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Global solvability of the mixed problem for first order functional partial differential equations

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Abstract. Global generalized solutions to the mixed (initial-boundary) problem for quasilinear hyperbolic systems of functional PDE's of the first order in two independent variables are investigated.

1. Introduction. Let $a_0, b, \Omega > 0$ be given constants. Let us denote by $|u|_n = \max_{1 \leqslant i \leqslant n} |u_i|$ the norm of u in R^n. Write $I_{a_0} = \{(x, y)\colon 0 \leqslant x \leqslant a_0, 0 \leqslant y \leqslant b\}$.

We seek global generalized (in the sense almost everywhere) solutions of hyperbolic systems of quasilinear functional partial differential equations in diagonal form

$$(1) \quad D_x z_i(x, y) + \lambda_i(x, y, z(x, y), (Vz)(x, y)) D_y z_i(x, y) = f_i(x, y, z(x, y), (Vz)(x, y)), \quad (x, y) \in I_{a_0},\ i = 1, \ldots, n,$$

with the initial conditions

$$(2) \qquad z(0, y) = \varphi(y), \quad y \in [0, b],$$

and the boundary conditions

$$(3) \quad \begin{aligned} z_i(x, 0) &= h_{0i}(x), \quad i \in J_0 = \{i\colon \operatorname{sgn} \lambda_i(0, 0, 0, 0) = 1\}, \\ z_i(x, b) &= h_{bi}(x), \quad i \in J_b = \{i\colon \operatorname{sgn} \lambda_i(0, b, 0, 0) = -1\},\ x \in [0, a_0], \end{aligned}$$

where $z(x, y) = (z_1(x, y), \ldots, z_n(x, y))$, $\varphi(y) = (\varphi_1(y), \ldots, \varphi_n(y))$, $D_x = \partial/\partial x$, $D_y = \partial/\partial y$.

The global solvability (with a different definition of generalized solution) of hyperbolic systems of partial differential equations of the first order has been investigated by various authors: the mixed problem for quasilinear equations by Filimonov [3], the mixed problem for semilinear equations by Abolinia and Myshkis [1], the Cauchy problem for quasilinear equations by Myshkis and Filimonov [5], the mixed problem for two equations in a very special form by Doktor [2], and the Cauchy problem by Johnson and Smoller [4].

In the present paper, the global solvability of problem (1)–(3) is ensured by the monotonic behavior of given functions and a growth restriction on the right-hand side. The functional operator V includes retarded arguments and Volterra hereditary operators.

2. Basic assumptions.

ASSUMPTION H_1. (i) The functions $\operatorname{sgn}\lambda_i(\cdot, 0, \cdot, \cdot)$, $\operatorname{sgn}\lambda_i(\cdot, b, \cdot, \cdot)$: $G_{a_0} = [0, a_0]\times\bar{\Omega}\times\bar{\Omega}\to\mathbf{R}$, $i = 1, \ldots, n$, are constant in G_{a_0}, where $\bar{\Omega} = [-\Omega, \Omega]^n \subset \mathbf{R}^n$

(ii) $\lambda_i(\cdot, y, u, v)$: $[0, a_0]\to\mathbf{R}$, $i = 1, \ldots, n$, are measurable for every $(y, u, v)\in G = [0, b]\times\bar{\Omega}\times\bar{\Omega}$.

(iii) $\lambda_i(x, \cdot)$: $G\to\mathbf{R}$, $i = 1, \ldots, n$, are continuous for a.e. $x\in[0, a_0]$.

(iv) There are a constant $\Lambda > 0$ and integrable functions l_j: $[0, a_0]\to\mathbf{R}_+ = [0, +\infty)$, $j = 1, 2, 3$, such that for all (y, u, v), $(\bar{y}, \bar{u}, \bar{v})\in G$, a.e. in $[0, a_0]$, we have

$$|\lambda_i(x, y, u, v)| \leqslant \Lambda,$$

$$|\lambda_i(x, y, u, v) - \lambda_i(x, \bar{y}, \bar{u}, \bar{v})| \leqslant l_1(x)|y-\bar{y}| + l_2(x)|u-\bar{u}|_n + l_3(x)|v-\bar{v}|_n,$$

$i = 1, \ldots, n$.

(v) There are constants $\varepsilon_0\in(0, b)$ and $\Lambda_0 > 0$, such that $\lambda_i(x, y, u, v) \geqslant \Lambda_0$ for $i\in J_0$, $y\in[0, \varepsilon_0]$, $(x, u, v)\in G_{a_0}$, and $-\lambda_i(x, y, u, v)\geqslant\Lambda_0$ for $i\in J_b$, $y\in[b-\varepsilon_0, b]$, $(x, u, v)\in G_{a_0}$.

ASSUMPTION H_2. (i) Assumptions (ii), (iii) of H_1 are satisfied by the functions f_i: $[0, a_0]\times G\to\mathbf{R}$, $i = 1, \ldots, n$.

(ii) There are a constant $F > 0$ and integrable functions k_j: $[0, a_0]\to\mathbf{R}_+$, $j = 1, 2, 3$, such that assumption (iv) of H_1 is satisfied by f_i with Λ replaced by F and l_j by k_j.

ASSUMPTION H_3. (i) There is a constant $H\geqslant 0$ such that for all $x, \bar{x}\in[0, a_0]$ we have

$$|h_{0i}(x) - h_{0i}(\bar{x})| \leqslant H|x-\bar{x}|, \quad i\in J_0, \quad |h_{bi}(x) - h_{bi}(\bar{x})| \leqslant H|x-\bar{x}|, \quad i\in J_b.$$

(ii) The compatibility conditions

$$h_{0i}(0) = \varphi_i(0), \quad i\in J_0, \quad h_{bi}(0) = \varphi_i(b), \quad i\in J_b,$$

are satisfied.

(iii) There is a constant $\Phi\geqslant 0$ such that for all $y, \bar{y}\in[0, b]$ we have

$$|\varphi(y) - \varphi(\bar{y})|_n \leqslant \Phi|y-\bar{y}| \quad \text{and} \quad \max_{[0,b]}|\varphi(y)|_n = \Phi_1 < \Omega.$$

We denote by $B(a)$ the set of all continuous functions z: $I_a\to\mathbf{R}^n$ such that $|z(x, y)|_n \leqslant \Omega$, $(x, y)\in I_a$.

We write $B(a, Q)$ for the set of all functions $z \in B(a)$ satisfying

$$(4) \quad |z(x, y)-z(x, \bar{y})|_n \leqslant Q|y-\bar{y}|, \quad z(0, y) = \varphi(y), \qquad \text{for all } (x, y), (x, \bar{y}) \in I_a.$$

ASSUMPTION H_4. (i) V: $B(a_0, Q) \to B(a_0)$.

(ii) There are integrable functions c, d: $[0, a_0] \to \boldsymbol{R}_+$, such that for every $z \in B(a_0, Q)$ we have

$$[\![(Vz)(x, \cdot)]\!] \leqslant c(x)[\![z(x, \cdot)]\!]+d(x), \quad \text{a.e. in } [0, a_0],$$

where

$$[\![z(x, \cdot)]\!] = \sup_{\substack{y, \bar{y} \in [0, b] \\ y \neq \bar{y}}} \frac{|z(x, y)-z(x, \bar{y})|_n}{|y-\bar{y}|}.$$

(iii) There is an integrable function m: $[0, a_0] \to \boldsymbol{R}_+$ such that for all $z, \bar{z} \in B(a_0, Q)$ we have

$$\|Vz-V\bar{z}\|_x \leqslant m(x)\|z-\bar{z}\|_x,$$

where $\|z\|_x = \sup_{(t,y) \in I_x} |z(t, y)|_n$, $I_x = [0, x] \times [0, b]$.

Remark 1. In particular, from assumption (iii) of H_4 it follows that V is an operator of Volterra type.

3. Preliminary lemmas. We consider, for $z \in B(a, Q)$, the problem

$$(5) \quad \begin{aligned} &D_t g(t; x, y) = \lambda_i\big(t, g(t; x, y), z(t, g(t; x, y)), (Vz)(t, g(t; x, y))\big) \\ &\qquad\qquad\qquad \text{for a.e. } t \in [0, x], \ (x, y) \in I_a, \ i = 1, \ldots, n, \\ &g(x; x, y) = y. \end{aligned}$$

From assumptions (ii)–(iv) of H_1, (ii) of H_4 and from $z \in B(a, Q)$ we conclude that the right-hand sides of system (5) satisfy the Carathéodory conditions. Thus, for every $z \in B(a, Q)$, there is a unique solution $g_i = g_i^z(t; x, y)$ of problem (5).

We denote by $\tau_i(x, y, z)$ the smallest value of t for which this solution is defined. Then $\big(\tau_i(x, y, z), g_i^z(\tau_i(x, y, z); x, y)\big)$ belongs to the boundary of I_a.

Let us introduce the following notations:

$$I_{\varphi i}^z = \{(x, y)\colon (x, y) \in I_a, \ \tau_i(x, y, z) = 0\},$$

$$I_{0i}^z = \{(x, y)\colon (x, y) \in I_a, \ \tau_i(x, y, z) > 0, \ g_i^z(\tau_i(x, y, z); x, y) = 0\},$$

$$I_{bi}^z = \{(x, y)\colon (x, y) \in I_a, \ \tau_i(x, y, z) > 0, \ g_i^z(\tau_i(x, y, z); x, y) = b\}.$$

Consider the ball $B(a, Q, r) = \{z\colon z \in B(a, Q), \max_{I_a} |z(x, y)-\varphi(y)|_n \leqslant r\}$, where $0 < r \leqslant \Omega-\Phi_1$. Obviously, for $z \in B(a, Q, r)$ we have $|z(x, y)|_n \leqslant r+\Phi_1 \leqslant \Omega$. Hence, for $z \in B(a, Q, r)$, the point $(x, y, z(x, y), (Vz)(x, y))$ with

$(x, y) \in I_a$ belongs to $I_a \times \bar{\Omega} \times \bar{\Omega}$. Thus, for every $z \in B(a, Q, r)$ the corresponding family of characteristics is defined.

We shall need the following

LEMMA 1. *If assumptions* (ii)–(iv) *of* H_1 *and* H_4 *are satisfied, then for all* $(x, y), (x, \bar{y}) \in I_a$, $u, v \in B(a, Q, r)$,

$$|g_i^u(t; x, y) - g_i^v(t; x, \bar{y})| \leqslant L_1(|y-\bar{y}| + L_2\|u-v\|), \quad i = 1, \ldots, n,$$

where

$$L_1 = L_1(a) = \exp\Big(\int_0^a \{l_1(t) + l_2(t)Q + l_3(t)[c(t)Q + d(t)]\}\,dt\Big),$$

$$L_2 = L_2(a) = \int_0^a [l_2(t) + l_3(t)m(t)]\,dt \quad \text{and} \quad \|z\| = \sup_{I_a} |z(x, y)|_n.$$

The proof follows, as in [6], from the previous inequalities and Gronwall's Lemma.

Now we consider in $B(a, Q, r)$ the operator S defined by

$$(Sz)_i(x, y) = (Rz)_i(x, y) + \int_{\tau_i(x,y,z)}^{x} f_i\big(t, g_i(t; x, y), z(t, g_i(t; x, y)), (Vz)(t, g_i(t; x, y))\big)\,dt,$$

$i = 1, \ldots, n$, where

$$(Rz)_i(x, y) = \begin{cases} \varphi_i\big(g_i(0; x, y)\big), & (x, y) \in I_{\varphi i}^z, \\ h_{0i}\big(\tau_i(x, y, z)\big), & (x, y) \in I_{0i}^z, \\ h_{bi}\big(\tau_i(x, y, z)\big), & (x, y) \in I_{bi}^z. \end{cases}$$

Put

$$K_1 = K_1(a) = \int_0^a \{k_1(t) + k_2(t)Q + k_3(t)[c(t)Q + d(t)]\}\,dt,$$

$$K_2 = K_2(a) = \int_0^a [k_2(t) + k_3(t)m(t)]\,dt,$$

$$Q^S = L_1\big(K_1 + \max\{\Phi, \Lambda_0^{-1}(H+F)\}\big), \qquad \mu = L_1 L_2\big(\Phi + \Lambda_0^{-1}(H+F) + K_1\big) + K_2.$$

Similarly to [6] we can prove the following

LEMMA 2. *Let Assumptions* H_1–H_4 *be satisfied, and let* $a \in (0, a_0]$ *be so small that*

(6) $\quad a \leqslant r(H + \Phi\Lambda + F)^{-1}, \; a \leqslant \varepsilon_0 \Lambda^{-1}, \; a < b(2\Lambda)^{-1}, \; Q^S \leqslant Q, \; \mu < 1.$

Then the operator S: $B(a, Q, r) \to B(a, Q, r)$ *is a contraction.*

Remark 2. Note that if r, Q are fixed and $a \to 0^+$ then $Q^S \to \max\{\Phi, \Lambda_0^{-1}(H+F)\}$ and $\mu \to 0$, since $L_2 \to 0$, $L_1 \to 1$, $K_2 \to 0$. Therefore, for arbitrary $r \in (0, \Omega - \Phi_1]$, if $Q > \max\{\Phi, \Lambda_0^{-1}(H+F)\}$, then for sufficiently small $a \in (0, a_0]$, all inequalities of (6) are satisfied.

4. Local existence theorem.

THEOREM 1. *Let Assumptions* H_1–H_4 *hold. Then, for any* $r \in (0, \Omega - \Phi_1]$, *and any sufficiently large constant* Q, *there are* $a \in (0, a_0]$ *and a function* $z\colon I_a \to \boldsymbol{R}^n$, $z \in B(a, Q, r)$, *which satisfies* (1) *a.e. in* I_a *and* (2), (3) *everywhere in* $[0, b]$, $[0, a]$, *respectively. Furthermore,* z *is unique in* $B(a, Q, r)$.

Proof. From Lemma 2, in view of the completeness of $B(a, Q, r)$, it follows that there exists $z \in B(a, Q, r)$ such that $Sz = z$. Proceeding as in [6] we can prove, using the group property of the characteristic lines and the chain rule for differentiation, that the fixed point z of S satisfies (1) a.e. in I_a and (2), (3) everywhere in $[0, b]$, $[0, a]$, respectively. This concludes the proof.

5. Global existence theorems. Let $D(a) = \{z\colon z \in B(a),\ z$ is a nondecreasing function of $y\}$. We write $D(a, Q)$ for the set of all functions $z \in D(a)$ satisfying conditions (4).

ASSUMPTION H_5. (i) The function $\varphi\colon [0, b] \to \boldsymbol{R}^n$ is nondecreasing.

(ii) The functions $h_{0i}\colon [0, a_0] \to \boldsymbol{R}$, $i \in J_0$, are nonincreasing, and $h_{bi}\colon [0, a_0] \to \boldsymbol{R}$, $i \in J_b$, are nondecreasing.

(iii) $\lambda_i\colon E_{a_0} = [0, a_0] \times [0, b] \times \boldsymbol{R}^n \times \boldsymbol{R}^n \to \boldsymbol{R}$, $i = 1, \ldots, n$, are nondecreasing functions of y, u, v, and, for every fixed $\Omega > 0$, satisfy Assumption H_1 with $l_j(x) = \text{const} = P$, $j = 1, 2, 3$.

(iv) $f_i\colon E_{a_0} \to \boldsymbol{R}$, $i = 1, \ldots, n$, are nondecreasing functions of y, u, v, for every fixed $\Omega > 0$ satisfy Assumption H_2 with $k_j(x) = \text{const} = P$, $j = 1, 2, 3$, and

(7) $f_i(x, y, u, v) \geqslant 0, \quad i \in J_0, \quad f_i(x, y, u, v) \leqslant 0, \quad i \in J_b, \quad (x, y, u, v) \in E_{a_0}.$

(v) There are an integrable function $p\colon [0, a_0] \to \boldsymbol{R}_+$ and a continuous nondecreasing function $q\colon \boldsymbol{R}_+ \to \boldsymbol{R}_+$, $q(0) > 0$, $\int_0^{+\infty} dt/q(t) = +\infty$, such that for every $z \in D(a_0, Q)$, we have

(8) $$|f_i(x, y, z(x, y), (Vz)(x, y))| \leqslant p(x)q(|z(x, y)|_n), \quad (x, y) \in I_{a_0},$$

$i = 1, \ldots, n$.

(vi) For every fixed $\Omega > 0$, the operator $V\colon D(a_0, Q) \to D(a_0)$ satisfies Assumption H_4 with $c(x) = d(x) = m(x) = \text{const} = s$.

Remark 3. From assumptions (iii), (vi) of H_5 we deduce that in Lemma 1 we can put $L_1 = 1$, provided $u = v \in D(a, Q)$. This follows at once from the fact that the absolute value of the difference of any two solutions of (5) is a nondecreasing function of y.

Remark 4. From assumption (iii) of H_5 it may be concluded that $J_0 \cap J_b = \emptyset$.

THEOREM 2. *Let Assumptions* H_3, H_5 *hold. Then there exists a unique generalized solution of problem* (1)–(3) *on the whole* I_{a_0}.

Proof. From Theorem 1 and the monotonicity of given functions it follows that the operator S maps the ball $D(a, Q, r) = \{z\colon z \in D(a, Q), |z(x, y) - \varphi(y)|_n \leqslant r\}$ into itself, provided conditions (6) are satisfied.

From assumption (v) of H_5 we see that

$$|z(x, y)|_n \leqslant \max\{\tilde{H}, \Phi_1\} + \int_0^x p(t) q(\|z\|_t) dt,$$

where $\tilde{H} = \max_{0 \leqslant x \leqslant a_0} \{\max\{\max_{i \in J_0} h_{0i}(x), \max_{i \in J_b} h_{bi}(x)\}\}$. Hence, by a theorem on integral inequalities, we get $|z(x, y)|_n \leqslant \omega$, where ω is defined by

$$\int_{\max\{\tilde{H}, \Phi_1\}}^{\omega} \frac{dt}{q(t)} = \int_0^{a_0} p(x) dx.$$

Therefore, if we take $\Omega > \omega$, then $|z(x, y)|_n \leqslant \Omega$ in an arbitrary existence domain I_a of a generalized solution.

The global uniqueness in I_{a_0} of a generalized solution of problem (1)–(3) can be proved by contradiction: if z and $\bar{z}$ are two solutions and $z \not\equiv \bar{z}$ in I_{a_0}, it suffices to move the origin to $\inf\{x\colon z(x, y) \not\equiv \bar{z}(x, y)\}$, and use the fact that S is a contraction operator.

Now we prove global existence. Note that if $l_1(x) = \text{const} = P$, then it is easy to show that we can take $\varepsilon_0 = \Lambda_0 P^{-1}$.

We shall construct a solution in the whole I_{a_0} step by step with respect to x. Putting $r = \Omega - \omega$ we apply Theorem 1, choosing suitable constants $Q = Q_1$ and $a = a_1$. Hence, if $a_1 < a_0$ then by moving the initial point to $x = a_1$ (i.e. by changing x to $\tilde{x} = x - a_1$) and using $z(a_1, y)$ for the initial data, we can apply Theorem 1 again with the same Ω and r. Let us denote the corresponding constants $\tilde{Q}$ and $\tilde{a}$ by Q_2 and a_2, respectively.

Further, if $a_1 + a_2 < a_0$ then by repeating this process we shall show that it is possible to choose the constants Q_k, a_k $(k = 1, 2, \ldots)$ so that the whole rectangle I_{a_0} will be covered.

Indeed, if we define the values of Q_k and a_k by suitable inequalities then we can regard H, Λ, F, r, P, s as fixed. We can take Q_{k-1} for the Lipschitz constant of the initial function at the kth step (for $k > 1$). Thus, the inequalities defining Q_k and a_k take the form (cf. Remark 3)

$$\Lambda a_k \leqslant \Lambda_0 P^{-1}, \quad \Lambda a_k < \frac{b}{2}, \quad a_k \leqslant \frac{1}{P(1+s)}, \quad a_k \leqslant \frac{1}{P(1+s)Q_k},$$

$$(H + Q_{k-1}\Lambda + F) a_k \leqslant r, \quad P(1+s)(Q_{k-1} + \Lambda_0^{-1}(H+F) + 3) a_k < 1,$$

$$Q_{k-1} + \Lambda_0^{-1}(H+F) + 2 \leqslant Q_k.$$

Let us introduce the following notations:

$$\alpha_1 = \min\left\{\frac{\Lambda_0}{\Lambda P}, \frac{b}{2\Lambda}, \frac{1}{P(1+s)}, \frac{r}{2(H+F)}, \frac{1}{2P(1+s)[\Lambda_0^{-1}(H+F)+3]}\right\},$$

$$\alpha_2 = \min\left\{\frac{r}{2\Lambda}, \frac{1}{2P(1+s)}\right\}, \quad \alpha_3 = \Lambda_0^{-1}(H+F)+2.$$

Put $Q_0 = \Phi$, $Q_k = Q_{k-1}+\alpha_3 = \Phi+k\alpha_3$, $k = 1, 2, \ldots$ Take

$$\alpha_4 \geqslant \max\{0, \alpha_2/\alpha_1 - \Phi - \alpha_3\}.$$

It can be easily verified that if

$$a_k = \frac{\alpha_2}{\Phi+\alpha_4+k\alpha_3}, \quad Q_k = \Phi+k\alpha_3, \quad k = 1, 2,$$

then all the necessary inequalities are satisfied. Since

$$\sum_{k=1}^{\infty} a_k = \sum_{k=1}^{\infty} \frac{\alpha_2}{\Phi+\alpha_4+k\alpha_3} = +\infty$$

and a_0 is finite, we get the existence of a solution in the whole I_{a_0}.

Remark 5. Assumption H_5 can be "reversed" (φ, λ_i, f_i, z nonincreasing, instead of nondecreasing, etc.). These monotonicity assumptions imply that the solution has no discontinuities. For instance, in gas dynamics, if they are violated a shock wave may occur, and global existence fails.

Now we consider system (1) in the semi-strip $I_\infty = \{(x, y)\colon 0 \leqslant x < +\infty, 0 \leqslant y \leqslant b\}$ with initial conditions (2) and boundary conditions (3) for $x \in \boldsymbol{R}_+$.

Assumption H_6. (i) The functions λ_i, f_i $E_\infty = I_\infty \times \boldsymbol{R}^n \times \boldsymbol{R}^n \to \boldsymbol{R}$, $i = 1, \ldots, n$, are measurable with respect to the first variable, locally bounded, locally Lipschitzian with respect to the last three variables with constants $l_j = k_j = P$, and nondecreasing with respect to the last three variables.

(ii) The functions $\operatorname{sgn}\lambda_i(\cdot, 0, \cdot, \cdot)$, $\operatorname{sgn}\lambda_i(\cdot, b, \quad)$: $\tilde{E}_\infty = \boldsymbol{R}_+ \times \boldsymbol{R}^n \times \boldsymbol{R}^n \to \boldsymbol{R}$, $i = 1, \ldots, n$, are constant in $\tilde{E}_\infty$, and

$$\liminf_{(\xi, y, \eta, \varkappa) \to (x, 0, u, v)} \lambda_i(\xi, y, \eta, \varkappa) > 0, \quad i \in J_0,$$

$$\limsup_{(\xi, y, \eta, \varkappa) \to (x, b, u, v)} \lambda_i(\xi, y, \eta, \varkappa) < 0, \quad i \in J_b.$$

(iii) The functions f_i, $i = 1, \ldots, n$, satisfy in E_∞ inequalities (7), (8) with a locally integrable function p.

(iv) The functions h_{0i}, h_{bi}: $\boldsymbol{R}_+ \to \boldsymbol{R}$, φ: $[0, b] \to \boldsymbol{R}^n$, are locally Lipschitzian, satisfy the compatibility conditions (ii) of H_3, and h_{0i}, $i \in J_0$, are nonincreasing and h_{bi}, $i \in J_b$, are nondecreasing.

As a corollary of Theorem 2 we can formulate the following.

THEOREM 3. *If Assumption* H_6 *is satisfied, then there exists on* I_∞ *a unique generalized solution of problem* (1)–(3).

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