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# Franciszek Leja

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Franciszek Leja was born on January 27th, 1885, at a village called Grodzisko Górne in southern Poland. His parents lived with their seven children on a four hectare farm. Although poor, they decided to educate one of their sons. Since his elder brother preferred to stay on the land, it was Franciszek who availed himself of the chance to study. However, the costs of his lodgings – a straw mattress on the floor – books and a school uniform were beyond his parents' means. At the age of fourteen Franciszek took up tutoring. His earnings, in addition to what his parents were able to give him, were still not enough to cover all the expenses of his schooling. Fortunately, thanks to the good offices of the Grodzisko parish priest, Franciszek benefited from a bequest made by a former vicar for the purposes of furthering the education of a gifted pupil from Grodzisko. What is more, when Franciszek, against the hopes and wishes of his father, abandoned the intention of becoming a priest and wanted to devote himself to mathematics, it was the same parish priest, a truly englightened man, who propitiated Franciszek's father and obtained his consent to the boy's lay studies.

So he entered the University of Lwów in 1904. The grant did not cover more than a quarter of his expenses, and so besides tutoring, he took on various jobs. For six months he was a bookkeeper; at another time an assistant manager in a small cigarette tube factory; for a time he was employed at collecting data in libraries about the history of the Church in the part of Poland occupied by Prussia at the time of the partitions.

The hardships Leja suffered during his studies were on a par with the general situation of his nation and his social class. At the end of 18th century just as the governing classes in Poland had become aware of the necessity of helping the peasants out of their misery, the country was partitioned between Russia, Austria and Prussia. Austria, which had captured the southern territories including Lwów and Cracow, was the most lenient of the invaders, but its policy further aggravated the poverty of Polish landholders and farm workers. At sixteen, Franciszek enters a secret organization diming at the preservation of Polish national awareness and the knowledge of Polish history. Also on holidays, when he was at home, he would carry on this activity. He was also a dedicated social worker, organizing, among other things, festivals for the benefit of the local library of the Rural School Society. He persuaded his neighbours to found a cooperative dairy society, which brought considerable profit to the village; it has been in operation to this day. He also organized an amateur orchestra, which still exists and flourishes.

On graduating from the University he was called up to the Austrian Army. He simulated illness so skilfully that they dimissed him after a few weeks. In the autumn of 1909 he was appointed to a teaching post in Drohobycz. He felt, however, that his studies had not given him much (the University of Lwów had only one professor of mathematics working full time), and therefore applied to the education authorities to be transferred to a university town so that he could continue his studies. His request was granted by the Regional School Council and Leja got a post at a grammar school – IV Gimnazjum in Cracow. His first publication, on the foundations of non-Euclidean geometry [B1], appeared in the annuals of that school. It drew the attention of Professor K. Żórawski of the Jagiellonian University. On his recommendation the Academy of Sciences in Cracow granted Leja a scholarship to study in Paris for one year. He attended lectures in mathematics and the humanities, at the Sorbonne in 1912/13. On his return to Cracow he started work on a doctor's thesis on the applications of continuous groups to differential equations. The outbreak of World War I caught him on a holidays at Grodzisko. He organized a ten-man "Bartosz Squad" (named after Bartosz Głowacki, a peasant hero in the Kościuszko Insurrection, which broke out after the second partition). Leja and his squad joined the Eastern Legion, one of the two Polish legions formed with Austria's consent to fight on her side. However, when the legionaries were requested to take an oath of allegiance to Austria, Leja left the service and returned to mathematics. He took his doctor's degree and got a post at the Jagiellonian University. At the same time he was teaching at the V Gimnazjum. He was one of the founders of the Polish Mathematical Society (1919) and its first secretary. In 1922 he passed his post-doctoral examination, and in 1923 he had a choice of two chairs in mathematics: at the University of Poznań and at the Warsaw Institute of Technology. He chose Warsaw, settled there and started lecturing at the Faculty of Chemistry, and also at the University of Warsaw. A year later he married Jadwiga Mizerska. They had no children but had the custody of a nephew. In 1936 Leja got a chair at the Jagiellonian University in Cracow, vacated by S. Zaremba, who had retired.

Soon after the Germans occupied Cracow in 1939 all the professors of the University and of other schools were invited by a German official named Müller, allegedly to listen to his lecture. Instead they were accused of opposing the German authorities through continuation of their work, arrested on the spot and taken to the concentration camp at Sachsenhausen. Many of them did not survive. Leja, who was used to hunger and cold, held out. He was among those who were freed from the camp through the intervention of the King of Sweden. Leja then returned to Grodzisko and, supporting himself and his wife on growing vegetables, he started to write a handbook of Calculus, intended as a contribution to the rehabilitation of Polish science after the war.

A few days after the liberation of Cracow Leja took up his duties at the University. He put a great deal of work in reestablishing its Mathematical Institute. It was owing to his efforts that the publication of the Annales de la Sociéte Polonaise de Mathématique was resumed (the first postwar volume appeared as early as 1945). In 1948 the present Institute of Mathematics of the Polish Academy of Sciences was founded and Leja became head of the Department of Function Theory in it. His activity in the Institute and at the Jagiellonian University contributed considerably to the development of complex analysis in Poland. He retired in 1960 but for a long time remained active doing research, conducting seminars and lecturing. At the ceremony to celebrate his 90th birthday he arrived at the Institute (having climbed its six floors on foot!) slight and lively as ever. About that time he gave the crowning touch to his university activity by offering to the University a gift of 200000 złotys (a professor's yearly salary) to be spent on prizes for outstanding students and young research workers.

Leja lived to the age of ninety four. "I am wondering why I have lived so long" he used to say. "Probably because I have often been hungry..."

He died on October 11th, 1979, and in accordance with his will he was burried at Grodzisko.

Leja's first attempts in research were made in the field of differential equation analysis by means of continuous groups. He found groups more interesting than the equations themselves – and he was the first to introduce the definition of an abstract topological group. This was done on 1925 (in a short communication in Ann. de la Soc. Polon. de Math. 3 (1925), p. 153)(*). Leja seemed to be aware of the value of his conception: he had his paper published in the most significant Polish journal [10]. But he had just become deeply interested in analytic functions, and so he did not continue his research in topological groups. He investigated the value distribution of analytic functions of several complex variables [4], their singular surfaces [5] and, above all, multiple power series. In [9] and [14] he obtained fundamental results concerning the shape of the convergence regions of such

(*) The same notion was introduced by O. Schreier in Abh. Math. Sem. Hamburg 4 (1926), 15–32.

series by a method of "hyperbolic curves". The regions in question turn out to be "hyperbolically convex". In [7], [13], [15], [22] some boundary problems were dealt with – the problem of Abel and that of the distribution of convergence and divergence points. Leja's papers were the first in this field. They showed, e.g., that within a certain part of the boundary of the domain of convergence of $\sum a_{mn} x^m y^n$ the divergence points cannot be isolated, while, on the other hand, the convergence points are capable of appearing in isolation.

Several papers, including [18] and [23], concern number series. Leja introduced the sum of a double series in the direction $(\alpha, \beta)$, i.e.,

$$S(\alpha, \beta) = \lim_{\lambda \to 0} \sum_{\alpha k + \beta l < \lambda} a_{kl},$$

which covers column, row and Cauchy's diagonal summation. He was also interested in summation methods for $\sum a_n z^n$ outside its covergence disc $K = \{z\colon |z| < r\}$ (Leja corrected here some mistakes – one due to G. Bouligand, and appearing in E. Borel, *Leçons sur les séries divergentes*, Paris 1928, and another one due to A. Zygmund). In particular, in [20] the following was shown:

Let

$$s_k(z) = \sum_{n=0}^{k} a_r z^n, \qquad c_j > 0, \ \sum c_j = \infty,$$

$$\sigma_n(z) = \sum_{j=0}^{n} c_j s_j(z) / \sum_{j=0}^{n} c_j,$$

$$\tau_n(z) = \sum_{j=0}^{n} c_j s_{n-j}(z) / \sum_{j=0}^{n} c_j.$$

Then it is possible for $\sigma_n$ to coverge for $|z| > r$, but only in a countable set with no limit point on $\partial K$, while by taking a suitable sequence $\{c_j\}$ one can obtain $\tau_n$ covergent in some open set out of $K$.

A good deal of work was done by Leja in holomorphic and polynomial sequences. Let $f_n$ be holomorphic in the open set $\Omega \subset C$ and $E \subset \Omega$. Then the least upper bound $\gamma = \gamma(E)$ of numbers $r > 0$ such that $\sum f_n(z) \lambda^n$ is uniformly convergent on the set

$$\{(z, \lambda) \in C^2\colon\ z \in E,\ |\lambda| \leqslant r\},$$

is called the *covergence factor* of $\{f_n\}$ in $E$. Then

$$\frac{1}{\gamma} = \limsup_{n \to \infty} \sqrt[n]{\sup_E |f_n(z)|}$$

holds.

Suppose $f_n$ is convergent or at least bounded on $F \subset \Omega$; what can be said about $\gamma(E)$ for related sets $E \subset \Omega$? In [24], [25], [28], [29], [31]–[34] Leja proved among other things that:

1° Let $f_n$ be an $n$-th degree polynomial and $\sup_n |f_n(z)| < \infty$ almost everywhere on a continuum $F$. Then $\gamma(E) > 0$ for any bounded set $E$. In particular, when $D$ is bounded and $F = \partial D$, we have $\gamma(D) \geqslant 1$.

2° Let $f_n$ be merely holomorphic, and $D$ a domain with its closure in $\Omega$. If $\sup_n |f_n(z)| < \infty$ almost everywhere on $\partial D$, $\gamma(D)$ equals 0 or 1. This generalizes the Hartogs lemma, which enabled him to obtain many results in several complex variables.

Now let $f_n$ be a homogeneous polynomial of degree $n$ of $k$ complex variables. Consider the series $\sum f_n$. In contrast to the case of $k = 1$, the general case was been little explored before Leja. It was not even known whether the covergence region is nonvoid (except for trivial cases). Some results were due to Hartogs; Leja gave a significant generalization of them in [30]. He proved that the covergence of the series $\sum f_n$ of homogeneous polynomials in a set $E$ implies its covergence in a neighbourhood of 0 provided $E$ is of positive triangle écart. On the other hand, he gave an example of covergence on a dense subset of $C^2$, with no inner point.

The term écart was introduced by Leja within his extremal points method. Originating from interpolation and potential theory, connected with the particular concept of transfinite diameter of Fekete, the method consists in general in what follows:

Let $\omega\colon T \times T \to \langle 0, \infty)$ be a continuous symmetric function of two points in a topological space $T$. Let

$$p^{(n)} = \{p_0, p_1, \ldots, p_n\}, \qquad V(p^{(n)}) = \prod_{0 \leqslant j < k \leqslant n} \omega(p_j, p_k).$$

The set $q^{(n)} = \{q_0, \ldots, q_n\} \subset E$ is called $\omega$-extremal for a compact $E$ if

$$V(p^{(n)}) \leqslant V(q^{(n)}) (p^{(n)} \subset E).$$

We have $v_n = V(q^{(n)})^{2/n(n+1)} \searrow v(E, \omega)$.

The number $v(E, \omega)$ was called the *$\omega$-écart of $E$*. With $T = C$, $\omega(p, q) = |p - q|$, the transfinite diameter (logarithmic capacity) is obtained, whereas $\omega(p, q) = \frac{1}{2}|\mathrm{re}\, p \cdot \mathrm{im}\, q - \mathrm{im}\, p \cdot \mathrm{re}\, q|$ leads to the triangle écart mentioned above. Leja then formed an extremal function

$$\Phi(x) = \Phi(x, E, \omega) := \lim_{n \to \infty} \sqrt[n]{\inf_{p^{(n)} \subset E} \big(\max_{0 \leqslant j \leqslant n} \Phi^{(j)}(x, p^{(n)})\big)}, \qquad x \in T,$$

$$\Phi^{(j)}(x, p^{(n)}) := \sum_{\substack{k \neq 0 \\ k \neq j}}^{n} \frac{\omega(x, p_k)}{\omega(p_j, p_k)}, \qquad j = 0, \ldots, n, \ x \in T.$$

With this tool, he obtained the following main results:

1° a criterion for a homogeneous polynomial series to converge in some open set;

2° the existence and construction of the Green function and conformal mapping of plane regions;

3° the same results for Dirichlet's problem (for $\Delta u = 0$), including the regularity criterion for boundary points;

4° a test for a holomorphic function to be univalent;

5° a new proof of Hartogs' theorem on separate analiticity.

More details may be found in [B13].

The extremal points method was used and evolved by Leja's mathematical school in Cracow; its tools were enriched and possibilities enlarged comprising, e.g., some important results in several complex variables. Also some specialists in other countries were influenced by Leja's method.

One of the most fruitful tools in Leja's method is the Polynomial Lemma. It is known in many formulations given by its author and other mathematicians. One of the original versions reads as follows:

Let us say that the set $E$ has the property $W$ at $a \in C$ whenever there is an $r > 0$ and a set $A \subset \langle 0, r\rangle$ of Lebesgue measure $r$ such that the values of $|a-z|$ on $E$ cover $A$. Note that any continuum $E$ has this property at any $a \in E$.

THE POLYNOMIAL LEMMA. *Suppose that*

1° *$E$ has the property $W$ at $a$, and*

2° *the family $F$ of polynomials is bounded at points of $E$:*

$$\sup\{|f(z)\colon f \in F\} < \infty \qquad (z \in E).$$

*Then*

$$\forall_{\varepsilon>0}\exists_{M>0}\exists_{\delta>0}\forall_{f\in F}\forall_{z\in C}\,|z-a| < \delta \Rightarrow |f(z)| < M(1+\varepsilon)^{s(f)},$$

*where $s(f)$ stands for the degree of $f$.*

To show the reader the strength of the lemma let us mention that it was utilized by Leja to give a new proof of Hartogs' theorem on separate analiticity as well as to strengthen considerably Hartogs' result concerning series of homogeneous polynomials.

Later, various authors proved its usefulness for holomorphic functions in topological linear spaces. Surprisingly, its proof is very easy – it only requires acquaintance with the very beginnings of Lebesgue measure theory and the Lagrange interpolation formula.

Leja showed considerable intuition in perceiving what was important and fruitful in mathematics. Thanks to that intuition, he offered to mathematics the concept of topological group; thanks to it he started research on several complex variables at the time when nobody else in Poland knew them. He also showed himself able to bring together a group of disciples ready to follow the directions he initiated as well as to study other topics in complex analysis.

Finally, let us mention Leja's excellent teaching ability. It manifested itself in particular in his textbooks. The one on Calculus (16 printings 1947–1979, 150000 copies), concise and rigorous and yet easy to read, is still widely used probably by all Polish students learning Calculus. His university textbook of complex analysis on the plane, republished five times (some 40000 copies), acquaints the student with all the main parts of function theory.

## List of publications

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