the economy is anticipated correctly. If investments performed on the basis of FUN permit the attainment of an above–the–

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[1] The term "aggregated" relates to model factors, which in a complex manner depend on the dynamics of change of financial results and BV/MV and E/MV indicators.

[2] The period 1991–1994 is omitted due to the lack of data.

[3] Campbell [1996] points out that the empirical applications of ICAPM should not involve the choice of important macroeconomic variables. Factors belonging to this model should relate to innovative variables, which forecast future and various possible ways of investing. On this basis Petkova [2006] proposes empirical implementation of the ICAPM model, in which the factors are market rate of return and four variables forecasting future means of investing. Petkova demonstrates that the proposed model better fulfils share valuation tests than the Fama and French model.

average rate of return, one may assume that there is a dependence between FUN and the resultants of known and unknown investment methods, which anticipate future states[4]. In other words, the FUN functional should define the variables of the state, permitting the securing of future payments out on performed investments.

Tests carried out by Urbanski [2006] demonstrate the possibility of taking investment decisions that permit the attainment of above–the–average rates of return on the basis of FUN at the WSE in 1996–2004. As a result, it was conjectured that there is a relationship between FUN and known and unknown variables forecasting the outcome of changing future investment methods.

The proposed model is presented in two main versions: as a two-factor model and a three-factor model. Section 1 discusses theoretical procedure for the description of rates of return and methods of forming and juxtaposing data. Section 2 describes the data and their transformation. Section 3 defines price equilibrium in the light of the proposed two-factor model. Section 4 defines equilibrium in the light of the proposed, three-factor model. Section 5 estimates the impact of characteristics of formed portfolios on the explanatory strength of the proposed model. Section 6 presents a visual assessment of the analysed ICAPM versions; application is made of the Jagannathan and Wang [1996] graphic model. The last part of the paper presents a summary and conclusions.

1. Econometric approach

The equilibrium analysis, performed in this paper, assumes that the rate of return on shares is described by the discreet ICAPM version of Merton [1973]. The author accepts and analyses ICAPM implementations which constitute an aggregated factor line model and attempts to describe rates of return on the basis of multi–dimensional indicator structure. This indicator, in keeping with Campbell's [1996] indications, takes into account assessment and pricing factors of securities and market factors in the light of changing future investment methods. These factors constitute explanatory variables of the proposed model.

Values of rates of return of shares are recorded in keeping with the matrix line regression equation (1)

$$\mathbf{r} = \mathbf{G}\mathbf{b} + \mathbf{e}, \tag{1}$$

where $\mathbf{r}$ is the vector of $m \cdot n$ rates of return, $\mathbf{G}$ is the $m \cdot n \times k$ matrix of aggregated explanatory variables, $\mathbf{b}$ is the vector of k regression coefficients and $\mathbf{e}$ is the vector of $m \cdot n$ residual components, m is the number of analysed portfolios during one investment period, n is the number of investment periods, k is the number of analysed explanatory variables plus 1.

[4] The author understands "above–the average rate of return" as a rate of return which is significantly higher than the average rate of return on the WIG (Index of the Warsaw Stock Exchange).

Equation (1) constitutes the econometric linear model, constructed on the basis of time–cross–section data. This model, in general terms, may be treated as an APT or ICAPM model, depending on the character of accepted explanatory variables. During each t period of time m portfolios are analysed, giving a rate of return for accepted explanatory variables. The regression coefficients vector is marked out on the basis of $m \cdot n$ equations.

In my research, an attempt is made to price securities on the basis of ICAPM theory. The explanatory variables of the aggregated model are constructed on the basis of the market rate of return RM, functional FUN presented by equation (2), and NUM and DEN functions constituting correspondingly the numerator and denominator of FUN. Research carried out by Urbanski [2006] shows that portfolios generated on the basis of FUN maximization permited the attainment of above-the--average rates of return on the Polish market in 1996–2004. In comparison to the work carried out by Fama and French [1993, 1995, 1996] and Lakonishok, Shleifer and Vishny [1994][5], one may assume that FUN may constitute good basis for the general description of rates of return.

$$\mathrm{FUN} = \frac{nor(\mathrm{ROE}) \cdot nor(\mathrm{AP}) \cdot nor(\mathrm{AZO}) \cdot nor(\mathrm{AZN})}{nor(\mathrm{MV/E}) \cdot nor(\mathrm{MV/BV})}\ , \tag{2}$$

where

$$\begin{gathered} \mathrm{ROE} = \mathrm{F}_1;\ \mathrm{AP} = \mathrm{F}_2 = \frac{\sum_{t=1}^{i} \mathrm{S}(Q_t)}{\sum_{t=1}^{i} \overline{\mathrm{S}(nQ_t)}};\ \mathrm{AZO} = \mathrm{F}_3 = \frac{\sum_{t=1}^{i} \mathrm{ZO}(Q_t)}{\sum_{t=1}^{i} \overline{\mathrm{ZO}(nQ_t)}}; \\ \mathrm{AZN} = \mathrm{F}_4 = \frac{\sum_{t=1}^{i} \mathrm{ZN}(Q_t)}{\sum_{t=1}^{i} \overline{\mathrm{ZN}(nQ_t)}},\ \mathrm{MV/E} = \mathrm{F}_5;\ \mathrm{MV/BV} = \mathrm{F}_6 . \end{gathered} \tag{3}$$

Variables F_j (j = 1, …, 6) are transformed to standardised numbers ranging $<a_j; b_j>$, in keeping with equation (4):

$$nor(\mathrm{F}_j) = \left[a_j + (b_j - a_j) \cdot \frac{\mathrm{F}_j - c_j \cdot \mathrm{F}_j^{\min}}{d_j \cdot \mathrm{F}_j^{\max} - c_j \cdot \mathrm{F}_j^{\min} + e_j} \right] . \tag{4}$$

In equation (3), the variables are defined as follows: ROE is return on equity $\sum_{t=1}^{i} \mathrm{S}(Q_t)$, $\sum_{t=1}^{i} \mathrm{ZO}(Q_t)$, $\sum_{t=1}^{i} \mathrm{ZN}(Q_t)$ are values accumulated from the beginning of the year (or for the last four quarters) of: net sales revenue, operating profit and net profit of the and of quarter i; $\sum_{t=1}^{i} \overline{\mathrm{S}(nQ_t)}$, $\sum_{t=1}^{i} \overline{\mathrm{ZO}(nQ_t)}$, $\sum_{t=1}^{i} \overline{\mathrm{ZN}(nQ_t)}$ are average

[5] Lakonishok, Shleifer and Vishny claim that by arranging shares according to sales dynamics and BV/MV and E/MV it is possible to better distinguish strong shares from weak shares, attaining considerable spread of average returns.

values, accu- mulated from the beginning of the year (or for the last four quarters) of: net sales revenue, operating profit and net profit at the end of quarter i over the last n years; MV/E is the relation of the current share price to the sum of net profits over the last four quarters per single share; MV/BV is the relation of the current share price to the average book value per single share over the last four quarters; a_j, b_j, c_j, d_j, e_j are variation parameters or may be accepted arbitrarily[6].

The constructed functional is dependent on company standing indicators, occurring in the numerator and company market pricing indicators ascribed to the denominator. The selection of companies to the portfolio is based on the criterion which is defined as optimal FUN value calculated for all listed companies. The constructed portfolio may contain open long positions or open short positions. For long positions the portfolio includes those companies for which FUN values are the highest. For short positions those companies are chosen for which FUN values are the lowest.

F_j variables are functions of company evaluation indicators (for $j = 1, \ldots, 4$) and functions of pricing indicators (for $j = 5, 6$). A given F_j may change considerably, thus influencing FUN value. For this reason it is necessary to transform all F_j variables to match appropriately defined standardised areas, in accordance with equation (4). Parameters a_j, b_j, c_j, d_j, e_j define the border of the F_j variable standardised area for six FUN variables (ROE, sales revenue, operating profit, net profit, MV/E, MV/BV). These parameters, for each of the six F_j, variables, are variation parameters in nature. The constructed portfolio contains N companies, for which FUN assumes N highest values (for long positions). Change of chosen variation parameters usually changes FUN values but does not always change the list of companies (or their order), which correspond to N FUN maximum values.

The solution of the model is found in two stages. The first stage involves searching the values of a_j, b_j, c_j, d_j, e_j parameters for which the value of the objective function will be optimal. In the second stage there is selection of N portfolio companies which correspond to N maximum (minimum) FUN values, calculated against optimal variation parameter values.

The objective function, dependent on a_j, b_j, c_j, d_j, e_j variation parameters, may be defined by the user. For example, this may be the effective rate of return on the N securities portfolio. In each case the analysis covers the concerns chosen historical periods.

Functional FUN is a gauge of securities which are assessed well by NUM and at the same time priced lowly by DEN. FUN contains a clear economic interpretation and may constitute a criterion for selecting securities for the portfolio. The investment is more attractive if the FUN value is greater.

[6] FUN discretization is performed for the forming of test portfolios. The modelling of investment is not the main purpose of this paper. For this reason it is not necessary to look for optimal variation parameter values. In modelling equilibrium on the share market it is possible to assume identical values for all parameters. In this paper the following parameter values are accepted arbitralily: $a_j = 1, b_j = 2, c_j = 1, d_j = 1, e_j = 0$, which leads to the transformation of the F_j ($j = 1, \ldots, 6$) to $\langle 1, 2 \rangle$.

The response variable is seen as an excess over the risk-free rate in the tested portfolios. The explanatory variables of (1) for portfolio i and period t solve (5):

$$x_{1it} = \text{RMO1}_t; x_{2it} = \text{RMO2}_t; x_{3it} = \text{HMLF}_t; x_{4it} = \text{HMLN}_t; x_{5it} = \text{LMHD}_t \quad (5)$$

where RMO1_t is the market factor, defined as excess market rate of return over the risk-free rate, not correlated with HMLF_t; RMO2_t is the market factor, defined as excess market rate of return over the risk-free rate, not correlated with HMLN and LMHD; HMLF_t is the difference between the rates of return from the portfolio with the highest and lowest FUN_t value in the period t, HMLN_t is the difference between the rates of return from the portfolio with the highest and lowest NUM_t value in the period t; LMHD_t is the difference between the rates of return from the portfolio with the lowest and highest DEN_t value in the period t. The market rate of return (RM) is evaluated by the percentage rate of return on the WIG index of the Warsaw Stock Exchange. The risk-free rate of return (RF) is evaluated by the 91-day Treasury bill rate of return.

FUN, NUM and DEN values are calculated for all analysed securities at the beginning of each investment period, in which the rate of return is to be described, according to equation (1). Investment periods must correspond to the analysed reporting periods; they cannot be shorter than quarterly periods and they cannot overlap. In reality, despite the need to accept periods which do not overlap, one may presume that there is a relationship of explanatory variables in neighbouring periods. FUN, NUM and DEN depend on the accumulated fundamental results from the beginning of the year, the ratio of market share price to profit per share from the last four quarters, the ratio of market share price to company book value, and the value of the ROE indicator.

2. Data and model digitization

The author analyses changes in rates of return on the basis of securities listed on the WSE main market for 1995–2005, with the exception of companies characterised by negative book value during the last reporting period. Data referring to the fundamental results of the inspected companies have been taken from the database drawn up by Notoria Serwis Co. Ltd. Data for the rates of return on securities was provided by the Warsaw Stock Exchange.

Analysis is carried out on quarterly rates of return of hypothetical portfolio investments performed on the day in which companies had the obligation to publish quarterly financial reports (in the analysed period this is between 40 and 60 days following the end of the quarter). FUN, NUM and DEN values are calculated for all analysed securities, whilst explanatory variables (5) are assigned to portfolios in which the companies are formed.

The inspected securities are divided into quintile portfolios built on the basis of FUN, NUM and DEN values. FUN, NUM and DEN function values for portfolios constitute average arithmetical values of these functions for various securities entering the portfolio. Rates of return on given portfolios are average weighted market capitalizations. Table 1 presents average rates of return values, average FUN, NUM and DEN values, average capitalization from 36 periods and their standard deviations, for given quintiles, constructed in terms of FUN, NUM and DEN.

Table 1

Data breakdown for the tested portfolios

Portfolios	Rate of return, %		Function value		Number of companies	
			Average	Std. dev.		
Panel FUN						
	Average	*p–value*[a]	FUN		Capitalization	
					Average, PLN '000s	*p–value*[a]
Max FUN	6.27	–	3.85	0.85	944 546	–
4	0.89	0.07	2.51	0.57	1 231 159	0.06
3	0.33	0.05	1.84	0.53	1 246 660	0.06
2	–3.57	0.00	1.11	0.56	635 122	0.02
Min FUN	–310	0.01	0.30	0.27	262 033	0.00
Panel NUM						
			NUM		Average, PLN '000s	*p-value*[a]
Max NUM	4.63	–	5.06	1.17	1 155 434	-
4	1.55	0.21	3.37	0.79	1 480 839	0.07
3	–0.40	0.08	2.55	0.63	1 054 739	0.32
2	–2.65	0.03	1.72	0.63	389 665	0.00
Min NUM	–1.95	0.07	0.60	0.42	231 053	0.00
Panel DEN						
			DEN		Average, PLN '000s	*p-value*[a]
Max DEN	0.51	–	3.16	0.42	464 363	-
4	–1.33	0.33	2.07	0.57	1 076 285	0.00
3	0.25	0.47	1.48	0.30	1 288 670	0.00
2	3.18	0.24	1.27	0.11	1 032 165	0.00
Min DEN	5.72	0.09	1.13	0.04	428 684	0.35

The table presents the average rate of return, FUN, NUM and DEN values and capitalization for tested portfolios. In the FUN panel the portfolios are classified with respect to the FUN value, in the NUM panel the portfolios are classified with respect to the NUM value, in the DEN panel the portfolios are classified with respect to the DEN value. Std. dev. is standard deviation. Negative–BV stocks are excluded from the portfolios. The sample period is from 1996 to 2005, 36 quarters. [a]This refers to the test of two averages for the accepted H_0 hypothesis that the expected value of the rate of return (or capitalization) from quintile 5 is equal to the expected value of the rate of return (or capitalization) from quintile *i* (on assumption of the alternative hypothesis that the quoted expected values are different).

Source: own research.

Capitalization values of portfolios formed on FUN, NUM and DEN are the highest for middle quintiles. Average capitalization of the portfolio with the highest FUN value amounts to PLN 944,546 thousand and significantly differs from the capitalization of the previous quintiles.

Correlation coefficient modules between explanatory variables do not exceed 0.38 ($HMLN_t$ i $HMLF_t$ are not jointly applied). Correlation coefficient modules between the response variable and explanatory variables range between 0.28 and 0.92. Correlation coefficient modules between the rate of return on the portfolio with *i* FUN value in the period *t*, r_{it} and $FUN_{i,t-1}$, $DEN_{i,t-1}$, $NUM_{i,t-1}$ functions, range from 0.13 to 0.22 [see Urbański 2008, p. 824, tab. 2].

The correlation of the $RM_t - RF_t$ market factor and of the $HMLN_t$ and $LMHD_t$ factors shows considerably high values. The correlation of the market factor and $HMLF_t$ is lower and equal to 0.14. There exists, therefore, the possibility of duplication of information. As a result, on the basis of the analysed variables a definition is given of the orthogonal market factor based on regressions (6) and (7).

$$\begin{aligned} &RM_t - RF_t = \alpha_1 + \beta_{HMLF} HMLF_t + e_t; t = 1,...,36 \\ &\alpha_1 = \underset{(-0.93)}{-0.03} \quad \beta_{HMLF} = \underset{(-0.25)}{0.21} \quad R^2 = 1.89\%, \end{aligned} \tag{6}$$

$$\begin{aligned} &RM_t - RF_t = \alpha_2 + \beta_{HMLN} HMLN_t + \beta_{LMHD} LMHD_t + e_t; t = 1,...,36 \\ &\alpha_2 = \underset{(-0.21)}{-0.01} \quad \beta_{HMLN} = \underset{(1.81)}{0.36} \quad \beta_{LMHD} = \underset{(-2.62)}{-0.55} \quad R^2 = 21.96\%. \end{aligned} \tag{7}$$

Under regressions equations (6) and (7) the values of parameters are included and *t*-statistics are given in brackets. Equation (6) has a low explanatory power. Equation (7), however, has quite a high explanatory power. The parameter values significantly differ from zero, whilst the value of the intercept $\alpha_2 = -0.01$ is statistically equal to zero.

The value of the orthogonalized market factor, correspondingly for regressions (6) and (7), is defined as follows:

$$RMO1_t = \alpha_1 + e_t, \tag{8}$$

$$RMO2_t = \alpha_2 + e_t. \tag{9}$$

A similar procedure concerning the orthogonalization of the market factor was applied by Fama and French [1993, pp. 27–31] for their five-factor model. The loadings of all of the tested HML, SMB, TERM and DEF variables differ significantly from zero. The determination coefficient of the analysed regression (by Fama and French) is $R^2 = 38\%$.

The response variable and the explanatory variables were subject to stationarity tests. The hypothesis as to the stationarity of variables is based on Ljung–Box and Dickey–Fuller tests [1979] [see: Jajuga 2000, p. 39; Suchecki 2000, pp. 20–21, pp. 110–112 and Ljung, Box 1978][7]. Dickey-Fuller tests confirm lack of unit elements

[7] Test findings are available from the author on request.

for each test case. Augmented Dickey-Fuller tests carried out for (k_i) lag, defined from minimising the modified Akaike criterion, indicate a lack of unit elements in 14 out of 19 tested cases. On the basis of the results one may accept the stationarity of the analysed variables.

ICAPM discreet implementation was tested in two passes. The first pass analysed the regression of time-series for the tested portfolios. Beta values were estimated; these are estimators of systematic risk connected with the accepted factors. In the second pass the values of beta loadings were estimated. Beta loadings define risk premiums for the adopted factors. The risk premium vector is estimated on the basis of panel data and the Fama and MacBeth [1973] method.

Regression parameters in the first pass were estimated by means of the GLS – generalized least squares method – according to Prais-Winsten procedure. The co-variance matrix of regression coefficients was also estimated by means of the Newey-West estimator.[8]

In the second pass, in time–cross–section estimation the lack of autocorrelation of the residuals was presumed.[9] The impact of heteroskedasticity is taken into account by means of the change of variables method. The co-variance matrix of risk premiums are estimated by means of the Newey-West estimator.

As far as Fama-MacBeth procedure is concerned, in the second pass the Prais-Winsten method was used. In each tested period (for cross-section data) first-order autocorrelation of the residual component was taken into account.

The impact of estimation errors of the true beta values in the first pass was taken into account by correcting the standard errors of beta loadings estimated in the second pass. With this purpose in mind Shanken's estimator [Shanken 1992] was applied. In order to assess the risk premium values, keeping up with the proposal of Jagannathan and Wang [1998], t-statistics were analysed without and with consideration of Shanken's corrections.

3. Equilibrium in the light of the aggregated two-factor model

Regressions analysed in the first and second test passes are given by equations (10) and (11):

$$r_{it} - \mathrm{RF}_t = \alpha_i + \beta_{i,\mathrm{HMLF}}\mathrm{HMLF}_t + \beta_{i,\mathrm{MO1}}\mathrm{RMO1}_t + e_{it}; \quad t = 1, \ldots, 36, \ \forall i = 1, \ldots, 15, \tag{10}$$

$$r_{it} - \mathrm{RF}_t = \gamma_0 + \gamma_{\mathrm{HMLF}}\hat{\beta}_{i,\mathrm{HMLF}} + \gamma_{\mathrm{MO1}}\hat{\beta}_{i,\mathrm{MO1}} + \varepsilon_{it}; i = 1,\ldots,15; t = 1,\ldots,36. \tag{11}$$

[8] Standard errors estimated by means of Newey-West estimator are similar and are available on request.

[9] Independent variables (betas) remain permanent for all periods, whilst dependent variables constitute rates of return which should by nature be random [see Cochrane 2001, p. 231].

The response variable of the above regressions constitutes the excess of returns of 15 test portfolios constructed on FUN, NUM and DEN values. Explanatory variables of regression (10) are the orthogonalized market factor $RMO1_t$, defined by equation (8), and the $HMLF_t$ factor. Explanatory variables of regression (11) are betas estimated in the first pass. The values of coefficients of regressions (10) were estimated by means of the GLS method with the application of the Prais–Winsten procedure with first-order autocorrelation. Table 2 presents the results of regressions (10) for the test portfolios.

Table 2

Time-series regression of excess stock returns on the orthogonalized stock-market factor (RMO1) and the mimicking returns for the FUN value (HMLF) factor

$$r_{it} - RF_t = \alpha_i + \beta_{i,HMLF} HMLF_t + \beta_{i,MO1} RMO1_t + e_{it};\ t = 1,...,36;\ \forall i = 1, \ldots, 15$$

Response variable: Excess returns on 15 stock portfolios formed on FUN value, NUM value and DEN value GRS–F = 1.64, *p-value*(GRS) = 0.15								
	α_i	*p-value*	$\beta_{i,\ HMLF}$	*p-value*	$\beta_{i,MO1}$	*p-value*	ρ_i	R^2, %
Portfolios formed on FUN, GLS method								
MIN, FUN_{1t}	0.01	0.72	–0.64	0.00	1.13	0.00	0.43	89.27
2) FUN_{2t}	–0.02	0.19	–0.36	0.01	0.82	0.00	0.15	73.07
3) FUN_{3t}	–0.02	0.14	0.23	0.08	0.92	0.00	0.03	78.83
4) FUN_{4t}	–0.04	0.00	0.49	0.00	0.90	0.00	–0.17	84.74
MAX, FUN_{5t}	0.02	0.09	0.48	0.00	1.09	0.00	–0.04	86.17
Portfolios formed on NUM, GLS method								
MIN, NUM_{1t}	0.02	0.47	–0.58	0.00	1.19	0.00	0.15	76.93
2) NUM_{2t}	0.00	0.85	–0.54	0.00	0.71	0.00	0.02	54.09
3) NUM_{3t}	–0.03	0.06	0.16	0.23	0.75	0.00	0.04	70.08
4) NUM_{4t}	–0.02	0.10	0.37	0.00	1.03	0.00	–0.09	88.74
MAX, NUM_{5t}	0.00	0.72	0.51	0.00	1.09	0.00	–0.18	85.19
Portfolios formed on DEN, GLS method								
MIN, DEN_{1t}	0.03	0.08	0.15	0.38	0.84	0.00	0.00	63.31
2) DEN_{2t}	0.00	0.89	0.28	0.03	0.91	0.00	–0.01	79.03
3) DEN_{3t}	–0.04	0.04	0.37	0.01	0.85	0.00	0.04	74.02
4) DEN_{4t}	–0.02	0.40	–0.03	0.79	1.14	0.00	0.28	86.29
MAX, DEN_{5t}	0.02	0.32	–0.19	0.15	1.23	0.00	0.12	86.31

This table presents the estimated parameters of the proposed two-factor model. $RMO1_t$ is the orthogonalized stock-market factor (see equation 8). $HMLF_t$ (high minus low), the rate of return on the mimicking portfolio for the FUN value, is the difference between the simple average of the returns on the two high-FUN portfolios (FUN_{5t} and FUN_{4t}) and the average of the returns on the two low-FUN portfolios (FUN_{1t} and FUN_{2t}). ρ_i is first-order autocorrelation. GRS-F is the F-statistic of Gibbons et al. [1989]. The Prais-Winsten algorithm is used for correction of autocorrelation. Negative-BV stocks are excluded from the portfolios. The sample period is from 1996 to 2005, 36 quarters.

Source: Urbański [2008, p. 825].

The stability of structural parameters is verified for each portfolio by means of the Chow test [Gujarati 1995, p. 263]. In 12 cases out of 15 tested portfolios there is no basis to reject the zero hypothesis which presumes the stability of parameters of regressions (10).[10]

Replacing RM-RF market excess by the RMO1 orthogonal market factor improves the significance of loading of the HMLF factor (despite the relatively low correlation between HMLF and RM-RF). For the majority of tested portfolios *t*-statistics grow in 10 to 15 cases. The values of intercepts, loadings of the market factor, the R^2 determination coefficient and the F-statistics, in both cases of applying RMO1 and RM–RF are identical. $\beta_{i,\mathrm{HMLF}}$ loadings values for cases of applying the orthogonalized RMO1 market factor, are shifted clearly in the direction of positive values. The findings of Fama and French [1993, pp. 27–31] for five-factor model tests on the American market were similar.

$\beta_{i,\mathrm{HMLF}}$ regression coefficients demonstrate periodic connections with FUN and NUM. For each FUN and NUM quintile HMLF regression coefficients increase monotonically from strongly negative values (for the smallest FUN and NUM quintiles) to strongly positive values for the largest quintiles. With the exception of medium quintiles $\beta_{i,\mathrm{HMLF}}$ coefficients differ from zero (see table 2, *p-value*s < 0.01).

Negative $\beta_{i,\mathrm{HMLF}}$ values for portfolios with low FUN and NUM values mean that for a market characterised by a growing HMLF value investments in these portfolios reveal decreasing rates of return. Similarly, positive $\beta_{i,\mathrm{HMLF}}$ coefficient values for portfolios with high FUN and NUM mean that for a market characterised by a growing HMLF value investments in these portfolios reveal increasing rates of return. In other words, investment in companies with the highest dynamics in financial results and high BV/MV and E/MV values should be the more profitable the more the market is characterised by a higher HMLF value.

The $\beta_{i,\mathrm{HMLF}}$ coefficient for the portfolio with maximum DEN attains negative value.[11] For portfolios with low DEN this coefficient attains positive values. This means that investment in companies from low DEN quintiles, in other words with high BV/MV and E/MV (value stocks), ought to give higher rates of return the higher the market is in terms of HMLF value. Similarly, investment in companies from high level DEN quintiles, in other words with low BV/MV and E/MV (growth stocks), ought to give lower rates of return the higher the market is in terms of HMLF value. Fama and French findings [1993, p. 24, tab. 6], showed similar response to HMLF.[12] Parameter values at the HML varied between –0.46 and 0.76.

In the majority of tested portfolios the model generates intercepts equal to zero which constitutes a positive ICAPM test. In 10 out of 15 cases intercepts do not differ significantly from zero (see table 2). This is borne out by the GRS–F statistic equal to 1.64 associated with a *p-value* of 0.15 which means that intercepts of

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[10] The results of these calculations are available from the author on request.

[11] For regression with the classic market factor $\beta_{5,\mathrm{HMLF}} = -0.42$, *p-value* $= 0.00$.

[12] HMLF defined in that paper was based on FUN values.

regressions (10) are equal to zero for all tested portfolios [see Gibbons et al., 1989]. Lewellen's research [1999] demonstrated that in 6 out of 37 analysed test portfolios intercept values were significantly different from zero, whilst Fama and French research [1993] demonstrated that only 3 out of 25 cases indicated significant differences.

Table 3 presents the results of regressions (11).

Table 3

The values of the risk premium vector (γ) estimated from second-pass full-sample regressions for the aggregated two-factor model

$$r_{it} - RF_t = \gamma_0 + \gamma_{HMLF}\hat{\beta}_{i,HMLF} + \gamma_{MO1}\hat{\beta}_{i,MO1} + \varepsilon_{it}; i = 1,...,15; t = 1,...,36$$

	γ_0	γ_{MO1}	γ_{HMLF}	R^2_{LL}, %	Q^A(F)
CT estimate^{PW}	–0.07	0.04	0.06	60.43	1.49 (0.20)
t-stat	–1.60	0.84	3.26		
p-value	0.11	0.40	0.00		
SH *t*-stat	–1.34	0.73	3.11		
p-value	0.18	0.46	0.00		
FM estimate^{PW}	–0.06	0.04	0.06	60.36	1.49 (0.20)
t-stat	–2.07	0.92	3.46		
p-value	0.05	0.36	0.00		
SH *t*-stat	–1.75	0.82	3.39		
p-value	0.09	0.42	0.00		

The table presents the estimated parameters of the proposed two-factor model. RF_t is the 91-day Treasury bill rate of return. $\hat{\beta}_{i,MO1}$ is the loading on the orthogonalized market factor estimated from a first-pass time-series regressions, for *i* portfolio. $\hat{\beta}_{i,HMLF}$ is the loading on the HMLF factor. The response variable is excess return on 15 stock portfolios formed on FUN, NUM and DEN values in period *t*. Q^A(F) reports F-statistic and its corresponding *p-value* indicated below in brackets for the test of Shanken [1985] that the pricing errors in the model are jointly zero. SH *t*-stat is the statistic of Shanken [1992] adjusting for error-s-in-variables. R^2_{LL} is a measure, following Lettau and Ludvigson [2001], showing the fraction of the cross-sectional variation in average returns that is explained by each model and is calculated as follows: $R^2_{LL} = [\sigma^2_c(\bar{r}_i) - \sigma^2_c(\bar{e}_i)]/\sigma^2_c(\bar{r}_i)$, where σ^2_c denotes a cross-sectional variance, and variables with bars over them denote time-series averages. CT estimate reports pooled time series and cross-sectional estimation. FM estimate reports Fama-MacBeth estimation that is formed by first running of the cross-sectional regression at each time series. ^{PW}reports GLS with Prais-Winsten procedure. The sample period is from 1996 to 2005, 36 quarters.

Source: own research.

Coefficients γ_{MO1} and γ_{HMLF} constitute systematic risk premium in terms of the market factor and the HMLF factor. Risk premiums and their standard errors are very similar for the classical appearance of the RM-RF market factor and the orthogonalized RMO1 factor. γ_{HMLF} values for the two estimating methods are significantly higher than zero. γ_{MO1} take on positive value but statistically equal to zero. This fact points towards the decisive impact of risk in terms of the HMLF factor on cross–section returns. Positive and significant risk premium in terms of HMLF means that investors expect growth in rates of return amounting to 6% quarterly per risk unit.

Q^A(F) statistics for Prais-Winsten procedure (^{PW}) take value 1.49 associated with a *p-value* of 0.20 which signifies that estimation pricing errors are jointly equal to zero. Also high R^2_{LL} coefficients indicate that the model constitutes a good description of cross-section rates of return in light of the ICAPM.

4. Equilibrium in the light of the aggregated three-factor model

Regressions analysed in the first and second test passes are given by equations (12) and (13):

$$r_{it} - \mathrm{RF}_t = \alpha_i + \beta_{i,\mathrm{HMLN}}\mathrm{HMLN}_t + \beta_{i,\mathrm{LMHD}}\mathrm{LMHD}_t + \beta_{i,\mathrm{MO2}}\mathrm{RMO2}_t + e_{it};$$
$$t = 1, \ldots, 36;\ \forall i = 1, \ldots, 15, \tag{12}$$

$$r_{it} - \mathrm{RF}_t = \gamma_0 + \gamma_{\mathrm{HMLN}}\hat{\beta}_{i,\mathrm{HMLN}} + \gamma_{\mathrm{LMHD}}\hat{\beta}_{i,\mathrm{LMHD}} + \gamma_{\mathrm{MO2}}\hat{\beta}_{i,\mathrm{MO2}} + \varepsilon_{it};$$
$$t = 1, \ldots, 36;\ i = 1, \ldots, 15. \tag{13}$$

The response variable of the above regressions is the excess of returns of 15 test portfolios constructed on FUN, NUM and DEN values. Explanatory variables of regression (12) are the orthogonal market factor defined by equation (9) and HMLN_t and LMHD_t factors. Explanatory variables of regressions (13) are betas estimated in the first pass.

The values of coefficients of regressions (12) are determined by means of the GLS method with the application of Prais-Winsten procedure with first order autocorrelation. Table 4 presents the values of coefficients of regression (12) for the test portfolios.

The stability of structural parameters is verified for each portfolio by means of the Chow test [Gujarati 1995, p. 263]. In 13 cases out of 15 portfolios there is no basis to reject the zero hypothesis which presumes the stability of parameters of regressions (12).[13]

Replacing RM-RF market excess with the RMO2 orthogonal market factor considerably improves the significance of HMLN and LMHD factor loadings (*t*-statistics grow in 12 out of 15 cases). Negative but insignificant $\beta_{i,\mathrm{LMHD}}$ beta values for portfolios formed on FUN and NUM are significantly negative once the RM-RF factor has been replaced by the RMO2 orthogonal market factor. For portfolios formed on DEN with the application of the RM-RF factor, $\beta_{i,\mathrm{HMLN}}$ betas are negative but insignificantly different from zero. This suggests a fall in the rate of return together with HMLN growth. As a result of applying the RMO2 orthogonal market factor, $\beta_{i,\mathrm{HMLN}}$ betas are significantly positive for all five quintiles. These results evidence the growth of rates of return together with HMLN growth for portfolios formed for DEN.

The results are similar to those obtained in the research carried out by Fama and French [1993, pp. 27–31]. When applying the RMO2 orthogonal market factor,

[13] The findings of these calculations are available from the author on request.

$\beta_{i,\text{HMLN}}$ loadings clearly shift towards positive values, whilst $\beta_{i,\text{LMHD}}$ loadings move in the direction of negative values.

Table 4

Time-series regression of excess stock returns on the orthogonalized stock-market factor (RMO2) and the mimicking returns for the NUM value (HMLN) and DEN value (LMHD) factors

$$r_{it} - \text{RF}_t = \alpha_i + \beta_{i,\text{HMLN}}\text{HMLN}_t + \beta_{i,\text{LMHD}}\text{LMHD}_t + \beta_{i,\text{MO2}}\text{RMO2}_t + e_{it};$$

$$t = 1, \ldots, 36; \ \forall i = 1, \ldots, 15$$

Response variable: Excess returns on 15 stock portfolios formed on FUN value, NUM value and DEN value
GRS-F = 1.61, *p-value*(GRS) = 0.17

	α_i	*p-value*	$\beta_{i,\text{HMLN}}$	*p-value*	$\beta_{i,\text{LMHD}}$	*p-value*	$\beta_{i,\text{MO2}}$	*p-value*	ρ_i	R^2, %
Portfolios formed on FUN, GLS method										
MIN, FUN_{1t}	–0.01	0.65	–0.22	0.03	–0.80	0.00	1.11	0.00	0.36	87.41
2) FUN_{2t}	–0.03	0.06	–0.04	0.73	–0.59	0.00	0.79	0.00	0.10	69.88
3) FUN_{3t}	–0.02	0.14	0.41	0.00	–0.52	0.00	0.90	0.00	0.02	78.93
4) FUN_{4t}	–0.03	0.01	0.58	0.00	–0.47	0.00	0.88	0.00	–0.13	83.81
MAX,FUN_{5t}	0.03	0.03	0.61	0.00	–0.59	0.00	1.08	0.00	–0.03	85.55
Portfolios formed on NUM, GLS method										
MIN, NUM_{1t}	0.00	0.93	–0.36	0.01	–0.58	0.04	0.01	0.00	0.10	82.19
2) NUM_{2t}	0.00	0.81	–0.33	0.04	–0.69	0.02	0.00	0.00	0.04	60.36
3) NUM_{3t}	–0.03	0.03	0.24	0.03	–0.24	4.85	0.00	0.00	0.13	72.17
4) NUM_{4t}	–0.01	0.16	0.58	0.00	–0.62	0.00	0.00	0.00	–0.13	89.64
MAX, NUM_{5t}	0.01	0.36	0.70	0.00	–0.64	0.00	0.01	0.00	–0.13	85.75
Portfolios formed on DEN, GLS method										
MIN, DEN_{1t}	0.01	0.30	0.22	0.03	0.10	0.35	1.08	0.00	–0.08	83.64
2) DEN_{2t}	–0.01	0.52	0.29	0.00	–0.12	0.18	1.04	0.00	–0.21	87.59
3) DEN_{3t}	–0.03	0.05	0.39	0.00	–0.31	0.02	0.91	0.00	0.05	74.13
4) DEN_{4t}	–0.01	0.50	0.29	0.00	–0.97	0.00	1.01	0.00	0.24	89.80
MAX, DEN_{5t}	0.01	0.37	0.27	0.02	–1.03	0.00	1.10	0.00	0.11	87.83

This table presents the estimated parameters of the proposed three-factor model. RF_t is the 91-day Treasury bill rate of return. RMO2_t is the orthogonalized stock-market factor (see equation 9). HMLN_t (high minus low) the rate of return on the mimicking portfolio for the NUM value, is the difference between the simple average of the returns on the two high-NUM portfolios (NUM_{5t} and NUM_{4t}) and the average of the returns on the two low-NUM portfolios (NUM_{1t} and NUM_{2t}). LMHD_t (low minus high) the rate of return on the mimicking portfolio for the DEN value, is the difference between the simple average of the returns on the two low-DEN portfolios (DEN_{1t} and DEN_{2t}) and the average of the returns on the two high-DEN portfolios (DEN_{5t} and DEN_{4t}). ρ_i is the first-order autocorrelation. GRS-F is the F-statistic of Gibbons et al. [1989]. The Prais-Winsten algorithm is used for correction of autocorrelation. Negative-BV stocks are excluded from the portfolios. The sample period is from 1996 to 2005, 36 quarters.

Source: Urbański [2008, pp. 826–827].

As in the case of the aggregated two-factor model HMLN, coefficients demonstrate links with FUN and NUM. For each quintile formed on FUN and NUM, the HMLN regression coefficients increase monotonically from strongly negative values for the smallest FUN and NUM quintiles to strongly positive values for the largest quintiles.

On the other hand, LMHD coefficients demonstrate links with DEN. For each quintile built in terms of DEN, the LMHD regression coefficients decrease monotonically from positive values for the smallest DEN quintiles to strongly negative values for the largest quintiles.[14]

In comparison with regression taking into account only market portfolio excess the values of intercepts are far nearer to zero.[15] Out of 15 intercepts, 10 statistically do not differ from zero. The aggregated three-factor model particularly well describes the rate of return for portfolios constructed on the basis of NUM and DEN, for which only the intercepts of medium quintiles differ significantly from zero. These results are borne out by the GRS–F statistic equal to 1.61, associated with a *p-value* of 0.17, which means that intercepts of regressions (12) are equal to zero for all tested portfolios. The R^2 coefficient attains very high values which come up to 90%.

The results of the analysis demonstrate that this model gives a fuller description of changes in rates of return in comparison with the aggregated two-factor model. The model makes the rates of return dependent on three market characteristics, i.e. on the HMLN and LMHD indicators and on the market factor. My research indicates that investment in companies with large FUN or NUM values gives growing rates of return for growing HMLN values and decreasing LMHD values. In turn, investment in companies with small FUN or NUM values is characterised by an increase in rates of return if the market demonstrates falling values both in HMLN and LMHD.

Investment in companies with large DEN (growth stocks, low BV/MV and E/MV) demonstrates growing rates of return for growing HMLN values and decreasing LMHD values. On the other hand, investment in companies with small DEN values (value stocks, high BV/MV and E/MV) demonstrates growing rates of return for growing HMLN and LMHD values.

Table 5 presents the values of coefficients of regressions (13).

Coefficients γ_{MO2}, γ_{HMLN} and γ_{LMHD} represent systematic risk premium in terms of the market factor and the HMLF and LMHD factors. The risk premium vector and R^2_{LL} and Q^A(F)-statistics for regressions (13) assume values which are very similar to one another. On the basis of calculated SH-*t* and *t*-statistics one may claim that the risk premium components: γ_{HMLN} and γ_{LMHD} are significantly greater than zero. Positive and significant risk premiums mean that investors expect growth in rates of return amounting respectively to 6% and 4% quarterly per risk unit.

[14] For the model with the orthogonal market factor, for the smallest DEN quintile $\beta_{1,\ LMHD}$ is insignificantly higher than zero, however for the model taking into account the classic market factor, the beta values for the first two quintiles are significantly positive (*p-value* = 0.00).

[15] Calculation findings for the classic CAPM, see Urbański [2007].

Table 5

The values of the risk premium vector (γ) estimated from second-pass full-sample regressions for the three-factor model

$$r_{it} - RF_t = \gamma_0 + \gamma_{HMLN}\hat{\beta}_{i,HMLN} + \gamma_{LMHD}\hat{\beta}_{i,LMHD} + \gamma_{MO2}\hat{\beta}_{i,MO2} + \varepsilon_{it};$$
$$i = 1, \ldots, 15;\ t = 1, \ldots, 36$$

	γ_0	γ_{MO2}	γ_{HMLN}	γ_{LMHD}	R^2_{LL}, %	Q^A(F)
CT estimate[PW]	–0.09	0.08	0.06	0.04	74.29	1.66 (0.15)
t–stat	–2.08	1.66	2.53	1.76		
p–value	0.04	0.10	0.01	0.08		
SH *t*–stat	–1.55	1.31	2.24	1.50		
p–value	0.12	0.19	0.03	0.13		
FM estimate[PW]	–0.09	0.08	0.06	0.04	74.61	1.66 (0.15)
t–stat	–2.38	1.71	3.17	2.44		
p–value	0.02	0.10	0.00	0.02		
SH *t*–stat	–1.75	1.35	3.08	2.38		
p–value	0.09	0.19	0.00	0.02		

This table presents the estimated parameters of the proposed three-factor model. RF_t is the 91-day Treasury bill rate of return. $\hat{\beta}_{i,MO1}$ is the loading on the orthogonalized market factor estimated from first-pass time-series regressions, for *i* portfolio. $\hat{\beta}_{i,HMLN}$ and $\hat{\beta}_{i,LMHD}$ are loadings on HMLN and LMHD factors. The response variable is excess return on 15 stock portfolios formed on FUN, NUM and DEN values in period *t*. Q^A(F) reports F-statistic and its corresponding *p-value* indicated below in brackets for the test of Shanken [1985] that the pricing errors in the model are jointly zero. SH *t*-stat is the statistic of Shanken [1992] adjusting for errors-in-variables. R^2_{LL} is a measure, follows Lettau and Ludvigson [2001], showing the fraction of the cross-sectional variation in average returns that is explained by each model and is calculated as follows: $R^2_{LL} = [\sigma^2_c(\bar{r}_i) - \sigma^2_c(\bar{e}_i)]/\sigma^2_c(\bar{r}_i)$, where σ^2_c denotes a cross-sectional variance, and variables with bars over them denote time-series averages. CT estimate reports pooled time series and cross-sectional estimation. FM estimate reports Fama-MacBeth estimation that is formed by first running of the cross-sectional regression at each time series. [PW] reports GLS with Prais-Winsten procedure. The sample period is from 1996 to 2005, 36 quarters.

Source: own research.

The value of γ_{MO2} is much the same as in the case of the two-factor model, it assumes positive values but they are statistically equal to zero. This fact points to the decisive impact of risk in terms of the HMLN and LMHD factors on cross-section returns. The value of γ_0 intercept, after taking into account Shanken's corrections, is statistically equal to zero.

Q^A(F)-statistics (for Prais-Winsten procedure (PW) and using the Newey-West estimator) assume respectively values 1.66 and 0.76, associated with *p-value*s of 0.15 and 0.67, which signifies that pricing errors in the model are jointly equal to zero. R^2_{LL} values are higher than in the case of the two-factor model.

5. The impact of characteristics of formed portfolios on the explanatory strength of the model

Sections 2 and 3 present a model indicating that the market rate of return and the proposed HMLF, HMLN and LMHD variables behave very well when explaining unconditional cross-section rates of return. On the other hand, Jagannathan and Wang [1998] argue that an important test for the correct model describing rates of return involves the inclusion of additional cross-section conversions of rates of return. This is particularly important as cross-section conversions of rates of return may be certain modelled functions, demonstrating prediction capacity, on the basis of which the forming of portfolios takes place.

An analogical phenomenon occurs in the case of procedures by Fama and French [1993], Petkova [2006] and in the model proposed in this paper. The prediction possibilities of FUN, NUM and to a lesser degree DEN, on the basis of which the following quintile portfolios are formed: $\mathrm{FUN}_{i=1-5,t}$, $\mathrm{NUM}_{i=6-10,t}$, $\mathrm{DEN}_{i=11-15,t}$ are shown by Urbański [2008, p. 821]. For this reason it seems to be necessary to check the correctness of the obtained findings in the presence of characteristics of portfolios formed in this manner.

The general form of the test for the two- and three-factor models is presented appriopriately by equation (14):

$$r_{it} - \mathrm{RF}_t = \gamma_0 + \gamma\hat{\beta}_i^{'} + \gamma_Z Z_{i,t-1} + \varepsilon_{it};\, i = 1,...,15;\, t = 1,...,36 \tag{14}$$

where γ is the vector of risk premium, $\hat{\beta}_i$ is the vector of systematic risk evaluated in the first pass, and $Z_{i,t-1}$ are FUN, NUM or DEN values for the $t-1$ period. Zero hypothesis is formulated as $H_0{:}\gamma_Z = 0$.

In practical terms, the following regressions for the two-factor model are analysed:

$$r_{it} - \mathrm{RF}_t = \gamma_0 + \gamma_\mathrm{M}\hat{\beta}_{i,\mathrm{M}} + \gamma_\mathrm{HMLF}\hat{\beta}_{i,\mathrm{HMLF}} + \gamma_\mathrm{FUN}\mathrm{FUN}_{i,t-1} + \varepsilon_{it};\quad i = 1, \ldots, 15;\, t = 1, \ldots, 36, \tag{15}$$

$$r_{it} - \mathrm{RF}_t = \gamma_0 + \gamma_\mathrm{M}\hat{\beta}_{i,\mathrm{M}} + \gamma_\mathrm{HMLF}\hat{\beta}_{i,\mathrm{HMLF}} + \gamma_\mathrm{NUM}\mathrm{NUM}_{i,t-1} + \varepsilon_{it};\quad i = 1, \ldots, 15;\, t = 1, \ldots, 36, \tag{16}$$

$$r_{it} - \mathrm{RF}_t = \gamma_0 + \gamma_\mathrm{M}\hat{\beta}_{i,\mathrm{M}} + \gamma_\mathrm{HMLF}\hat{\beta}_{i,\mathrm{HMLF}} + \gamma_\mathrm{DEN}\mathrm{DEN}_{i,t-1} + \varepsilon_{it};\quad i = 1, \ldots, 15;\, t = 1, \ldots, 36 \tag{17}$$

and for the three-factor model:

$$r_{it} - \mathrm{RF}_t = \gamma_0 + \gamma_\mathrm{M}\hat{\beta}_{i,\mathrm{M}} + \gamma_\mathrm{HMLN}\hat{\beta}_{i,\mathrm{HMLN}} + \gamma_\mathrm{LMHD}\hat{\beta}_{i,\mathrm{LMHD}} + \gamma_\mathrm{FUN}\mathrm{FUN}_{i,t-1} + \varepsilon_{it};\quad i = 1, \ldots, 15;\, t = 1, \ldots, 36, \tag{18}$$

$$r_{it} - \mathrm{RF}_t = \gamma_0 + \gamma_\mathrm{M}\hat{\beta}_{i,\mathrm{M}} + \gamma_\mathrm{HMLN}\hat{\beta}_{i,\mathrm{HMLN}} + \gamma_\mathrm{LMHD}\hat{\beta}_{i,\mathrm{LMHD}} + \gamma_\mathrm{NUM}\mathrm{NUM}_{i,t-1} + \varepsilon_{it};\quad i = 1, \ldots, 15;\, t = 1, \ldots, 36, \tag{19}$$

$$r_{it} - \mathrm{RF}_t = \gamma_0 + \gamma_\mathrm{M}\hat{\beta}_{i,\mathrm{M}} + \gamma_\mathrm{HMLN}\hat{\beta}_{i,\mathrm{HMLN}} + \gamma_\mathrm{LMHD}\hat{\beta}_{i,\mathrm{LMHD}} + \gamma_\mathrm{DEN}\mathrm{DEN}_{i,t-1} + \varepsilon_{it};\quad i = 1, \ldots, 15;\, t = 1, \ldots, 36. \tag{20}$$

In regressions (15–20) $\widehat{\beta}_{i,\mathrm{M}}$ is the loading on the classical market factor estimated from first-pass time-series regressions, for i portfolio. $\mathrm{FUN}_{i,t-1}$ is the vector whose coordinates are: $\mathrm{FUN}_{1,t-1}, \ldots, \mathrm{FUN}_{5,t-1}, \mathrm{FUN}_{1,t-1}, \ldots, \mathrm{FUN}_{5,t-1}, \mathrm{FUN}_{1,t-1}, \ldots, \mathrm{FUN}_{5,t-1}$, $\mathrm{NUM}_{i,t-1}$ is the vector whose coordinates are: $\mathrm{NUM}_{1,t-1}, \ldots, \mathrm{NUM}_{5,t-1}, \mathrm{NUM}_{1,t-1}, \ldots, \mathrm{NUM}_{5,t-1}, \mathrm{NUM}_{1,t-1}, \ldots, \mathrm{NUM}_{5,t-1}$; $\mathrm{DEN}_{i,t-1}$ is the vector whose coordinates are: $\mathrm{DEN}_{1,t-1}, \ldots, \mathrm{DEN}_{5,t-1}, \mathrm{DEN}_{1,t-1}, \ldots, \mathrm{DEN}_{5,t-1}, \mathrm{DEN}_{1,t-1}, \ldots, \mathrm{DEN}_{5,t-1}$, for portfolios formed on FUN, NUM and DEN.

Regression parameter values which test the two-factor and three-factor models are presented in tables 6 and 7.

Table 6

Time-cross-section regressions demonstrating explanatory strength of portfolio characteristics, constituting the aggregated two-factor model tests

$$r_{it} - \mathrm{RF}_t = \gamma_0 + \gamma_{\mathrm{M}}\hat{\beta}_{i,\mathrm{M}} + \gamma_{\mathrm{HMLF}}\hat{\beta}_{i,\mathrm{HMLF}} + \gamma_{\mathrm{FUN}}\mathrm{FUN}_{i,t-1} + \varepsilon_{it}; i = 1,...,15; t = 1,...,36;$$
$$R^2_{LL} = 61.20\%$$

Panel A	γ_0	γ_{M}	γ_{HMLF}	γ_{FUN}
Parameter	–0.07	0.07	0.07	–0.01
t–stat	–1.75	1.40	3.27	–0.98
p–value	0.08	0.16	0.00	0.33

$$r_{it} - \mathrm{RF}_t = \gamma_0 + \gamma_{\mathrm{M}}\hat{\beta}_{i,\mathrm{M}} + \gamma_{\mathrm{HMLF}}\hat{\beta}_{i,\mathrm{HMLF}} + \gamma_{\mathrm{NUM}}\mathrm{NUM}_{i,t-1} + \varepsilon_{it}; i = 1,...,15; t = 1,...,36;$$
$$R^2_{LL} = 61.67\%$$

Panel B	γ_0	γ_{M}	γ_{HMLF}	γ_{NUM}
Parameter	–0.07	0.06	0.07	–0.00
t–stat	–1.67	1.26	3.13	–0.68
p–value	0.10	0.21	0.00	0.49

$$r_{it} - \mathrm{RF}_t = \gamma_0 + \gamma_{\mathrm{M}}\hat{\beta}_{i,\mathrm{M}} + \gamma_{\mathrm{HMLF}}\hat{\beta}_{i,\mathrm{HMLF}} + \gamma_{\mathrm{DEN}}\mathrm{DEN}_{i,t-1} + \varepsilon_{it}; i = 1,...,15; t = 1,...,36;$$
$$R^2_{LL} = 58.72\%$$

Panel C	γ_0	γ_{M}	γ_{HMLF}	γ_{DEN}
Parameter	–0.06	0.01	0.05	0.01
t–stat	–1.34	0.23	2.33	1.07
p–value	0.18	0.82	0.02	0.29

Time cross-section estimation is made on the basis of panel data. In the first pass betas are estimated according to GLS with application of the Prais-Winsten procedure. In the second pass there is only heteroskedasticity correction by means of transformation of variables. Panel A demonstrates whether the lagged $\mathrm{FUN}_{i,t-1}$ contains explanatory strength in the aggregated two-factor model. Panel B shows whether the lagged $\mathrm{NUM}_{i,t-1}$ contains explanatory strength in the aggregated two-factor model. Panel C demonstrates whether the lagged $\mathrm{DEN}_{i,t-1}$ contains explanatory strength in the aggregated two-factor model. R^2_{LL} is an informal determination coefficient of Lettau and Ludvigson [2001], demonstrating the share of cross–sectional changes in the rate of return explained by the model. Negative-BV stocks are excluded from the portfolios. The sample period is from 1996 to 2005, 36 quarters.

Source: own research.

Table 7

Time-cross-section regressions demonstrating explanatory strength of portfolio characteristics, constituting the aggregated three-factor model tests

$$r_{it} - \mathrm{RF}_t = \gamma_0 + \gamma_{\mathrm{M}}\hat{\beta}_{i,\mathrm{M}} + \gamma_{\mathrm{HMLN}}\hat{\beta}_{i,\mathrm{HMLN}} + \gamma_{\mathrm{LMHD}}\hat{\beta}_{i,\mathrm{LMHD}} + \gamma_{\mathrm{FUN}}\mathrm{FUN}_{i,t-1} + \varepsilon_{it};$$
$$i = 1, \ldots, 15;\ t = 1, \ldots, 36;\ \mathrm{R}^2_{LL} = 75.93\%$$

Panel A	γ_0	γ_{M}	γ_{HMLN}	γ_{LMHD}	γ_{FUN}
Parameter	–0.09	0.07	0.05	0.05	0.00
t–stat	–2.09	1.43	1.48	1.49	0.21
p–value	0.04	0.15	0.14	0.14	0.84

$$r_{it} - \mathrm{RF}_t = \gamma_0 + \gamma_{\mathrm{M}}\hat{\beta}_{i,\mathrm{M}} + \gamma_{\mathrm{HMLN}}\hat{\beta}_{i,\mathrm{HMLN}} + \gamma_{\mathrm{LMHD}}\hat{\beta}_{i,\mathrm{LMHD}} + \gamma_{\mathrm{NUM}}\mathrm{NUM}_{i,t-1} + \varepsilon_{it};$$
$$i = 1, \ldots, 15;\ t = 1, \ldots, 36;\ \mathrm{R}^2_{LL} = 78.41\%$$

Panel B	γ_0	γ_{M}	γ_{HMLN}	γ_{LMHD}	γ_{NUM}
Parameter	–0.10	0.07	0.04	0.05	0.00
t–stat	–2.15	1.34	1.24	1.73	0.58
p–value	0.03	0.18	0.22	0.08	0.56

$$r_{it} - \mathrm{RF}_t = \gamma_0 + \gamma_{\mathrm{M}}\hat{\beta}_{i,\mathrm{M}} + \gamma_{\mathrm{HMLN}}\hat{\beta}_{i,\mathrm{HMLN}} + \gamma_{\mathrm{LMHD}}\hat{\beta}_{i,\mathrm{LMHD}} + \gamma_{\mathrm{DEN}}\mathrm{DEN}_{i,t-1} + \varepsilon_{it};$$
$$i = 1, \ldots, 15;\ t = 1, \ldots, 36,\ \mathrm{R}^2_{LL} = 88.25\%$$

Panel C	γ_0	γ_{M}	γ_{HMLN}	γ_{LMHD}	γ_{DEN}
Parameter	–0.09	0.01	0.01	0.08	0.03
t–stat	–2.12	0.24	0.27	2.77	2.25
p–value	0.03	0.81	0.79	0.00	0.03

Time cross-section estimation is made on the basis of panel data. In the first pass betas are estimated according to GLS with application of the Prais-Winsten procedure. In the second pass there is only heteroskedasticity correction by means of transformation of variables. Panel A shows whether the lagged $\mathrm{FUN}_{i,t-1}$ contains explanatory strength in the aggregated three-factor model. Panel B shows whether the lagged $\mathrm{NUM}_{i,t-1}$ contains explanatory strength in the aggregated three-factor model. Panel C shows whether the lagged $\mathrm{DEN}_{i,t-1}$ contains explanatory strength in the aggregated three-factor model. R^2_{LL} is an informal determination coefficient of Lettau and Ludvigson [2001], demonstrating the share of cross-sectional changes in the rate of return explained by the model. Negative-BV stocks are excluded from the portfolios. The sample period is from 1996 to 2005, 36 quarters.

Source: own research.

Panel A in tables 6 and 7 shows whether the lagged $\mathrm{FUN}_{i,t-1}$ variable contains additional explanatory strength, respectively in the aggregated two-factor and three-factor models. Similarly, panels B and C depict the impact of lagged $\mathrm{NUM}_{i,t-1}$ and $\mathrm{DEN}_{i,t-1}$.

On the basis of the obtained results one should note that for the two-factor model, supplemented to include the analysed characteristics, the impact of HLMF variable loadings and the market excess rate of return are very similar. In every case lagged FUN, NUM and DEN loadings are statistically equal to zero, which means that the explanatory power of the model has not changed and by this virtue the correctness of the assumed zero hypothesis is confirmed. R^2_{LL} coefficient values stand at a similar level, amounting to about 60%.

In the case of the three-factor model, significant impact is noted only for the DEN variable which contains the smallest prediction capacity to describe rates of return in comparison with FUN and NUM [see Urbański, 2008, p. 821, tab. 1]. Values of the R^2_{LL} coefficient after supplementing the lagged DEN model increase from 74% to 88%. After supplementing the lagged FUN and NUM models, R^2_{LL}values remain at a similar level. Lagged FUN and NUM loadings are insignificantly different from zero.

6. Visual assessment of the analysed ICAPM implementations

Figure 1 presents the visual assessment of the tested versions of the proposed model in comparison with the classic CAPM (applied by Jagannathan and Wang [1996]). The figure presents pricing errors in each of the tested portfolios, marked with numbers 1 to 15. Portfolios 1 to 5 are formed on FUN values, from 6 to 10 on NUM values, and from 11 to 15 on DEN values. On the basis of the obtained results one should note that the classic CAPM version contains the greatest pricing errors. The Rsq coefficient attains high negative values, whilst R^2 assumes the lowest value equal to 4.88%. The aggregated three-factor model is characterised by the smallest pricing errors. Rsq is equal to 66.15%.[16]

Conclusion

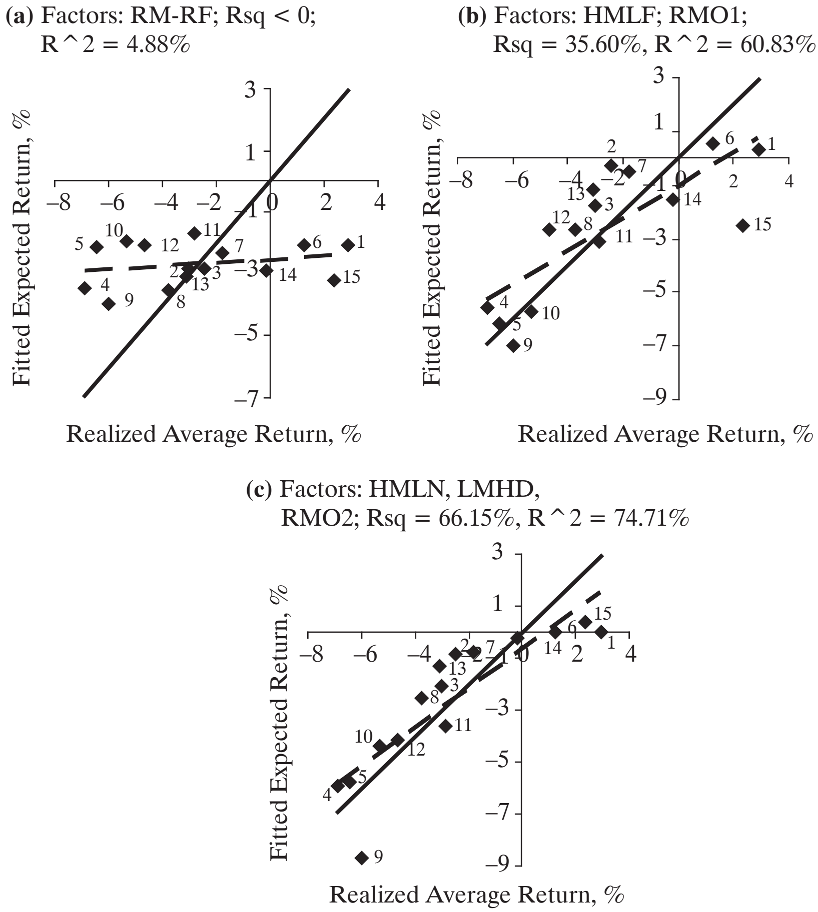
This paper presents the aggregated factor line model describing rates of return on shares listed on the Warsaw Stock Exchange in the years preceding Poland's accession to the European Union. The basic difference between the proposed pricing procedure and the methods hitherto used consists in the taking into account the known and also unknown parameters of different future investor decisions. The outcome of various types of investment behaviour is foreseen as a result of the maximization of the FUN functional (see section 1).

Explanatory variables are constructed on the basis of the market rate of return, the FUN functional and the NUM and DEN functions. The proposed variables depend on profit structure over a three-year historical period and on company book-to-market value.

[16] Rsq is the R^2 coefficient on condition that the simple regression does not contain an intercept and is inclined towards the abscissa of a point at an angle of 45 degrees. R^2 represents the real regression.

Figure 1

Fitted expected returns versus realized average returns: (a) CAPM, (b) the aggregated two-factor model, (c) the aggregated three-factor model



The figure shows the pricing errors for each of the 15 portfolios. Each number of scatter points represents one portfolio, 1–5 portfolios formed on FUN, 6–10 portfolios formed on NUM, and 11–15 portfolios are formed on DEN. For each portfolio i, the realized average return is the time-series average of the portfolio return and the fitted expected return is the value for the expected return, $E[r_i]$, in the following regression model: $E[r_i] = \gamma_0 + \sum_{l=1}^{L} \gamma_l \beta_{il}$ where β_{il} are the slope coefficients in the first-pass OLS regression of the returns' excess of the portfolios in respect of the used factors, γ_0 is the expected return on a "zero-beta" portfolio, γ_l is the l component of risk premium vector, γ_0 and γ_l are estimated from a second-pass OLS regression. If the model fits perfectly, all the points would lie along the (45-degree line). Rsq is R-squared statistics measure of the success of the regression in predicting the values of the fitted expected return against their realized average returns if the regression does not have an intercept and contains loading restriction equal one (45-degree line). The broken line and R^2 represent the actual regression. RM is the percentage return on the Warsaw Stock Exchange WIG index. RF is the 91-days Treasury bill rate. RMO1 and RMO2 are orthogonalized stock-market factors not correlated with HMLF and HMLN and LMHD. HMLF, HMLN and LMHD are factors of the proposed aggregated model.

Source: own research.

Aggregated two-factor model tests demonstrate that for portfolios formed on FUN and NUM, $\beta_{i,\text{HMLF}}$ betas increase monotonically from strongly negative values, for the smallest FUN and NUM quintiles, to strongly positive values for the largest quintiles. For portfolios formed in terms of DEN, $\beta_{i,\text{HMLF}}$ increase from strongly negative values for the largest DEN quintiles to positive values towards decreasing quintiles (see table 2). Investments in companies with the highest FUN and NUM should turn out to be more profitable the more that the market is characterised by greater HMLF value. Investment in companies with low DEN should give higher rates of return the more that the market is characterised by higher HMLF value. The risk premium vector estimated in the second pass points towards the clear impact of risk connected with the HMLF factor (see table 3). Positive and significant risk premium in terms of HMLF means that investors expect growth in rates of return amounting to 6% quarterly per risk unit. The zero value of the intercept γ_0 and high R^2_{LL} indicate that the model better describes rates of return in comparison with the classic CAPM, giving a good description of cross-section rates of return in the light of ICAPM. [17]

Tests of the first pass of the aggregated three–factor model demonstrate that investment in companies with large FUN or NUM values give an increase in the rate of return for growing HMLN and decreasing LMHD. Investments in companies with small FUN or NUM are characterised by an increase in rates of return if the market demonstrates falling values both in HMLN and LMHD. Investments in portfolios formed on DEN and characterised by small DEN values bring growing rates of return for the market characterised by rising LMHD and HMLN. Investments in companies with large DEN values bring growing rates of return for the market characterised by increasing HMLN and decreasing LMHD (see table 4). GRS-F statistic value is similar to the two-factor model which permits the value of intercepts to be noted as equal to zero. The R^2 coefficient attains very high values, in many cases exceeding 80%. Risk premium values in terms of HMLN and LMHD, determined in the second pass, are significantly different from zero and correspondingly equal to 6% and 4%. Risk premium in terms of the market factor is much of the same as in the case of the two-factor model, statistically equal to zero. Zero values of the γ_0 intercept, high $R^2_{LL} = 75\%$ and pricing errors insignificantly different from zero demonstrate that the aggregated three–factor model adequately describes the cross–section rates of return in light of the ICAPM.

The test of the model, according to Jagannathan and Wang [1998], involves the inclusion in the second pass regressions of modelled functions: FUN, NUM and DEN. The zero hypothesis test shows that the model functions included in regression do not increase the explanatory power of the model.

The obtained results demonstrate that for the two-factor model supplemented to include lagged FUN, NUM and DEN, the impact of HLMF and RM-RF loadings is very similar (see table 4). In each case loadings of lagged FUN, NUM and DEN are statistically equal to zero, which means that the explanatory power of the model

[17] Calculation results for the classic CAPM are available from the author on request.

does not change. The R^2_{LL} coefficient values stand at a similar level, amounting to about 60%. By this virtue the correctness of the assumed zero hypothesis is confirmed.

In the three-factor model, significant impact is noted only of the lagged DEN variable which contains the smallest prediction capacity of describing rates of return in comparison with FUN and NUM. The value of the R^2_{LL} coefficient increases from 74% to 88%. Lagged FUN and NUM loadings turn out to be insignificantly different from zero. The zero hypothesis is not shown in the event of supplementing the model by the lagged DEN variable.

When comparing the proposed aggregated model with the classic CAPM one should note that it constitutes a more precise description of rates of return in the light of ICAPM. This is borne out by the value of the informal R^2_{LL} determination coefficient of Lettau and Ludvigson [2001] and the results of two fundamental assessment tests of the equilibrium model: a test of the efficiency of a given portfolio of Gibbons et al. [1989] and Shanken's pricing errors test [1985].

R^2_{LL} is the lowest for the classic CAPM (R^2_{LL} < 10%). R^2_{LL} coefficient for the aggregated two-factor model amounts to around 60%, and for the aggregated three--factor model it is 75%.

The classic CAPM generates intercepts significantly different from zero. This is supported out by GRS-F statistics values and the γ_0 coefficient defined in the second pass. This is the proof that this procedure does not define well rates of return on the Polish market in light of the ICAPM. The proposed aggregated model generates zero intercepts.

Shanken's Q^A(F)-statistics values [1985] are proof that the classic CAPM generates pricing errors which are different from zero. Pricing errors of the aggregated two and three–factor versions are insignificantly different from zero.

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Zmiany stóp zwrotu akcji notowanych na Giełdzie Papierów Wartościowych w Warszawie

Streszczenie

W artykule przedstawiony został ekonometryczny model CAPM opisujący zmiany stóp zwrotu akcji notowanych na Giełdzie Papierów Wartościowych w Warszawie. Badane są wartości składowych wektorów ryzyka systematycznego i premii za ryzyko w latach poprzedzających wejście Polski do Unii Europejskiej. Algorytm opisujący stopy zwrotu oparty jest na pracach Famy i Frencha oraz jest wynikiem własnych propozycji autora. Zmienne objaśniające modelu uwzględniają znane i nieznane parametry przyszłych inwestycji. Uzyskane wyniki porównywane są z symulacjami opartymi na klasycznej wersji CAPM. Wykonane testy wykazały dobre dopasowanie proponowanego modelu do danych empirycznych. Test Jagannathana i Wanga (1998) dodatkowo potwierdził poprawność modelu. Uzyskane wyniki umożliwiają sformułowanie praktycznych wskazań, użytecznych dla inwestorów oraz doradców zarządzających portfelami inwestycyjnymi.

Słowa kluczowe: model wyceny CAPM • model Famy-Frencha • portfel rynkowy • metoda Famy-MacBetha

CROSS-SECTION CHANGES OF RATES OF RETURN ON THE SHARES TRADED ON THE WARSAW STOCK EXCHANGE

Summary

The paper presents an econometric CAPM model, which describes changes of rates of return on the shares traded on the Warsaw Stock Exchange. It analyses the components of systematic risk and risk premium vectors in the years preceding Poland's accession to the European Union. The algorithm used for the description of rates of return combines hitherto Fama and French research and results from author's own considerations. It takes into account both the known and unknown parameters of future investments. The obtained results are compared with the simulation outcome based on the classic CAPM. The conducted tests have demonstrated good fit of the proposed model. Jagannathan and Wang test (1998) also confirms the correctness of the model. The results lead to a number of conclusions which may be useful for investors and portfolio managers.

Keywords: CAPM pricing model • Fama-French model • market portfolio • Fama-MacBeth method

ИЗМЕНЕНИЯ НОРМ ДОХОДНОСТИ АКЦИЙ, КОТИРУЕМЫХ НА БИРЖЕ ЦЕННЫХ БУМАГ В ВАРШАВЕ

Резюме

В статье представлена эконометрическая модель CAPM, описывающая изменения нормы доходности акций, котируемых на Бирже ценных бумаг в Варшаве. Исследуются величины составляющих векторов систематического риска и премии за риск в период, предшествующий вступлению Польши в Евросоюз. Алгоритм описывающий нормы окупаемости опирается на труды Фамы и Френча, а также на предложения автора. Объясняющие переменные модели учитывают известные и неизвестные параметры будущих инвестиций. Полученные результаты сопоставляются с имитациями, опирающимися на классический вариант CAPM. Проведенные тесты выявили хорошую увязку предлагаемой модели и эмпирических данных. Тест Джаганнатана и Ванга (1998) дополнительно подтвердил корректность модели. Полученные результаты позволяют сформулировать практические указания, полезные для инвесторов и консультантов, управляющих инвестиционными портфелями.

Ключевые слова: модель оценки CAPM • модель Фамы-Френча • рыночный портфель • метод Фамы-Макбета