

where  $f$  is continuous and satisfies the inequality

$$(17) \quad |f(t, x_1, \dots, x_n)| \leq \sum a_i(t)|x_i| + b(t).$$

Here  $a_i(t)$  and  $b(t)$  are non-negative locally integrable functions on the interval  $(a, b)$ . Together with equation (16) consider the differential inequality

$$(18) \quad |x^{(n)}| \leq \sum a_i(t)|x^{(n-i)}|,$$

where the coefficients  $a_i(t)$  are the same as in (17).

Opial proved that if  $x(t) \equiv 0$  is the only solution of inequality (18) which satisfies (12) for each choice of points  $t_i$  satisfying (11), then for each sequence of points in (13) and each sequence of positive integers satisfying (14) there is at least one solution of (16) satisfying the boundary conditions

$$x^{(j)}(t_i) = r_{ij}, \quad i = 1, \dots, m; j = 0, \dots, p_i - 1,$$

where  $r_{ij}$  are arbitrary but fixed real numbers.

What is particularly worth noticing in this result is the implication: the uniqueness of the solution of the homogeneous problem (18)–(12) implies the existence for a rather large class of non-linear boundary value problems.

Opial's results and ideas have stimulated many mathematicians in the world and his influence on the Kraków mathematical center cannot be overestimated.

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