

*SOME OLD AND NEW PROBLEMS
IN THE INDEPENDENCE THEORY*

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0. Introduction. About sixty years ago a similarity was noticed between the linear dependence in linear spaces and the algebraic dependence in the field theory. During the last one hundred years several notions of “independence” or “freedom” have appeared in mathematics. Since they have some common features, there appeared a tendency to find general schemes containing all those notions as particular cases. Such a scheme permits simplifications and unifications of proofs of various results lying even in rather distant branches of mathematics, e.g. dependence relations in some topological investigations (see [47], [48], [217], [50] and [579]) and in electrical networks (see [329] and also [500], [229], [67], [399], [527], [263]). It also allows a global analysis and inspires new investigations in some, interesting from this and other points of view, concrete cases.

An abstract notion of independence has been so far based either on the Set Theory (started independently by Whitney [538], van der Waerden [524], Menger [326], [327], and Nakasawa [339]) or on the Universal Algebra (Ph. Hall, unpublished, and Marczewski [302*], [305*], [312*] and [313*] — this idea goes back to the important paper of Birkhoff [29]). Both these directions are interrelated and in both of them there is a tendency to create very general schemes in order to enclose all notions of independence as special cases.

This article is devoted mainly to the algebraic scheme, which is closer to the work of Marczewski himself. Professor Marczewski has played a pioneering role in this area and has inspired many further investigations. Among quoted papers, those closely related to his investigations will be marked with an asterisk (e.g. [312*]). Bibliography of papers concerning the algebraic scheme is fairly complete (to the best of my knowledge) and the most representative research on the set-theoretical scheme will be also noted in the next section and in the bibliography.

The inspiration to write this article has come from Professor W. Narkiewicz and the article itself owes much to conversations with Professors M. Sekanina, B. Csákány, W. Narkiewicz, G. Eigenthaler and H. Lausch. I want to express my gratitude to Professor Sekanina for his help in the preparation of Section 8, and to Professor Csákány for calling my attention to papers of Lovász, Megyesi and Szodoray.

Notions and notation not defined here can be found in [532*], [305*], [174*], [193*], [262*] and [86*] or in special references. Problems which have appeared in Colloquium Mathematicum will be sometimes described by their consecutive number and the volume (e.g. P 776 of J. Płonka, CM 24 (1972)).

I. THE SET-THEORETICAL SCHEME

1. Matroids. A short survey. In most cases the set-theoretical scheme either gives an axiomatic treatment of the notion of dependence of an element a on a subset Z of a considered universum A , or enumerates the properties of a distinguished family of subsets (called *independent subsets*) of A . These conceptions originated in the thirties of this century. In §33 of the second edition (1937) of vol. I of his "Moderne Algebra" [524], van der Waerden lists four basic properties of the relation " a depends on (a finite set) Z " (while in the first edition [523] (in 1930) this notion is not considered). These properties are satisfied by the linear dependence and by the algebraic dependence, and from them one can derive all usually used properties of these two notions. Independently, in 1935, Whitney [538] introduced the notion of a matroid or of an independence space, which axiomatizes, in fact, the same structure.

The pair $\mathfrak{M} = (A; \mathbf{J})$ is called a *matroid* (or an *independence space*), if

- (0) A is a finite set,
- (1) $\emptyset \in \mathbf{J} \subset 2^A$,
- (2) $X \in \mathbf{J}, Y \subset X \Rightarrow Y \in \mathbf{J}$,
- (3) $\nexists X, Y \in \mathbf{J}, \text{card}(X) = \text{card}(Y) + 1 \Rightarrow (\exists a \in X \setminus Y)(Y \cup \{a\}) \in \mathbf{J}$.

If $X \in \mathbf{J}$, it is called *independent*, otherwise X is called *dependent*.

A commonly used tool in the theory of matroids is the notion of a *rank function* $\varrho: 2^A \rightarrow \mathbb{Z}$ defined in the following way:

- (4) $\varrho(X) = \max \{ \text{card}(Y) \mid Y \subset X, Y \in \mathbf{J} \}$.

Indeed, Whitney in his fundamental paper [538] has defined matroid in terms of its rank function. One can consider a generalization of matroids to the infinite sets (*infinite matroids*, [392], see also [5], [43], [66], [113*], [216], [402], [553], [597], [618], [760*], [331] (p. 90), [532*] (p. 385-401)).

There are also known several other ways of introducing the notion of a matroid (see, e.g., [208], [393], [479], [112*], [403], [113*], [394],

[116*], [806], [330], [317], [93*], [281], [542], [553a], [397], [532*], [549], [595], [594*]). Related notions are: independence structures, independence functions, abstract linear dependence, abstract dependence, transitive dependence relations, general vector spaces, basic systems, exchange systems, spanning systems, closure spaces, independent transversals, common transversals, transversal matroids, cycle systems, circuit spaces, gammoids, polymatroids, tabloids, pregeometries, v^* -algebras, etc. Surveys of the matroid theory can be found in the books: [69]-[71], [93*], [129], [199], [263], [331], [397], [504], [532*], [549], [553a], [594], [595], [682], [707], [776] and, for example, in articles: [94*], [227], [281], [394], [542], [702], [703], [709], [717*], [744], [782], [806].

V. Dlab has generalized in [111*] (see also [115*] and [118*]) the notion of dependence relation in a way such that he has obtained linear dependence in groups (which was considered independently by Szele [478] and Dlab [106]; see also [14], [336], [110], [165], [166]) as a special case.

In these axiomatic treatments of dependence relation an important role is played by a kind of "exchange axiom", which is usually used for proving "Steinitz' exchange (or "augmentation") theorem" (it appeared first in Grassmann's book [188], see also [463] and [222]), Hamel's theorem on existence of a basis, the theorem that bases of a linear space have the same cardinality, and some others. An axiomatic treatment of dependence relations is very elegant and is of great didactic value. For these reasons it is used in several text-books (e.g. [525], [368], [547], [476], [232], [86*], [253], [28]).

In the theory of matroids there are natural assumptions that the set A is finite, or that the family J is of a finite character, i.e.

(5) $X \in J$ iff every finite $Y \subset X$ belongs to J .

This leads to many combinatorial considerations, see for example the books: [71], [89], [93*], [331], [398], [493], [532*], [549], [553a], [595], [682], [701], [812], [813]. And, on the other hand, the theory of matroids or of independence spaces can be applied to some combinatorial problems (see [127], [128], [130], [132], [263], [270], [333], [348], [363], [391], [446], [461], [501], [502], [529], [530], [549], [683], [746], [809], [810] and [834]).

Some earlier classical results belong essentially to the theory of matroids. For example, the theorem of Rado [391], [392] on independent transversals gives, in a special case, necessary and sufficient conditions for a family of finite sets to have a transversal. This had been proved earlier by Ph. Hall [206] and M. Hall, Jr. [205]. It seems worth to note that the Knaster-Tarski theorem [255] is equivalent to the Perfect-Pym theorem [365] on transversals. An important theorem relating transversal theory to the theory of independence spaces was first proved by Edmonds and Fulkerson [132] in the finite case and independently by Mirsky and

Perfect [333] in the general case. Edmonds' subsequent discovery that most of the known efficient solutions to combinatorial problems can be associated with a matroid structure (see [131], [263]) has led to considerable interest in matroids during recent years. D. J. A. Welsh in [532*] wrote: "Since then interest in matroids and their applications in combinatorial theory has accelerated rapidly. This is probably due to the discovery independently by Edmonds and Fulkerson (1965) and Mirsky and Perfect (1967) of new important class of matroids called transversal matroids. It is in the field of transversal theory that matroids seem so far to have had the most effect (measured in terms of new results obtained or easier proofs found of known results). The beauty and importance of matroids is perhaps best appreciated by the study of two covering and packing theorems of Edmonds [127], [128]. These results give as easy corollaries earlier difficult and intricate theorems of graph theory due to Tutte [503] and Nash-Williams [346], [347], a theorem about vector spaces due to Horn [218] and several results in transversal theory proved earlier by Higgins [215], Ore [359] and others. They illustrate perfectly the principle that mathematical generalization often lays bare the important bits of information about the problem at hand."

Matroid theory has been applied to solve problems in generalized assignment, operations research, theory of optimization, control theory, network theory, flow theory, generalized flow theory or linear programming, coding theory and telecommunication network design (see [263], [527], [699], [704], [731], [740], [741], [749], [757], [779], [796], [814], [833]).

Interpretations of matroids (or, in other words, abstract dependence relations) in lattice theory (or in projective geometry) are given by G. Birkhoff (1935, [30]; see also [31]), S. Mac Lane (1936, [289]; and 1938, [290]) and T. Nakasawa (1936, [339]). It is worth to notice that K. Menger, F. Alt, G. Bergman and O. Schweiber developed closely related ideas (in a connection to the projective geometry) independently during the years 1928-1935 (see [34], [326], [26], [327]). These interpretations lead to considerations of special geometric lattices (matroid lattices, exchange lattices, relatively complemented semimodular lattices; see for related topics: [33], [34*], [93*]-[95], [103]-[105], [119*], [121*], [123], [157], [202], [203], [236], [252], [254], [282], [318], [371*], [403], [406], [407], [409], [438], [476], [501], [539], [540], [564], [567], [575], [576], [581], [583], [588], [592], [594*], [606], [629], [645], [646], [667], [680], [681], [711], [714]-[719], [748], [765], [772]-[778], [815], [832]).

The existence of matroids which cannot be represented, in an obvious sense, in vector spaces (it was Whitney's problem) was shown by Mac Lane [289] (see also [265], [225], [393], [394], [674], [373], [504], [585], [586], [675], [226], [227], [331], [542], [271], [680], [681], [247], [606], [38], [435], [532*] and [595]). The so-called Fano matroid of rank 3

(see [595], p. 276) is an example of a matroid which is not representable over the field of rational numbers (it is representable only over any field of characteristic two). P. Vámos discovered a simple example of a matroid which is not representable over any field (see [331], p. 116-118, [532*], p. 140, [227], p. 156). Vámos' construction is based on a set of 8 elements and the number 8 is the best possible since J. C. Fournier ([585], [586]) has shown that every matroid on a set of 7 elements is *linear* (i.e., it is representable in a certain linear space). Still open is the following (see [532*]):

PROBLEM 1.1 (H. Whitney, 1935). Under what conditions on a matroid \mathfrak{M} does there exist a (not necessarily commutative) field F and a vector space V over F such that \mathfrak{M} is isomorphic to some matroid induced on a subset of V by linear independence over F ?

The mentioned Vámos' matroid is also an example of a matroid which is not a direct sum (see, e.g., [283], [532*]) of representable (in linear spaces) matroids, another such example is given by L. Lovász (it is based on a set of 10 elements). In 1964 W. Narkiewicz asked (Problem no. 716 in [357*]) if it is possible. The following question still remains open:

PROBLEM 1.2. Which matroids are direct sums of some linear matroids (maybe over distinct fields)? (P 1086)

Some other representation problems (e.g., for transversal matroids, for gammoids, for graphic matroids, for algebraic matroids) were considered in several papers; see for example: [640], [333], [66], [373], [317], [318], [99], [65], [331], [552], [556], [555], [601], [565], [566], [501], [504], [631], [599], [584], [650], [542], [228], [532*], [700], [702], [742], [824]. In this connection it seems interesting to consider the following:

PROBLEM 1.3. Which matroids are representable in v^* -algebras or in v^{**} -algebras? (For the definition of v^* -algebras and v^{**} -algebras see [340*], [342*], [511*] and [193*].) (P 1087)

Some open problems concerning the theory of matroids (or related theories) can be found, e.g., in the book of Welsh [532*] and in papers of Rado [394] (Problems P 530 - P 532), of Higgs [216] (Problem P 668), of Bruter [68], of Welsh [531], of Kelly and Rota [755] and of Woodall [683].

Since the notion of independence, which was a prototype for the matroids, came from the classical algebra (as independence with respect to generating), where only finitary operations were considered, in this theory a finiteness assumption appears naturally (see [372]). But the consideration of infinitary operations is also of some interest (see, e.g., [450] and [420*]). It is very natural to ask for a theory of independence spaces without any finiteness assumption (P 531 of Rado [394]). Such investigations are due to Dlab [114*] (see also Appendix in [115*]) and Higgs [216], p. 215.

In addition to the above let us mention some papers which are close to algebraic considerations: [13], [27], [40]-[43], [58], [62], [82], [116*], [118*], [222], [235], [251], [334], [402], [438]-[441], [479], [480], [499*], [536], [594*], [655], [773], [800] and [808].

2. \mathcal{C} -independence and related topics. A general dependence relation on the set A can be defined by a distinguished family D of subsets X of A such that

(6) $X \in D$ iff some finite subset Y of X belongs to D .

A subset X belonging to D is called *D-dependent*, and $X \notin D$ is called *D-independent*. An element $a \in A$ depends (with respect to D) on $X \subset A$ iff either $a \in X$ or there exists a finite D -independent subset Y of X such that $Y \cup \{a\} \in D$. We define an operator \mathcal{D} on subsets of A by putting

(7)
$$\mathcal{D}(X) = \{a \in A \mid a \text{ depends on } X\}.$$

If $\mathcal{D}(X) = A$ and $X \notin D$, then X is called an *irredundant basis* with respect to D or a *\mathcal{D} -base*. It is clear, by (6), that the operator \mathcal{D} is of a finite character. If, moreover, \mathcal{D} has the idempotence property,

(8)
$$\mathcal{D}(\mathcal{D}(X)) = \mathcal{D}(X) \quad \text{for each } X \subset A,$$

then \mathcal{D} is called an *algebraic closure operator* (see, e.g., [86*], [91], [134], [594*]). One often considered the case in which one has the idempotence property (8) and the following *exchange property*:

(9) if $y \notin \mathcal{D}(X)$ and $y \in \mathcal{D}(X \cup \{z\})$, then $z \in \mathcal{D}(X \cup \{y\})$

(see, e.g., [532*], p. 8-9, [595], p. 280-283, and [86*], p. 253). That case relates the theory of closure operators with the theory of (infinite) matroids.

It is well-known (the Birkhoff-Frink Theorem, [35]) that for every algebraic closure operator \mathcal{D} on A there exists a general algebra \mathfrak{A} , with A as a carrier, in which $\mathcal{D}(X) = \mathcal{C}(X)$, for every $X \subset A$, where \mathcal{C} is the generating operator in \mathfrak{A} (see also [80*] for some specification).

These facts show that matroids (independence spaces) can be interpreted in general algebras. The role of the required assumptions can be visible from results of [1], [10], [55], [56], [42*], [86*], [112*], [115*], [116*], [372], [378*], [465], [532*], [548] and [806].

It is worth to add that in the paper [548] one can also find some applications to the mathematical theory of systems engineering (generating sets of inputs for dynamic systems; see [544], [788]) and to the algebraic theory of machines and languages (irreducible elements in Krohn and Rhodes theory; see [9], p. 41).

If \mathcal{C} is a general closure operator on the set A (i.e., a mapping from 2^A into 2^A which is extensive, isotone and idempotent; see [483], [33],

p. 49, [52], p. 107, [193*], p. 27, 46, 49, [414], [310*], [96], and [7*]), then one can define \mathcal{C} -independence of subsets X of A in the following way:

(10) X is \mathcal{C} -independent iff $(\forall x \in X) (x \notin \mathcal{C}(X \setminus \{x\}))$.

In the opposite case the set A is called \mathcal{C} -dependent.

We shall denote the set of all \mathcal{C} -independent subsets of A by $\mathcal{C}\text{-Ind}(A)$ (or by $\mathcal{C}\text{-Ind}(\mathfrak{A})$ if the operator \mathcal{C} is the generating operator of an algebra $\mathfrak{A} = (A; F)$). \mathcal{C} -dependence does not, in general, satisfy condition (6) (where we put $D = \{X \mid X \notin \mathcal{C}\text{-Ind}(A)\}$), but if the operator \mathcal{C} is of a finite character, then it does. The families $\mathcal{C}\text{-Ind}(\mathfrak{A})$ for arbitrary algebras \mathfrak{A} were characterized by R. McKenzie (unpublished) and Truöl [498*]. The irredundant bases for clone operations on finite sets were considered, e.g., in [81], [745] (p. 19), [747] (for a "dual" notion see also [816]).

Using a closure operator it is possible to define also some other kinds of dependence (see, for example, [415], [111*], [310*], [118*], [8*], [214*], p. 65 and 66, [51*], [211*]). Some of them (e.g., [111*] and [118*]) do not satisfy the exchange property. In the paper [211*] there are considered some kinds of dependence relations with respect to a closure operator (in a unary algebra), which is not a generating operator in this algebra, and considered dependence relations are not of a finite character. It would be interesting to know the answer to the following problem:

PROBLEM 2.1. Which of the dependence relations mentioned above are of a finite character, and for which is the exchange property satisfied? (P 1088)

J. Schmidt in [415] has considered the following kinds of independence (stronger than the \mathcal{C} -independence):

A subset X of A is called \mathcal{C}_i -independent if $\mathcal{C}(\emptyset) \cap X = \emptyset$ and the following condition (C_i) (for $i = 1, 2$ or 3) is fulfilled:

$$(C_1) \quad (\forall x \in X) [\mathcal{C}(\{x\}) \cap \mathcal{C}(X \setminus \{x\}) = \mathcal{C}(\emptyset)],$$

$$(C_2) \quad (\forall Y, Z \subset X) [Y \cap Z = \emptyset \Rightarrow \mathcal{C}(Y) \cap \mathcal{C}(Z) = \mathcal{C}(\emptyset)],$$

$$(C_3) \quad (\forall Y, Z \subset X) [\mathcal{C}(Y \cap Z) = \mathcal{C}(Y) \cap \mathcal{C}(Z)].$$

The \mathcal{C}_1 -independence is called also a *direct independence* (see [121*]). A subset of a group can be a \mathcal{C}_1 -base not being simultaneously a free set of generators. It follows from a negative solution of problem P 156 from Colloquium Mathematicum (see ibidem 4 (1957), p. 239-240). Problems P 701 and P 702 of [8*] are concerned with Cartesian products of \mathcal{C}_i -independent sets for $i = 2$ and $i = 3$.

The following questions are still open:

PROBLEM 2.2. Characterize the families of \mathcal{C}_i -independent subsets of arbitrary algebras ($i = 1, 2$ or 3). (P 1089)

PROBLEM 2.3. For which algebras does the \mathcal{C}_i -independence coincide with the \mathcal{C} -independence ($i = 1, 2$ or 3)? (P 1090)

In [335] the so-called *\mathcal{C} -dependent algebras* were considered (in these algebras every two-element set is \mathcal{C} -dependent). In a similar way one can define \mathcal{C}_i -dependent algebras.

PROBLEM 2.4. Characterize \mathcal{C}_i -dependent algebras in the language of the lattice of subalgebras. It is known (see [335]) that the lattice of all subalgebras of a \mathcal{C} -dependent algebra is a chain. Characterize these chains for the \mathcal{C}_i -dependence. (P 1091)

For every closure structure (A, \mathcal{C}) one can consider the set K of these cardinal numbers κ for which there is an irredundant \mathcal{C} -base X with $\text{card}(X) = \kappa$. An especially interesting case is that of (A, \mathcal{C}) finitely based (for equational bases of varieties see, e.g., [286], [16], [242], [323], [636]-[638], [649], [672], [673], [676], [789]; Problems no. 4.5 and 8.1-8.4 of [241] are concerned with finitely based lattice varieties). If the structure is finitely based, then it is known that, under some natural assumption about the closure operator \mathcal{C} , the set K is an interval $[k_0, \omega)$ (see [214*], [173], [488], [638]). There are also similar considerations for equational bases of varieties (see [487], [201], [214*], and Problems no. 34 and 35 of [193*]). For example, finitely based varieties of groups can have only the intervals $[1, \omega)$ or $[2, \omega)$.

PROBLEM 2.5 (G. Grätzer, Problem no. 36 of [193*]). Find an algebraic characterization of the smallest element of the set K for equational bases of varieties.

PROBLEM 2.6. What about K for irredundant \mathcal{C}_i -bases ($i = 1, 2$ or 3)? (P 1092)

The problem no. 63 of Mazur and Ulam from the New Scottish Book [357*] was concerned with irredundant bases for groups. It was solved by V. Dlab in [107] and [109] (see also [108]). The paper [259] contains a definition of a strongly irredundant \mathcal{C} -basis for Boolean algebras. This notion can be also defined in a similar way for arbitrary general algebras.

PROBLEM 2.7. What about K for strongly irredundant \mathcal{C} -bases (\mathcal{C}_i -bases)? (P 1093)

II. MARCZEWSKI'S ALGEBRAIC INDEPENDENCE

3. M -independence. In 1958 Professor Edward Marczewski introduced a general notion of independence (called also *algebraic independence*; see [302*], [305*], [312*]), which contained as special cases majority of independence notions used in various branches of mathematics. In particular, it included linear independence (of vectors, of points, and of numbers), algebraic independence of numbers, set-theoretical independence, logical implicational independence, "liberty" of events (see [322], p. 86), independence of polynomials (the above examples can be found in [302*])

and [304*]) and independence of continuous functions (defined with using a Jacobian; see [473*]), w -independence of functions (defined with using a Wronskian; see [296*]), etc. In the sequel we shall call this notion of independence an M -independence. In 1964 the information came to Wrocław that a notion similar to the M -independence (and equivalent to the notion of an M -basis) appeared earlier in Philip Hall's lectures on Universal Algebra (see [351*], [428*]).

The notion of the M -independence (see below) was inspired by the notion of the set-theoretical independence introduced and applied by G. Fichtenholz and L. Kantorovitch ([156], p. 78) and independently discovered by E. Marczewski. The notion of the set-theoretical independence (or the independence in Boolean algebras), as well as its variants, have interesting applications in measure theory, probability theory, logic, topological algebra and functional analysis, and have been treated by several authors (see [156], [209], [297*], [299*], [300*], [442*], [443*], [278], [279], [445*], [288], [338*], [522], [495*], [259], [568], [610], and [679]). It is worthy to remark that ramified sets, in the sense of D. Kurepa, form a family of sets such that any two members of the family are set-theoretically dependent (see [616] and [617]). The problems no. P 71, P 72 and P 73 of [401] deal with the set-theoretical independence (and are still open). Let us also mention the papers [6*], [304*] and [526*] on the set-theoretical independence.

One can also consider a so-called m -independence in m -Boolean algebras (with infinitary operations), where m is some cardinal number (see [495], p. 150), and, more generally, α -independence in α -lattices (see [219]). It is well-known that if the m -Boolean algebra $\mathfrak{B}_{m,n}$ is free in the class of all m -Boolean algebras, with a set G ($\text{card}(G) = n$) of m -free generators, then G is m -independent (Theorem 31.8 of [495]).

PROBLEM 3.1 (see P 779 of T. Traczyk, OM 24 (1972)). What about the converse implication in the above-quoted theorem?

It is known that it is true for finitary Boolean algebras, i.e. if G is an n -element independent set (in the sense of Boolean algebras) of a free Boolean algebra with n free generators, then G is a set of free generators. The answer for this problem is positive in the case $m = \aleph_0$ and, more generally, if $\mathfrak{B}_{m,n}$ is a free m -representable Boolean algebra (comp. [445]).

It is worth to notice that the so-called stochastic independence and the independence with respect to a measure are not special cases of M -independence (see, e.g., [302*], p. 736, and [142*], p. 227).

Now we shall recall some fundamental notions.

Let $\mathfrak{A} = (A; F)$ be a *general algebra* (in other terminologies — a universal algebra or an abstract algebra). Let $T(\mathfrak{A})$ denote the set of all term operations (algebraic operations in the sense of Marczewski, see [305*] and [417*]) of the algebra \mathfrak{A} , i.e. the smallest set of operations

on the set A containing all trivial operations (projections) $e_k^{(n)}$ ($k = 1, 2, \dots, n$; $n = 1, 2, \dots$) and fundamental operations $f \in F$, and closed with respect to compositions. Of course, $T(\mathfrak{U})$ is a clone in the sense of Ph. Hall (see [86*], [489*]). Let $T^n(\mathfrak{U})$ denote the set of all n -ary term operations. We shall identify two algebras \mathfrak{U}_1 and \mathfrak{U}_2 on the same set A with $T(\mathfrak{U}_1) = T(\mathfrak{U}_2)$. \mathfrak{U}_1 is a reduct of \mathfrak{U}_2 iff $T(\mathfrak{U}_1) \subset T(\mathfrak{U}_2)$ ([486]). If $\mathfrak{U} = (A; T^{(n)}(\mathfrak{U}))$ and $\mathfrak{U} \neq (A; T^{(n-1)}(\mathfrak{U}))$, then \mathfrak{U} has the arity equal to n . If $f(x, x, \dots, x) = x$ for every $x \in A$, then f is an idempotent operation on A . By $\mathcal{I}(\mathfrak{U})$ we shall denote the maximal idempotent reduct of \mathfrak{U} .

A subset I of the set A is called *M-independent* if for every finite $n \leq \text{card}(I)$, for every $f, g \in T^{(n)}(\mathfrak{U})$ and for any system of different elements $a_1, \dots, a_n \in I$ the equality

$$(11) \quad f(a_1, \dots, a_n) = g(a_1, \dots, a_n)$$

implies $f = g$ in A .

The following conditions (for $I \neq \emptyset$) are equivalent:

- (a) I is *M-independent*;
- (b) each mapping $p: I \rightarrow A$ can be extended to a homomorphism of the subalgebra $\mathcal{C}(I)$ of \mathfrak{U} into the algebra \mathfrak{U} ;
- (c) $\mathcal{C}(I)$ is a *K-free* algebra freely generated by I with respect to the class $K = \mathcal{HSP}\{\mathfrak{U}\}$.

Other equivalent definitions and general considerations on the *M-independence* can be found in [305*], [312*], [314*], [102*], [193*], [198*], [101*], [422*], [429*] and [430*]. One can also investigate this notion in infinitary algebras (see, e.g., [420*]) and in partial algebras ([75*], [76*]); in so-called quasi-algebras (see [453*]), and also in topological algebras and relational structures ([337*], [153*], [338*], [580], [642*], [643*] and [615*]).

In [312*], [193*] and [198*] one can find surveys of results, related to the *M-independence*. It is worth to add that if C is a subalgebra of an algebra $\mathfrak{U} = (A; F)$ and we consider a new algebra $\mathfrak{U}_C = (A; F \cup \{f_c \mid c \in C\})$, where $f_c(x) = c$ (for all $x \in A$), then the *M-independence* in \mathfrak{U}_C is equivalent to the \mathfrak{B} -algebraic independence over C in the sense of Lausch and Nöbauer ([262*], p. 58).

M-independent sets were investigated in several classes of general algebras, for example: in some reducts of Boolean algebras [305*], [174*], [175*], in Post algebras [494*], in De Morgan algebras [697*], in Plonka's sums of Boolean algebras [389*], in distributive quasi-lattices [386*], in lattices and semilattices [475*], [308*], [477*], in FUS-algebras [384*], in diagonal and generalized diagonal algebras [379*], [382*], in unary algebras [234*], [145*], [499*], in some groupoids related to Steiner triple systems [183*] (see also: [472*] and [519*]), and in so-called separable variable algebras [151*], [174*]. It is worth to notice that the problem P 387,

posed by G. Szász in [475*] and concerning the M -independence in complemented lattices, seems to be still open. Among the problems P 522 - P 528 posed by E. Marczewski in [312*] only P 524 and P 527 are still open, and P 522 has only a partial answer.

E. Marczewski has raised the following question:

PROBLEM 3.2 (E. Marczewski, P 522 of [312*]). Let P_n be a projective n -dimensional space. Does there exist an algebra $(P_n; F)$ such that

(α) a set $I \subset P_n$ is linearly independent in P_n iff I is M -independent in the algebra $(P_n; F)$?

This problem has the negative solution [144*] under additional but natural assumption that

(β) every subalgebra of $(P_n; F)$ is a subspace of P_n .

It is interesting to know the answer in general case without the assumption (β).

The following problem seems to be interesting (for its connections with the additive number theory):

PROBLEM 3.3 (W. Narkiewicz, 1962). Investigate M -independence in the algebra $\mathfrak{N} = (2^{N_0}; +)$, where $+$ is the complex sum on subsets of $N_0 = \{0, 1, 2, \dots\}$.

Now we shall denote by $\text{Ind}(\mathfrak{A}, M)$ the set of all M -independent sets in an algebra \mathfrak{A} . If $I \in \text{Ind}(\mathfrak{A}, M)$, then I is \mathcal{C}_3 -independent (see Section 2, and [305*]). Several authors have investigated set-theoretical properties of $\text{Ind}(\mathfrak{A}, M)$ and connections of the M -independence in \mathfrak{A} with the closure operator in \mathfrak{A} . There were also considered some special elements of \mathfrak{A} , namely: self- M -independent elements, quasi-constants, semi-constants, constants, and relations between them (see [354*], [184*], [180*], [176*]). Problems P 877, P 878 and P 879 of [180*] deal with quasi-constants and semi-constants.

It follows directly from the definition of M -independence that the family $\text{Ind}(\mathfrak{A}, M)$ is hereditary and of a finite character for each algebra \mathfrak{A} . Hence, in the study of set-theoretical properties of the family $\text{Ind}(\mathfrak{A}, M)$, it is sufficient to consider only finite sets belonging to it. This remark relates the theory of matroids to algebraic considerations. S. Świerczkowski has shown ([471*]) that if $J \neq \emptyset$ is a hereditary system of finite subsets of a set A , then there exist a set $B \supset A$ and an algebra \mathfrak{B} on B such that, for all finite $X \subset B$, X is M -independent iff $X \in J$.

The main question is:

PROBLEM 3.4 (E. Marczewski, 1958; see Problem no. 52 of [193*]). For which hereditary families J of finite subsets of a given set A does there exist an algebra $\mathfrak{A} = (A; F)$ such that J is the family of all finite M -independent sets of the algebra \mathfrak{A} ?

We know only some sufficient and some necessary conditions (see [471*], [508*], [142*] and also [312*], [193*]). The following specification of Problem 3.4 seems to be interesting:

PROBLEM 3.5. Describe the family $\text{Ind}(\mathfrak{A}, M)$ for any fixed variety K ($\ni \mathfrak{A}$) of algebras (e.g., of groups, of rings, of lattices etc.). (P 1094)

Families $\text{Ind}(\mathfrak{A}, M)$ are completely characterized for unary algebras (see [145*], [499*], and also [234*] for mono-unary algebras).

PROBLEM 3.6. Characterize the family of functions $f: 2^A \rightarrow \{0, 1\}$ defined by

$$(12) \quad f(I) = \begin{cases} 1 & \text{if } I \in \text{Ind}(\mathfrak{A}, M), \\ 0 & \text{if } I \notin \text{Ind}(\mathfrak{A}, M), \end{cases}$$

for any algebras \mathfrak{A} (or for algebras \mathfrak{A} from some variety K). (P 1095)

We shall also recall the following problems:

PROBLEM 3.7 (G. Grätzer; Problem no. 49 of [193*]). Characterize those algebras which have one or both of the following two properties:

- (A) every maximally M -independent set is a minimal generating set,
- (B) every minimal generating set is a maximally M -independent set.

PROBLEM 3.8 (E. Marczewski; P 524 of [312*]). For which classes of algebras \mathfrak{A} is it true that if there exist an n -element independent set and a set of n generators, then every n -element subset of A is independent iff it is a set of generators?

The answer is positive for finite algebras, for linear spaces and, more generally, for so-called v^* -algebras (see [466*], [312*], and for related results [245*] and [248]).

4. M -bases. An M -independent generating set in \mathfrak{A} is called an M -basis of \mathfrak{A} (and \mathfrak{A} is called a *based algebra*). For example, the trivial n -ary operations on A form a basis of the algebra of all n -ary term operations. Algebras in which either all n -element sets are bases or the whole set A is a basis were completely described by Świerczkowski and others ([467*], [468*]; see also [190*], [509*], [511*] and [490*]) and by Marczewski and Urbanik ([315*], [316*]), respectively. Automorphisms and weak automorphisms of algebras with bases were considered in [496*] and [377*], respectively.

If \mathfrak{A} is a unitary module over a commutative ring with the unity, then all bases of \mathfrak{A} have the same cardinal numbers (see [269] and [84]). But it is not true in general (for modules see: [141], [266], [267], [268] and [400], and for general algebras: [244], [245*], [248], [184*], [220*]). If an algebra has two bases with different cardinal numbers, then all bases of \mathfrak{A} are finite (see [167]), their cardinal numbers form an infinite arithmetical progression ([305*], and also [466*], [184*], [469*], [470*], [371*]) and there exists in \mathfrak{A} an infinite M -independent set (J. Dudek,

an unpublished result from 1968, and [125*]). Any arithmetical progression is the set of numbers of elements of all bases of a certain algebra (it is a solution of Marczewski's problem no. 464 of [357*]; see [469*], [470*] and also [184*]). It is worth to add that the problems P 628, P 629 and P 630 of [153*] concerning bases with different cardinal numbers (in topological algebras) are still open, but problems P 525 and P 526 of [312*] have the answers. J. Schmidt [421*] has had more general considerations for algebras with infinitary operations. His problem P 485 bears an analogy to the Marczewski-Świerczkowski theorem on arithmetical progression (for M -bases). It was solved independently by Grätzer [192*] and by Burmeister [72*] (for related topics see [73*] and [74*]; in the latter paper two other problems are posed (p. 335), connected with these considerations).

Denote by $S(\mathfrak{A})$ the set of all n such that there exists, in \mathfrak{A} , an essentially n -ary term operation, i.e. an operation depending on all its variables. Marczewski [312*] has raised the conjecture that $S(\mathfrak{A}) = \{1, 2, \dots\}$ for all algebras with bases of different numbers of elements. Partial results in this direction are obtained by Narkiewicz [344*] and Dudek [124*] and [125*]. Recently J. Dudek has shown that $S(\mathfrak{B})$ is infinite for the maximal idempotent reduct $\mathfrak{B} = \mathcal{J}(\mathfrak{A})$ of the considered algebra \mathfrak{A} , and for that reduct the conjecture is true for algebras with one- and n -element ($n > 1$) bases. In general, the questions below are still open:

PROBLEM 4.1 (E. Marczewski, P 527 of [312*]). Is it true that $S(\mathfrak{A}) = \{1, 2, \dots\}$ for all algebras which have bases with different cardinal numbers?

PROBLEM 4.2 (K. Urbanik). What is the arity of \mathfrak{A} if \mathfrak{A} has bases with different cardinal numbers?

PROBLEM 4.3. What is the arity of $\mathcal{J}(\mathfrak{A})$ under the same assumption about \mathfrak{A} ? (**P 1096**)

PROBLEM 4.4. Is any M -basis a maximally \mathcal{C}_3 -independent set (see Section 2)? (**P 1097**)

PROBLEM 4.5 (S. Fajtlowicz, 1968). Can an algebra with bases of different cardinal numbers be a subalgebra of an algebra, in which all bases have the same finite cardinal numbers?

If an algebra $\mathfrak{A} = (A; F)$ has two bases with cardinal numbers m and n , then there exist $f_i, g_j \in T(\mathfrak{A})$ ($i = 1, \dots, m; j = 1, \dots, n$) such that the following identities hold (see [470*] and also [244], [184*], [469*]):

$$(13) \quad f_i(g_1(x_1, \dots, x_m), \dots, g_n(x_1, \dots, x_m)) = x_i \quad (i = 1, \dots, m),$$

$$(14) \quad g_j(f_1(x_1, \dots, x_n), \dots, f_m(x_1, \dots, x_n)) = x_j \quad (j = 1, \dots, n).$$

It is worth to remark that the varieties $K_{n,m}$, $K_{n,m}^1$ and $K_{n,m}^2$ defined by both (13) and (14) or by one only (for $n, m = 1, 2, \dots$) were

intensively investigated by several authors, see e.g. [2*], [3*], [4*], [25*], [85*], [125*], [233*] (p. 165-167), [295*] (p. 318-322), [319*], [320*], [321*], [457*]-[460*], [469*], [489*], [693*], [811*] and [817*]. Recently Dudek [125*] observed that for any $K_{1,m}$ Marczewski's conjecture for $S(\mathfrak{A})$ is true.

Let algebras \mathfrak{A}_1 and \mathfrak{A}_2 be of the same similarity type and have bases E_1 and E_2 , respectively. Then for every homomorphism h (or, more generally, for every weak homomorphism, see [182*] and [181*]) of \mathfrak{A}_1 into \mathfrak{A}_2 , we can define a generalized matrix as the system

$$\mathfrak{M}_h = \langle f_a | a \in E_1, f_a \in T(\mathfrak{A}_2), \text{ such that there are } b_1, \dots, b_n \in E_2 \\ \text{with } h(a) = f_a(b_1, \dots, b_n) \rangle.$$

This notion is a natural generalization of that in linear algebras (see [287], [434*] and [179*] for related topics).

PROBLEM 4.6 (some partial results are in [179*]). Work out the theory of generalized matrices. (P 1098)

In connection with Problem 3.3 and with the Arithmetical Progression Theorem on M -bases (the Marczewski-Świerczkowski Theorem, see above), we pose

PROBLEM 4.7. Give a set-theoretical description of the family of all M -bases of any based general algebra \mathfrak{A} . (P 1099)

5. Near linear properties of the M -independence. *EIS* and *JIS*. A very interesting and deeply developed line of research is an investigation of algebras in which the M -independence has the main properties of the linear independence. This line has brought to considerations of several special classes of algebras. *v*-algebras (called also *Marczewski's algebras*) were introduced by E. Marczewski in [303*] and investigated by himself and K. Urbanik [506*] (see also [505*], [541*]; an example is given in [19*]). v^* -algebras were introduced, under inspiration of Marczewski, by Nariewicz [340*] (they were investigated in [341*], [507*], [190*], [509*], [535*], [735*], and in [514*] where Fajtlowicz's problem no. 792 from [357*] is solved). He also introduced in [342*] the notion of v^{**} -algebras (called also v^*_* -algebras) as algebras in which the \mathcal{C} -independence is exactly the M -independence (see also: [343*] and [511*]). For every v^* -algebra the family $\text{Ind}(\mathfrak{A}, M)$ forms an independence space in the sense of the matroid theory (see [532*], p. 387). It is very surprising that if a v^{**} -algebra has a basis, then all bases have the same cardinality [342*]. This is the first instance of a result of this kind in a situation where we do not have any exchange property. Every absolutely free algebra is a v^{**} -algebra [511*]. G. H. Wenzel has introduced the class of v' -algebras as such v^{**} -algebras in which every maximally independent set is a basis (see [193*], p. 219). The class of v_* -algebras was introduced by K. Urbanik in [510*]. There

were also considered *separable variables algebras* introduced by E. Marczewski in [223*] and investigated by S. Fajtlowicz, K. Głazek and K. Urbanik (and also called *quasi-linear algebras*; [152*], [151*] and [174*]). Surveys of related results can be found in [511*], [193*] and [198*].

PROBLEM 5.1. Investigate and characterize algebras in which the \mathcal{C}_s -independence (see Section 2) is exactly the M -independence. (P 1100)

PROBLEM 5.2 (S. Fajtlowicz, 1968). Let $(A; \mathcal{D})$ be a set A with a generalized closure operator \mathcal{D} of a finite character. Find a sufficient and necessary condition for the existence on A of a v^{**} -algebra $\mathfrak{A} = (A; F)$ such that for every $X \subset A$ the following equivalence holds:

(15) $\mathcal{D}(X) = X$ iff X is a subalgebra of \mathfrak{A} .

PROBLEM 5.3 (K. Urbanik, P 529 of [511*]). Establish and prove a representation theorem for v^{**} -algebras.

This problem seems to be rather difficult; for partial results see [342*] and [511*].

The representation theorems for v -algebras ([506*]), for v^* -algebras ([507*], [509*] and [511*]), for v_* -algebras ([510*]) and for separable variables algebras ([152*]) are complicated and have some common ideas.

PROBLEM 5.4. Give a common representation theorem for all these algebras (see above) with a possible simple proof. (P 1101)

Let V, V^*, V', V_*, V^{**} and SV denote the classes of all v -, v^* , v' -, v_* -, v^{**} -algebras and separable variables algebras, respectively.

PROBLEM 5.5 (G. Grätzer; Problem no. 59 of [193*]). Find all operators \mathcal{O} on classes K of algebras for which $\mathcal{O}(K) \subset K$, where $K = V, V^*$ or V^{**} .

PROBLEM 5.6. The same for $K = V', V_*$ or SV . (P 1102)

The following two properties (EIS and JIS) of general algebras, expressed in the language of M -independent sets, are fulfilled in linear spaces but they do not hold in general.

We say, following E. Marczewski (see [223*], [312*] and Problem no. 486 of [357*]), that a general algebra \mathfrak{A} satisfies the condition of exchange of M -independent sets (\mathfrak{A} has the *EIS-property*) iff for any non-empty subsets I_1, I_2 and J of A we have $I_2 \cup J \in \text{Ind}(\mathfrak{A}, M)$ whenever $I_1 \cup J, I_2 \in \text{Ind}(\mathfrak{A}, M)$, $I_1 \cap J = \emptyset$ and $I_2 \subset \mathcal{C}(I_1)$.

It is well-known that if we replace the condition $I_2 \subset \mathcal{C}(I_1)$ by $\mathcal{C}(I_2) = \mathcal{C}(I_1)$, then the obtained assertion is true in each general algebra (see Theorem 2.4 (ii) of [305*]). A more general property $\text{EIS}_{\mathfrak{A}}$ was considered by S. Fajtlowicz in [143*].

The algebra \mathfrak{A} has the *JIS-property* (the joining M -independent sets property, see [151*]) if, for any two M -independent sets I and J such that $\mathcal{C}(I) \cap \mathcal{C}(J) = \mathcal{C}(I \cap J)$, $I \cup J \in \text{Ind}(\mathfrak{A}, M)$.

Let *EIS* and *JIS* denote the classes of all algebras satisfying the EIS-property or JIS-property, respectively. Then we have $SV \subset JIS \subset EIS$ (see [151*]). Examples of a trivial algebra and a Boolean algebra show that neither the first nor the second inclusion cannot be replaced by equality. Every finite unary algebra belongs to *JIS* [145*]. The EIS-property was extensively investigated by S. Fajtlowicz, E. Marczewski, J. Płonka and others, and the JIS-property was not.

The following algebras have EIS-property: algebras having at most 6 elements ([379*], [381*]), algebras without constants having at most 11 elements ([383*]), groups having at most 728 elements ([223*]), *v*-algebras, Abelian groups and, more generally, separable variables algebras ([223*]), Boolean algebras and, more generally, Post algebras ([223*], [494*]), unary algebras, semilattices, distributive lattices, diagonal and generalized diagonal algebras ([379*], [381*], [382*], [383*]), homogeneous algebras, the algebra $\mathfrak{A}^{(n)}$ (of all *n*-ary term operations of \mathfrak{A}) whenever \mathfrak{A} is finite [143*] and full idempotent reducts $\mathcal{J}(\mathfrak{G})$ of Abelian groups \mathfrak{G} ([186*], [187*]). On the other hand, there exist algebras without EIS-property: precisely one (up to isomorphism) 7-element algebra, a 12-element algebra without algebraic constants [379*] and precisely one 729-element group [223*]. Recently S. Fajtlowicz ([149*], [150*]) proved that one can find a lattice without the EIS-property in every variety of lattices in which there is at least one non-distributive lattice. This result extends his earlier positive answer to the problem P 523 of [223*] (and simultaneously negative one to the problem no. 742 of [357*]). If every finitely generated subalgebra satisfies the EIS-property, then the whole algebra satisfies it too [223*]. An interesting connection between the EIS-property of \mathfrak{A} and a so-called amalgamation property in the class $\mathcal{HSP}(\mathfrak{A})$ has been found by B. Jónsson (see [240*], and also *Colloquium Mathematicum* 15 (1966), p. 180, and 22 (1971), p. 165; for the definition of this property see [163], [239], [18]; in this last paper there is a partial answer to Fajtlowicz's problem P 644).

The following questions are still open:

PROBLEM 5.7 (S. Fajtlowicz; a part of P 644). A variety *K* is called *equationally complete* if the set of equations fulfilled in this variety is maximal consistent. Does any algebra of such a variety have the EIS-property?

PROBLEM 5.8 (J. Płonka; P 776, CM 24 (1972)). Is it true that any reduct of an algebra with the EIS-property also has that property?

PROBLEM 5.9 (J. Płonka; see P 777 of J. Płonka, CM 24 (1972)). Does Płonka's sum of a direct system of algebras with EIS also have this property? (For the definition of Płonka's sum, see [385*], [387*], and [408].)

PROBLEM 5.10. Find all operators \mathcal{O} on classes K of algebras for which $\mathcal{O}(K) \subset K$, where $K = JIS$ or $K = EIS$. (P 1103)

PROBLEM 5.11 (S. Fajtlowicz, 1965). Does there exist an algebra with the EIS-property, which has bases with different cardinal numbers?

PROBLEM 5.12. What is the connection between the JIS-property and the amalgamation property? (P 1104)

PROBLEM 5.13. Which algebras with EIS do not have the JIS-property? (P 1105)

6. Some numerical constants related to the M -independence. In [306*] Professor E. Marczewski has introduced and investigated some numerical constants for all finite algebras \mathfrak{A} (with more than one element) in the following way: if $\alpha(\mathfrak{A}) = \text{card}(A)$ and $\gamma^*(\mathfrak{A})$ is the smallest number with the property

$$(16) \quad (\forall X \subset A) (\text{card}(X) = \gamma^*(\mathfrak{A}) \Rightarrow \mathcal{G}(X) = A),$$

and, moreover, if $\gamma(\mathfrak{A})$ is the minimal number of generators of \mathfrak{A} , $\iota(\mathfrak{A})$ — the maximal number of M -independent elements in \mathfrak{A} , $\iota_*(\mathfrak{A})$ — the greatest number with the property

$$(17) \quad (\forall I \subset A) (\text{card}(I) = \iota_*(\mathfrak{A}) \Rightarrow I \in \text{Ind}(\mathfrak{A}, M)),$$

and, finally,

$$\tau(\mathfrak{A}) = -1 \quad \text{if } \alpha(\mathfrak{A}) = 1 \text{ or } \mathcal{G}(\emptyset) \neq \emptyset,$$

and if $\mathcal{G}(\emptyset) = \emptyset$ and $\text{card}(A) \neq 1$, then

$$\tau(\mathfrak{A}) = \begin{cases} 0 & \text{if there exist non-trivial unary term operations,} \\ n & \text{if all term operations in } n \text{ variables are trivial and} \\ & n \text{ is the maximal number with this property,} \\ \infty & \text{if no such finite } n \text{ exist.} \end{cases}$$

We shall write \varkappa instead of $\varkappa(\mathfrak{A})$ for $\varkappa = \alpha, \gamma^*, \gamma, \iota, \iota_*$ or τ , if it does not lead to confusions.

These constants are invariant under equivalences of algebras (and under isomorphisms). For every finite non-trivial algebra we have $\alpha \geq \gamma^* \geq \gamma \geq \iota \geq \iota_* \geq \tau$ and $\tau = \iota_*$ or $\tau = \iota_* - 1$ (see [306*], [312*]). If $\iota_*(\mathfrak{A}) = \alpha(\mathfrak{A}) \neq 2$, then \mathfrak{A} is trivial [468*]. All two-element algebras with $\iota_* = 2$ have been described by E. Marczewski and K. Urbanik (up to equivalence) [316*]. If three out of the numbers $\gamma^*, \gamma, \iota, \iota_*$ are equal, then all of them are equal [306*]. Świerczkowski in paper [472*] has given an interesting discussion of algebras with $\tau = \iota_* - 1$. The problem arises which values are admissible for the number α of elements of such an algebra with $\iota_* = 0, 1, 2$ or 3 . For $\iota_* = 0$ or 1 all positive integers are admissible. For $\iota_* = 2$ and 3 the problem was reduced by Świerczkowski

to some open questions about families of finite sets, called Steiner systems. One of the fundamental questions concerning the considered constants is the evaluation of the number ι by α or conversely. It was studied by E. Marczewski, J. Płonka, S. Fajtlowicz and G. H. Wenzel (see [309*], [312*], [380*], [381*], [388*], [533*], [534*], [193*] and [198*]). Other numerical constants associated with general algebras were defined and investigated, under inspiration of E. Marczewski, by S. Fajtlowicz, J. Płonka and K. Urbanik (see [143*], [379*], [380*], [384*], [388*], [512*], [513*], [514*]). In [489*] one can find (among others) the result that if the variety K has a non-trivial finite algebra, then K has finite algebras \mathfrak{A} with arbitrarily large finite $\gamma(\mathfrak{A})$ and $\iota(\mathfrak{A})$.

PROBLEM 6.1 (E. Marczewski 1962, see Problem no. 50 of [193*]). Characterize the six-tuples which can be represented as

$$(18) \quad \langle \alpha(\mathfrak{A}), \gamma^*(\mathfrak{A}), \gamma(\mathfrak{A}), \iota(\mathfrak{A}), \iota_*(\mathfrak{A}), \tau(\mathfrak{A}) \rangle$$

for some finite algebra \mathfrak{A} .

These six-tuples (18) were completely described for finite homogeneous algebras and quasi-trivial algebras by E. Marczewski in [311*] (see also [312*]), and for free unary algebras by G. H. Wenzel in [533*] (see also [534*]). G. H. Wenzel also gave a complete solution of the characterization problem of the 15 possible pairs of the above constants [534*]. In [514*] the constants $\gamma(\mathfrak{A})$ and $\iota(\mathfrak{A})$ were considered also for infinite algebras, the inequality $\iota(\mathfrak{A}) \leq \gamma(\mathfrak{A})$ was established for any v^* -algebra.

We shall recall the following problems:

PROBLEM 6.2 (G. Grätzer, Problem no. 51 of [193*]). Characterize the six-tuples (18) for v -, v^* -, v^{**} -algebras.

PROBLEM 6.3 (G. Grätzer, Problem no. 60 of [193*]). Define and discuss (18) for infinite algebras.

PROBLEM 6.4 (G. Grätzer, Problem no. 61 of [193*]). Define and discuss (18) for infinitary algebras and relate these to the characteristic (see p. 76 of [193*]).

In [154*] the notion of a rank of algebras was investigated. For every family L of subsets of a fixed set A we put

$$(19) \quad \nu(L) = \sup \{\text{card}(X) \mid X \in L\},$$

and we say that L has the property (ω) if every set belonging to L is finite and $\nu(L) = \aleph_0$. For example, if B is the family of all bases in an algebra \mathfrak{A} having bases with different cardinal numbers, then B has the property (ω) .

We say that L has a *rank* if 1° every set belonging to L is contained in a maximal one, and 2° all maximal sets in L have the same cardinality (see [154*]).

No family with a rank has the property (ω) .

We say that an algebra \mathfrak{A} has a *rank* if the family $\text{Ind}(\mathfrak{A}, M)$ has a rank. Analogously, \mathfrak{A} is said to have the property (ω) if the family $\text{Ind}(\mathfrak{A}, M)$ has this property [154*]. We put $\iota(\mathfrak{A}) = \nu(\text{Ind}(\mathfrak{A}, M))$. Of course, if $\text{Ind}(\mathfrak{A}, M)$ has a rank, then \mathfrak{A} with the family $\text{Ind}(\mathfrak{A}, M)$ is a supermatroid in the sense of [532*] (p. 354; see also [126]). It is known (see [154*]) that if \mathfrak{A} is a finitely generated algebra or a v^* -algebra (or, more generally, an algebra with a basis), then \mathfrak{A} has not the property (ω) . On the other hand, there are an Abelian semigroup, a distributive lattice and a Boolean algebra with the property (ω) (see [154*]).

We shall recall some problems:

PROBLEM 6.5 (P 667 of [154*]). Do there exist v^{**} -algebras with the property (ω) ?

PROBLEM 6.6 (P 665 of [154*]). Do there exist groups, rings and separable variables algebras with the property (ω) ?

PROBLEM 6.7 (P 666 of [154*]). In which varieties of groups (except Abelian) has every free group a rank?

PROBLEM 6.8 (S. Fajtlowicz, 1968). Is the following implication true: if every subalgebra of a finite algebra \mathfrak{A} has a rank, then \mathfrak{A} satisfies the condition JIS?

It is worth to add that this implication is true if we consider the EIS-property instead of JIS (see [154*], p. 191).

III. GENERALIZATIONS

7. Q -independence. The important scheme of M -independence is not enough wide to cover the stochastic independence, the independence in projective spaces, and the linear independence in groups. Some notions weaker than the M -independence were developed (see [312*], [193*]). J. Schmidt introduced in [418*] the *independence-in-itself* (called also the *local independence*, see [193*]; this independence we shall call the *S-independence*), S. Świerczkowski dealt (in [474*]) with *weak independence* (it will appear in this article as the *S₀-independence*). Further, G. Grätzer used such a "weak independence" (in [189*], [194*] and [193*]) to include the linear independence in Abelian groups (for subsets which do not contain the zero element). This notion will be called here the *G-independence*. K. Głazek introduced the notions of *H_{*}-independence*, *R-independence* and *A₁-independence* [174*], and also of *H_{*}^I-independence* [176*].

As a common way of defining all these notions Professor E. Marczewski proposed in [312*] (p. 173; see also [313*]) a notion of inde-

pendence with respect to a family Q of mappings, and also called it the Q -independence. This notion was investigated by E. Marczewski (in [313*]) and by K. Głazek (in [174*], [175*]). Likewise, besides the notions mentioned above, the notion of \mathfrak{B} -algebraic independence over \mathfrak{A} , introduced in [262*], p. 58, can be also considered (as was remarked by G. Eigen-thaler) as a special case of the Q -independence.

Let $\mathfrak{A} = (A; F)$ be a general algebra, and let $M(X, A)$ denote the family of all mappings $p: X \rightarrow A$, where $X \subset A$. Further, let $H(X, \mathfrak{A})$ denote the set of mappings $p: X \rightarrow A$ (for $X \subset A$) which possess an extension (necessarily unique) to a homomorphism of the subalgebra $\mathcal{C}(X)$, generated in \mathfrak{A} by the subset X , into A .

Take $Q \subset M(A) = \bigcup \{M(X, A) \mid X \subset A\}$, and let us put $H = H(\mathfrak{A}) = \bigcup \{H(X, \mathfrak{A}) \mid X \subset A\}$.

A set $I \subset A$ is called *independent with respect to the family Q* or, shortly, *Q -independent* if $Q(I, A) \subset H(I, \mathfrak{A})$ ([312*], [313*]).

Let $\text{Ind}(\mathfrak{A}, Q)$ denote the set of all Q -independent subsets of \mathfrak{A} . E. Marczewski ([305*], [313*]) has proved that $I \in \text{Ind}(\mathfrak{A}, Q)$ iff for every $p \in Q(I, A)$ and each natural $n \leq \text{card}(I)$, if $a_1, \dots, a_n \in I$, $f, g \in T^{(n)}(\mathfrak{A})$ and

$$(11) \quad f(a_1, \dots, a_n) = g(a_1, \dots, a_n),$$

then

$$(20) \quad f(p(a_1), \dots, p(a_n)) = g(p(a_1), \dots, p(a_n)).$$

It is worthwhile to remark that if $Q_1 \subset Q_2 \subset M(A)$, then $\text{Ind}(Q_2) \subset \text{Ind}(Q_1)$ and that for every $Q \subset M(A)$ we have the inclusions $\text{Ind}(\mathfrak{A}, M) \subset \text{Ind}(\mathfrak{A}, Q) \subset \text{Ind}(\mathfrak{A}, H) = 2^A$ [313*]. Most of results, quoted in this section, can be found in [174*]. For every family $Q \subset M(A)$, the greatest family \bar{Q} of mappings exists such that $\text{Ind}(\mathfrak{A}, \bar{Q}) = \text{Ind}(\mathfrak{A}, Q)$. A family Q such that $Q = \bar{Q}$ will be shortly called *maximal*. For an algebra \mathfrak{A} and every family J of subsets of A such that

$$(21) \quad \text{Ind}(\mathfrak{A}, M) \subset J \subset 2^A$$

there exists (see [174*], Theorem 1) a family of mappings $Q \subset M(A)$ satisfying the equality

$$(22) \quad \text{Ind}(\mathfrak{A}, Q) = J.$$

We have also interesting corollaries:

For an arbitrary family $J \subset 2^A$ there exist a subfamily $Q \subset M(A)$ and an algebra $\mathfrak{A} = (A; F)$ such that (22) holds. For every algebra

$\mathfrak{A} = (A; F)$ there exist families $Q_i \subset M(A)$ ($i = 0, 1, 2, 3$) such that

$$(23) \quad \mathcal{C}\text{-Ind}(\mathfrak{A}) = \text{Ind}(\mathfrak{A}, Q_0),$$

$$(24) \quad \mathcal{C}_i\text{-Ind}(\mathfrak{A}) = \text{Ind}(\mathfrak{A}, Q_i) \quad \text{for } i = 1, 2, 3.$$

The family Q_0 in (23) can be found by some construction given in [174*]. It gives an answer to a problem posed by Marczewski in 1968. Some problems related with it, formulated by M. Sekanina, will be given in the next section. As it was observed by S. Fajtlowicz, for every generalized closure operator \mathcal{D} of a finite character given on 2^A , there exist an algebra $\mathfrak{A} = (A; F)$ and a family of mappings $Q_0 \subset M(A)$ such that for any $X \subset A$ the subalgebra generated by X is equal to $\mathcal{D}(X)$ and equality (23) holds. The problem No. 1 of [174*] (p. 16) is related to these considerations and is still open. We can pose the following problems:

PROBLEM 7.1. Let a family $Q \subset M(A)$ be uniformly defined (in the obvious sense) for algebras \mathfrak{A} of a class K (or even for all algebras). Characterize the families $\text{Ind}(\mathfrak{A}, Q)$. (P 1106)

PROBLEM 7.2. For a fixed algebra \mathfrak{A} , let the family $\text{Ind}(\mathfrak{A}, M)$ have a rank (see Section 6). For which $Q \subset M(A)$ does the family $\text{Ind}(\mathfrak{A}, Q)$ have also a rank? (P 1107)

PROBLEM 7.3. For finite algebras \mathfrak{A} one can define the constant numbers $\iota(\mathfrak{A}, Q)$ and $\iota_*(\mathfrak{A}, Q)$ in a similar way as for the M -independence. Characterize the six-tuples which can be represented as

$$(18') \quad \langle \alpha(\mathfrak{A}), \gamma^*(\mathfrak{A}), \gamma(\mathfrak{A}), \iota(\mathfrak{A}, Q), \iota_*(\mathfrak{A}, Q), \tau(\mathfrak{A}) \rangle$$

for some finite algebra \mathfrak{A} and suitably chosen uniformly defined Q such that $\text{Ind}(\mathfrak{A}, Q) \subset \mathcal{C}\text{-Ind}(\mathfrak{A})$ for each algebra \mathfrak{A} . (P 1108)

PROBLEM 7.4. Give an axiomatic description of the operator $Q \rightarrow \bar{Q}$ in a fixed algebra \mathfrak{A} . (It is known that the operator $Q \setminus H \rightarrow \bar{Q} \setminus H$ is a topological closure on $M(A) \setminus H(\mathfrak{A})$, see [174*].) (P 1109)

PROBLEM 7.5. For any algebra \mathfrak{A} one can construct, in a rather artificial way, a family Q_i such that $\text{Ind}(\mathfrak{A}, Q_i) = t\text{-Ind}(\mathfrak{A})$, where $t\text{-Ind}(\mathfrak{A})$ denotes the family of all t -independent sets (for the definition see [186*] and also [187*] and [390*]). Does there exist a uniform method of such a construction, valid for all algebras? (P 1110)

PROBLEM 7.6. Can one find (see Problem 3.2)

1° an algebra $\mathfrak{P} = (P_n; F)$ on an arbitrary n -dimensional projective space P_n such that

(β) every subalgebra of \mathfrak{P} is a subspace of P_n ;

2° a family $Q_P \subset M(P_n)$ such that

(α') Q_P -independence in \mathfrak{P} is exactly the independence usually defined in the projective space P_n ? (P 1111)

K. Głazek proved in [174*] that the algebra $\Omega(\mathfrak{U})$ of all maximal families $Q = \bar{Q} \subset M(A)$ of mappings defined on \mathfrak{U} , with the set-theoretical join and meet and with a complementation defined by the equality

$$(25) \quad Q' = (M(A) \setminus Q) \cup H,$$

is a complete atomic Boolean algebra. The problems are:

PROBLEM 7.7. Is it true that if algebras \mathfrak{U}_1 and \mathfrak{U}_2 on the same set A have the same Boolean algebras $\Omega(\mathfrak{U}_1) = \Omega(\mathfrak{U}_2)$, then $\mathfrak{U}_1 = \mathfrak{U}_2$ (i.e., $T(\mathfrak{U}_1) = T(\mathfrak{U}_2)$)? (P 1112)

PROBLEM 7.8. Are two arbitrary algebras \mathfrak{U} and \mathfrak{B} with isomorphic Boolean algebras $\Omega(\mathfrak{U})$ and $\Omega(\mathfrak{B})$ weak isomorphic (for the definition of the weak isomorphism see [182*] and [181*])? (P 1113)

PROBLEM 7.9. Do a generalized closure operator $\mathcal{D}: 2^A \rightarrow 2^A$ of a finite character and an additive closure operator $Q \mapsto \tilde{Q}$ for families $Q \subset M(A)$ determine (up to equivalence) an algebra \mathfrak{U} on the set A such that \mathcal{D} is the closure with respect to generating of subalgebras, families $Q = \tilde{Q}$ are the maximal ones forming the Boolean algebra $\Omega(\mathfrak{U})$, and $\tilde{\emptyset} = H(\mathfrak{U}) = (H(\mathfrak{U}))^\sim$? (P 1114)

In [174*] (Corollary 6) an interesting connection is given between the properties of hereditariness and of the finite character for families $\text{Ind}(\mathfrak{U}, Q)$. Namely, if the family $\text{Ind}(\mathfrak{U}, Q)$ is hereditary, then the family $\text{Ind}(\mathfrak{U}, \tilde{Q})$ is of a finite character.

PROBLEM 7.10. Is the converse implication true? (P 1115)

PROBLEM 7.11. Characterize families $Q \subset M(A)$ of mappings defined on an algebra \mathfrak{U} in such a way that suitable Exchange Theorems (see Theorem 2.4 (ii) of [305*] and Theorem 9 of [174*]) are true. (P 1116)

PROBLEM 7.12. The same for Arithmetical Progression Theorems (see Theorem 2.4 (iv) of [305*] and Theorem 8 of [174*]). (P 1117)

Some sufficient conditions for these two types of theorems (for the Q -independence) were found by K. Głazek in [174*]. The Problem no. 2 of [174*] deals with the second theorem.

8. \mathcal{C} -independence as a special case of Q -independence. Let $\mathfrak{U} = (A; F)$ be a fixed general algebra. Following [174*] we can define the family $Q_C^* \subset M(A)$ by putting

$$(26) \quad Q_C^*(X, A) = H(X, \mathfrak{U}) \cup P_X^*,$$

where P_X^* is the set of mappings $p_{(a,b)}: X \rightarrow A$ which are defined for all $a, b \in A$ by

$$(27) \quad p_{(a,b)}(x) = \begin{cases} x & \text{for } x = a \text{ or if } x \in \mathcal{C}(X \setminus \{x\}), \\ b & \text{in other cases.} \end{cases}$$

Then we have

$$(28) \quad \mathcal{C}\text{-Ind}(\mathfrak{U}) = \text{Ind}(\mathfrak{U}, Q_C^*).$$

Define also the set P_X of all mappings $p_{(a,b)}: X \rightarrow X$ for each $a, b \in X$ by (27) and put $Q_C(X, A) = H(X, \mathfrak{U}) \cup P_X$. Then a subset I of A with at least two elements is \mathcal{C} -independent iff it is Q_C -independent.

We shall describe some categorical aspects of these results. For the definitions from the Theory of Category the reader should refer to [291], [433]; for the definition of weak homomorphism see [182*], [181*] and [353*]. The following considerations and problems are due to M. Sekanina:

\mathcal{A} will denote the category of non-indexed general algebras with the injective weak homomorphisms as morphisms. In the sequel \mathcal{S}_0 and \mathcal{M} are the following categories:

Category \mathcal{S}_0 . Objects. If A is a set and $X \subset A$, then Q_X means some subset of X^X . All possible pairs $\langle A; \{Q_X | X \subset A\} \rangle$, where A runs through all possible sets, and $\{Q_X | X \subset A\}$ runs through all possible systems of mappings, form the class of objects of \mathcal{S}_0 .

Morphisms of \mathcal{S}_0 are "carried" by injective mappings between sets, i.e. when $f: \langle A; \{Q_X | X \subset A\} \rangle \rightarrow \langle A'; \{Q'_Y | Y \subset A'\} \rangle$, then f is an injection of A into A' and the following condition is supposed to be fulfilled:

$$(29) \quad (\forall X \subset A) (\forall g' \in Q'_{f(X)}) (f^{-1}g'f \in Q_X).$$

Category \mathcal{M} . Objects as in \mathcal{S}_0 , but X^X replaced by A^X .

Morphisms as in \mathcal{S}_0 , but (29) replaced by

$$(30) \quad (\forall Y \subset f(A)) (\forall g \in Q_{f^{-1}(Y)}) (fgf^{-1} \in Q_Y).$$

Let $\mathfrak{U} = (A; F)$ be an algebra (i.e., an object of \mathcal{A}). Put

$$F_1(\mathfrak{U}) = \langle A; \{P_X | X \subset A\} \rangle.$$

If $f \in \text{Mor } \mathcal{A}$, put $F_1(f) = f$. Then F_1 is an embedding of \mathcal{A} in \mathcal{S}_0 . Put $F_2(\mathfrak{U}) = \langle A; \{P_X^* | X \subset A\} \rangle$, and $F_2(f) = f$. F_2 is also an embedding of \mathcal{A} in \mathcal{M} (Sekanina's letter [431*]). In this sense, constructions in Section 6 of Glazek's paper [174*] are categorical.

There are some more or less natural questions connected with this fact:

PROBLEM 8.1 (M. Sekanina, 1978). Study the properties of categories \mathcal{S}_0 and \mathcal{M} .

PROBLEM 8.2 (M. Sekanina, 1978). Are F_1 and F_2 full embeddings, i.e. given f in \mathcal{S}_0 (or \mathcal{M}) as morphism between $F_1(\mathfrak{U})$ and $F_1(\mathfrak{U}')$ ($F_2(\mathfrak{U})$ and $F_2(\mathfrak{U}')$, resp.), is f an injective weak homomorphism from \mathfrak{U} into \mathfrak{U}' ? (One expects a negative answer.)

PROBLEM 8.3 (M. Sekanina, 1978). If the answer to Problem 8.2 is negative, are there any full embeddings of \mathcal{A} into \mathcal{S}_0 and \mathcal{M} ? (Maybe general results of L. Kučera [612] can be used here.)

PROBLEM 8.4 (M. Sekanina, 1978). Find a categorical settling for those questions using all mappings and all weak homomorphisms (not only injective ones).

9. Independence with respect to special families Q of mappings. Most interesting are some special cases of the Q -independence. Of course, if $Q = M(A)$, we obtain the M -independence (see Section 3). If we take $S(X, A) = \mathcal{C}(X)^X$ instead of $Q(X, A)$, then we get the S -independence introduced by J. Schmidt in [418*] and called also the *local independence* (see [193*], [519*], [520*] and [174*]). A set $I \subset A$ is S -independent in an algebra $\mathfrak{A} = (A; F)$ iff I is M -independent in the algebra $(\mathcal{C}(I); F)$. The family $\text{Ind}(\mathfrak{A}, S)$ is hereditary and of the finite character. The following problem is still open:

PROBLEM 9.1 (G. Grätzer, Problem no. 53 of [193*]). Characterize the system of locally independent sets in an algebra.

R. M. Vancko ([517*], [518*], [520*]) and, independently, K. Truöl [498*] have obtained the required characterization for all finite algebras. Some partial results are obtained also by J. Schmidt in [418*]. It is worthwhile to add that R. M. Vancko has described the spectrum of the classes L_k of algebras which are freely generated by k elements and in which every k -element subset is locally independent. His results are related to Świerczkowski's considerations in [468*].

PROBLEM 9.2. For which algebras is the S -independence equivalent to the M -independence for subsets consisting of at least two elements? (P 1118)

This holds for linear spaces, affine spaces (and, more generally, for v^{**} -algebras), torsion-free groups and regular reducts of Boolean algebras (see [174*]).

If we put $S_0(X, A) = X^X$ instead of $Q(X, A)$, then we obtain the S_0 -independence introduced by S. Świerczkowski in [474*]. We have $\text{Ind}(\mathfrak{A}, M) \subset \text{Ind}(\mathfrak{A}, S) \subset \text{Ind}(\mathfrak{A}, S_0)$, and if $I \in \text{Ind}(\mathfrak{A}, S_0)$ with $\text{card}(I) \geq 2$, then $I \in \mathcal{C}\text{-Ind}(\mathfrak{A})$.

PROBLEM 9.3 (Problem no. 3 of [174*], p. 32, and Problem no. 1 of [175*]). For which algebras is the S_0 -independence equivalent to the S -independence for subsets consisting of at least two elements?

This holds for v^{**} -algebras (and thus for linear spaces and affine spaces), for torsion-free groups and for regular reducts of Boolean algebras (see [174*]). We do not know whether in quasi-linear algebras \mathfrak{A} (see [152*], [174*]) if $\mathcal{C}(\emptyset) = \{0\}$, then $\text{Ind}(\mathfrak{A}, S) = \text{Ind}(\mathfrak{A}, S_0)$. The problem

no. 5 of [174*] (on a characterization of S_0 -independent subsets of quasi-linear algebras) is also still open.

Investigations of the S_0 -independence are still very far from an end. For example, we do not know any description of the S_0 -independence for arbitrary quasi-linear algebras. It seems to be interesting to know the answers to Problems 7.1-7.3 in the particular cases $Q = S$ and $Q = S_0$.

A mapping $p: X \rightarrow A$, where $X \subset A$, is called k -diminishing if for every $f, g \in T^{(k)}(\mathfrak{A})$ and for each $a_1, \dots, a_k \in X$ the equality $f(a_1, \dots, a_k) = g(a_1, \dots, a_k)$ implies $f(p(a_1), \dots, p(a_k)) = g(p(a_1), \dots, p(a_k))$.

Denote by $G^{(k)}(X, A)$ the set of all k -diminishing mappings from X into A and put $G^{(1)} = G$. One can consider $G^{(k)}$ -independence. Of course, if $\text{card}(I) \leq k$ and $I \subset A$, then $I \in \text{Ind}(\mathfrak{A}, G^{(k)})$. The 1-diminishing mappings and the G -independence were considered by E. Marczewski in [313*]. This last notion is equivalent to the weak independence introduced by G. Grätzer in [189*] and [194*]. For further investigations of the G -independence see [193*], [174*], [374*], [521*] and [231*]. An algebra \mathfrak{A} is independent over a class K in the sense of Don Pigozzi [374*] if \mathfrak{A} is generated by a set of elements weakly independent in K (in the sense of Grätzer [194*]). If I is a subset of an Abelian group and $0 \notin I$, then I is G -independent iff it is linearly independent (in the sense, e.g., of [478], [165]). The linear independence in a multiplicative semigroup A with the unity e (introduced in [230*], [231*]) coincides with the G -independence in the monoid $(A; \cdot, e)$. A subset I of an Abelian group is S_0 -independent iff the set $\{a - b \mid a, b \in I\}$ is G -independent and all elements have a finite order ([174*], p. 34). The problem no. 4 of [174*] on a characterization of G -independent subsets in quasi-linear algebras is still open. In an Abelian group the set $\{a_1, \dots, a_n\}$ is G -independent iff $\mathcal{C}(\{a_1, \dots, a_n\}) \simeq \mathcal{C}(\{a_1\}) \times \dots \times \mathcal{C}(\{a_n\})$ ([193*], [194*]). In [751*], [752*], [521*] and [720*] it is proved that the class of algebras in which that equivalence holds is precisely the class of semimodules over semirings. He solved, in this way, Grätzer's problem P 606 (in [194*]; and simultaneously the problem no. 56 in [193*]).

We shall recall some Grätzer's problems about G -independence from [194*] and [193*]:

PROBLEM 9.4 (G. Grätzer, Problem no. 54 of [193*]; see also P 603 of [194*]). Characterize the system of G -independent sets in an algebra.

PROBLEM 9.5 (G. Grätzer, Problem no. 55 of [193*] and problems P 602 and P 604 of [194*]). Characterize the set of cardinal numbers of G -bases of an algebra. Can an algebra $\mathfrak{A} = (A; F)$ without algebraic constants have a finite and an infinite G -basis? Even if F is finite? ⁽¹⁾

⁽¹⁾ Note added in proof. The both questions have positive answers (see G. Grätzer, *Universal algebra*, 2nd edition, New York 1979).

PROBLEM 9.6 (see P 605 of [194*]). For which algebras \mathfrak{A} is it true that $\text{Ind}(\mathfrak{A}, M) \neq \text{Ind}(\mathfrak{A}, G) \subset \mathcal{C}\text{-Ind}(\mathfrak{A})$?

One can also consider families R, A_1, A_2, H_* and H_*^I of mappings with respect to which suitable notions of the Q -independence are interesting. These families are defined in the following way:

For all $X \subset A$,

$R(X, A)$ consists of all injective mappings from X into A ,

$A_1(X, A) = \{p \mid p \in A^X, (\exists f \in T^{(1)}(\mathfrak{A})) (\forall x \in X) (f(x) = p(x))\}$,

$A_2(X, A) = \{p \mid p \in A^X, (\exists g \in P^{(1)}(\mathfrak{A})) (\forall x \in X) (g(x) = p(x))\}$, where $P^{(1)}(\mathfrak{A})$ denotes the set of all unary polynomials of \mathfrak{A} in the sense of [262*].

Further, $H_* \subset M(A)$ is the smallest family containing $H(\mathfrak{A})$ and closed with respect to restrictions and "sticking together" of mappings on disjoint subsets (see [174*], p. 28).

Let I be a fixed family of subalgebras B of the algebra $\mathfrak{A} = (A; F)$. Then H_*^I is a subfamily of H_* such that $p \in H_*^I(X, A)$ iff

$$(31) \quad (\forall B \in I) (\exists h \in \text{Hom}(B, A)) (\forall x \in X \cap B) (p(x) = h(x)).$$

It is known that for the H_* -independence the Exchange Theorem holds, and for the R -independence the Arithmetical Progression Theorem holds. The R -independence is equivalent to the M -independence in quasi-linear algebras (in particular: in Abelian groups and linear spaces), in v^{**} -algebras \mathfrak{A} with an infinite carrier A , and in regular reducts of Boolean algebras. It is worthwhile noting that there exists a two-element v^{**} -algebra for which $\text{Ind}(\mathfrak{A}, R) \neq \text{Ind}(\mathfrak{A}, M)$ (see [174*]).

PROBLEM 9.7 (Problems no. 6 of [174*] and no. 3 of [175*]). For which algebras \mathfrak{A} , $\text{Ind}(\mathfrak{A}, R) = \text{Ind}(\mathfrak{A}, M)$?

It seems interesting to investigate these notions in the following directions:

PROBLEM 9.8. Work out the notions of Q -independence for $Q = S_0, G^{(k)}$ ($k > 1$), A_2, R and H_* . (P 1119)

In particular:

PROBLEM 9.9. Characterize the families $\text{Ind}(\mathfrak{A}, Q)$ for $Q = S_0, G^{(k)}, A_1, A_2, R$ or H_* in an arbitrary algebra \mathfrak{A} . (P 1120)

PROBLEM 9.10. Characterize the sets of cardinal numbers of Q -bases of an algebra for $Q = S_0, S, G^{(k)}, A_1, A_2$ or H_* . (P 1121)

PROBLEM 9.11. Give a description of Q -dependent algebras for $Q = M, S, S_0, G^{(k)}, A_1, A_2$ or H_* (\mathfrak{A} is called Q -dependent if every two-element set in \mathfrak{A} is Q -dependent). (P 1122)

PROBLEM 9.12. Describe algebras \mathfrak{A} in which the whole set A is Q -independent for $Q = S, S_0, G^{(k)}, A_1, A_2$ or H_* . (P 1123)

10. Independence of subalgebras. In 1972, Professor E. Marczewski has proposed the notion of *independence of subalgebras* of a general algebra (see [180*]). This notion is connected with the notion of a free product of algebras (see, e.g., [193*], p. 184) similarly as the M -independence of elements is connected with the notion of a free generated algebra. Recall that an algebra $\mathfrak{A} = (A; F)$ is a K -free product of its subalgebras B and C if $A = \mathcal{C}(B \cup C)$ and, for every pair of homomorphisms $h_B: B \rightarrow D$ and $h_C: C \rightarrow D$, there is a homomorphism $h: A \rightarrow D$ such that $h|_B = h_B$ and $h|_C = h_C$, where $\mathfrak{D} = (D; F)$ is an arbitrary algebra from the class K of algebras of the same similarity type as the algebra \mathfrak{A} . This definition makes sense also for algebras without determined similarity types (i.e., non-indexed algebras) whenever A, B and C are subalgebras of a certain algebra $\mathfrak{A}_0 = (A_0; F_0)$ and $K \subset \text{Sub}(\mathfrak{A}_0)$ (where $\text{Sub}(\mathfrak{A}_0)$ denotes the set of all subalgebras of an algebra \mathfrak{A}_0). The concept of a K -free product was introduced (in the case $\mathfrak{A} \in K$) by R. Sikorski in [444] (for general considerations and related topics see [17], [21]-[24], [86*], [87], [90], [168], [185], [193*], [200], [214], [237], [239], [243], [249], [261], [280], [284], [319*], [351], [352], [360], [361], [369], [371*], [413], [416], [437], [441], [450], [452], [456], [491], [492], [551]) and it has been applied by various authors to such structures as loops, semigroups, groups, lattices, Boolean algebras, rings etc.

A set I of subalgebras of an algebra $\mathfrak{A} = (A; F)$ is independent if, for every family of homomorphisms $h_B: B \rightarrow A$, $B \in I$, there exists a (necessarily unique) homomorphism

$$h: \mathcal{C}(\bigcup \{B | B \in I\}) \rightarrow A$$

such that $h|_B = h_B$ for every $B \in I$. We use the notation $I \in \text{ind}(\text{Sub}(\mathfrak{A}))$ if I is an independent set of subalgebras of \mathfrak{A} .

The following conditions are equivalent (see [176*] and [180*]):

- (a) $I \in \text{ind}(\text{Sub}(\mathfrak{A}))$;
- (b) the subalgebra $\mathcal{C}(\bigcup \{B | B \in I\})$ of the algebra \mathfrak{A} is a K -free product of subalgebras $B \in I$ for any class K of algebras such that $\mathfrak{A} \in K \subset \text{Sub}(\mathfrak{A})$;
- (c) $\{h(B) | B \in I\} \in \text{ind}(\text{Sub}(\mathfrak{A}))$ for every injective homomorphism $h: \mathcal{C}(\bigcup \{B | B \in I\}) \rightarrow A$;
- (d) for any family of homomorphisms $h_B: B \rightarrow A$ ($B \in I$) if $f, g \in T^{(n)}(\mathfrak{A})$ and $a_i \in B_i \in I$ ($i = 1, \dots, n$; $n = 1, 2, \dots$), then the implication

$$\begin{aligned} f(a_1, \dots, a_n) &= g(a_1, \dots, a_n) \\ &\Rightarrow f(h_{B_1}(a_1), \dots, h_{B_n}(a_n)) = g(h_{B_1}(a_1), \dots, h_{B_n}(a_n)) \end{aligned}$$

holds;

- (e) $\bigcup \{B | B \in I\} \in \text{Ind}(\mathfrak{A}, H_*^I)$ (see Section 9).

In particular we have $\{\mathcal{C}(\{a\}) \mid a \in I\} \in \text{ind}(\text{Sub}(\mathfrak{A}))$ iff $I \in \text{Ind}(\mathfrak{A}, H_*)$. If $\{a\} \in \text{Ind}(\mathfrak{A}, M)$ for all elements $a \in I$, and $\{\mathcal{C}(\{a\}) \mid a \in I\} \in \text{ind}(\text{Sub}(\mathfrak{A}))$, then $I \in \text{Ind}(\mathfrak{A}, M)$. Some problems connected with the notion of the independence of subalgebras are posed in [176*] (Problems no. 5-9). If \mathfrak{A} is a unary algebra, then any set of pairwise disjoint subalgebras of \mathfrak{A} is independent. The independence of subalgebras in Boolean algebras or independence of fields of sets (see [445*], Section 13) is another special case of our notion. An indexed set $\{B_i\}_{i \in T}$ of subalgebras of a Boolean algebra $\mathfrak{A} = (A; \cup, \cap, ')$ is independent if

$$\bigcap_{i \in T_0} b_i \neq 0$$

for an arbitrary finite set $T_0 \subset T$ and $b_i \neq 0$, $b_i \in B_i$. This notion (and the notion of almost independence) was also considered in infinitary Boolean algebras and had applications to measure theory (see for example [20], [212*], [259], [298*], [301*], [436], [442*], [464*] and [610]).

There also appeared some notions of independence of algebras (and, in particular, of subalgebras of an algebra) connected with the direct product of general algebras. Let us recall that a family $\{\mathfrak{A}_1, \dots, \mathfrak{A}_n\}$ of algebras, from a class K of algebras with the same similarity type, is called *independent*, if there exists a term $f(x_1, \dots, x_n)$ such that for every $i = 1, 2, \dots, n$ and each $x_1, \dots, x_n \in A_i$ the identity $f(x_1, \dots, x_n) = x_i$ holds. This notion was introduced by A. L. Foster in 1955 ([159]; see also [670]). It is stronger than the notion of "pairwise independence" (see [164], [160] and also [578]). Other kinds of independence of algebras were defined by Foster ([161], "independence in the small" or "local independence of algebras"; see also [121] and [375]), by Foster and Pixley ([162], "weak independence of algebras"), by Pixley ([376], "local weak independence of algebras"), by Eigenthaler and Wiesenbauer ([133], [727]), by Pfanzagl ([366], "stochastic independence of subalgebras"), by Dlab ([118*], [121*], "direct independence of subalgebras"), by Ol'sanskiĭ ([358], "independence of verbal subgroups of a free group"), and by Ljapin ([274], [275], [325], "independence of subsemigroups"). The last two notions can be generalized, in a natural way, to the range of general algebras. Also the notion of "independence of ideals of a ring" is considered in ring theory (see, e.g., [260], p. 253). Recently, L. Bukovský introduced in [568] (see also [593]) the notion of "local independence of Boolean algebras".

Two general questions are:

PROBLEM 10.1. For which algebras is Marczewski's independence of subalgebras equivalent to one of the notions mentioned above?
(P 1124)

PROBLEM 10.2. Establish relations between the notions of independence of algebras (mentioned just above) in the case of subalgebras of a fixed algebra and Marczewski's independence of subalgebras. (P 1125)

11. General algebraic scheme. The Q -independence covers a very large class of independence notions. However, in some special cases, finding an algebra \mathfrak{A} and a suitable family Q is possible only in a rather artificial way, e.g. for the stochastic independence. On the other hand, there appear in the Universal Algebra some kinds of independence, which are defined in a natural way, similarly as the notion of freedom of Birkhoff [29], [32] (see, for example, [420*], [453*], [454*], [428*], [430*] and [295*]). It is worthwhile to add that one can also consider these notions for infinitary algebras, partial algebras, quasi-algebras and so-called algebraic systems, but here we shall only consider the usually used (finitary full) algebras.

Now we shall describe the most general algebraic scheme.

Let $\mathfrak{A} = (A; F)$ and $\mathfrak{B} = (B; G)$ be two algebras. We shall define, following T. Fujiwara ([169], [170]), some large class $F(A, B; \pi_B^A)$ of F -mappings between algebras \mathfrak{A} and \mathfrak{B} . Let $F(A, B; \pi_B^A) \subset B^A$. The expression

$$(\pi_f^h) \quad h(f(x_1, \dots, x_n)) \\ = g(h_1(x_1), \dots, h_m(x_1), h_1(x_2), \dots, h_m(x_2), \dots, h_1(x_n), \dots, h_m(x_n)),$$

where $f \in F$, $g \in T(\mathfrak{B})$, $h, h_1, \dots, h_m \in F(A, B; \pi_B^A)$ and $x_1, \dots, x_n \in A$, is called a *basic mapping formula*.

For example, we can consider the following formulas:

$$(32) \quad h(f(x_1, \dots, x_n)) = g(h_1(x_1), \dots, h_n(x_n)),$$

$$(33) \quad h(f(x_1, \dots, x_n)) = g(h(x_1), \dots, h(x_n)).$$

For a simplification we shall assume that the right-hand side of (π_f^h) is uniquely determined by the pair (f, h) , where $f \in F$ and $h \in F(A, B; \pi_B^A)$. Then h is called an *F-mapping between \mathfrak{A} and \mathfrak{B}* if the suitable basic formulas (π_f^h) are satisfied for each $f \in F$ (for some $g \in T(\mathfrak{B})$, $h_1, \dots, h_m \in F(A, B; \pi_B^A)$ and for every $x_1, \dots, x_n \in A$).

If the basic mapping formulas have the form (32) (or the form (33)) for every f , then h is said to be a *semi-weak homotopy* (a *semi-weak homomorphism*, respectively; see [453*] and [146*], where semi-weak homomorphisms appear under different name). If, moreover, for every $g \in T(\mathfrak{B})$ there exists an $f \in T(\mathfrak{A})$ such that (32) or (33) holds, then h is called a *weak homotopy* or a *weak homomorphism* (see [182*] and [181*]), respectively. $\text{SWHom}(\mathfrak{A}, \mathfrak{B})$ (or $\text{SWHom}(A, B)$) denotes the set of all semi-weak homomorphisms of \mathfrak{A} into \mathfrak{B} and $\text{WHom}(\mathfrak{A}, \mathfrak{B})$ (or $\text{WHom}(A, B)$) denotes the

set of all weak homomorphisms of \mathfrak{A} into \mathfrak{B} . As a particular case we can consider the algebras \mathfrak{A} and \mathfrak{B} of the same similarity type, and if g in (33) is the operation corresponding to f , then, of course, h is a usually used homomorphism ($h \in \text{Hom}(\mathfrak{A}, \mathfrak{B})$).

Consider an algebra $\mathfrak{A} = (A; F)$ and a class K of algebras (maybe of different similarity types or even non-indexed algebras). Let the set π_B^A of basic mapping formulas, and the sets $F(\mathcal{C}(X), B; \pi_B^A) \subset B^{*(X)}$ be defined for every $\mathfrak{B} \in K$ and for each $X \subset A$. Further, let $Q(X, B) \subset B^X$, where $X \subset A$.

A set $I \subset A$ is called *independent relatively to a class K of algebras* with respect to a family Q of mappings under a set π of basic mapping formulas (or, shortly, *Q_K^π -independent*) if for every $\mathfrak{B} \in K$ and $p \in Q(I, B)$ there exists $h \in F(\mathcal{C}(I), B; \pi_B^A)$ such that $h|_I = p$.

In the case of $F(\mathcal{C}(X), B; \pi_B^A) = \text{SWHom}(\mathcal{C}(X); B)$, $Q(X, B) = B^X$ ($X \subset A$) and $K = \{\mathfrak{B}\}$ we obtain \mathfrak{B} - P -independence considered by J. Słomiński in [453*] and [454*] (also for so-called quasi-algebras). If a class K contains only the algebra \mathfrak{A} , $F(\mathcal{C}(X), A; \pi_A) = \text{Hom}(\mathcal{C}(X), A)$ and $Q(X, A) \subset M(A)$, then we get the Q -independence (and, in particular, the M -independence). If a class K consists of algebras of the same similarity type as \mathfrak{A} , then, by taking B^X instead of $Q(X, B)$ and $\text{Hom}(\mathcal{C}(X), B)$ instead of $F(\mathcal{C}(X), B; \pi_B^A)$, we obtain the K -independence considered (under different names) by J. Schmidt, P. Burmeister, T. K. Hu and A. I. Mal'cev (see [74*]-[76*], [78*], [79*], [220*], [273], [285], [295*], [419*]-[430*], [780*], [781*]); this notion was considered for infinitary partial algebras and for algebraic systems in Mal'cev's sense). If K forms a category with free algebras, then our K -dependence is exactly the *standard dependence* (relative to K), which is defined in Cohn's book ([86*], Section 2 of Chapter 7). If X is a K -independent set of generators of \mathfrak{A} , then \mathfrak{A} is a *free algebra* relatively to the class K (or, shortly, *K -free algebra*). In a particular case, if K consists of all algebras with the same similarity type as \mathfrak{A} , then \mathfrak{A} is (*absolutely*) *free* in the well-known sense. This last notion was introduced by G. Birkhoff (in an equivalent form, see [29] and [32]; but it is worth to add that it also appears implicitly in some earlier papers, e.g. free Boolean algebras were considered by G. Boole [44], free distributive lattices by Th. Skolem [447] and [448], free unars (i.e., algebras with only one unary operation) by R. Dedekind [97]). For further references see: on free products in Section 10, and also [11], [36], [53], [67], [100], [122], [148*], [155], [171], [195]-[197], [213], [260], [272], [277], [285], [292]-[294], [370], [427*], [432], [451*], [457*], [481], [482], [485], [497], [587], [605], [670], [678], [684]. Since the notions of freedom can be also defined using a logical notion of satisfiability of equations by elements of an algebra, the considerations of independence are connected with the word problem (let us mention some

remarkable papers of T. Evans, Ph. Hall, P. M. Whitman and others, see e.g. [45], [88], [137], [138]-[140], [207], [224], [257], [537], [557], [582], [600], [607], [630], [653]).

Let $I \subset A$ and $\mathfrak{A} = (A; F)$ be an (indexed) algebra. We can consider, following J. Schmidt, the largest class $\text{ind}_{\mathfrak{A}}(I)$ of algebras \mathfrak{B} of the same type as \mathfrak{A} such that I is a K -independent set, where $K = \{\mathfrak{B}\}$. One can prove that $\text{ind}_{\mathfrak{A}}(I)$ is a variety (see [419*], [420*]). This variety was also considered (in a more general situation) by P. Burmeister ([74*], [75*], [76*]), J. Schmidt ([418*], [420*], [423*], [425*], [428*], [429*], [430*]) and J. Słomiński ([453*], [454*]).

In particular, if for some $I \subset A$ the variety $\text{ind}_{\mathfrak{A}}(I)$ consists of all algebras of the same similarity type as \mathfrak{A} , then $\mathcal{C}(I)$ is an absolute free algebra; and this situation can be intrinsically described by so-called generalized Peano axioms (see [428*]). The extremally opposite special case is obtained if $\text{ind}_{\mathfrak{A}}(I)$ consists precisely of the algebras \mathfrak{B} with $\text{card}(B) \leq 1$; we may call I *absolutely dependent*. We do not know answers to the following problems:

PROBLEM 11.1 (J. Schmidt, see [428*]). Give the inner characterization of absolutely dependent sets.

PROBLEM 11.2. When does the set $\{\text{ind}_{\mathfrak{A}}(I) \mid I \in \text{Ind}(\mathfrak{A}, M)\}$, for a fixed algebra \mathfrak{A} , form a lattice (a modular lattice or a distributive lattice) of varieties? (**P 1126**)

PROBLEM 11.3. Investigate the Q_K^* -independence. (**P 1127**)

PROBLEM 11.4. Characterize the families of K -independent sets of an arbitrary algebra. (**P 1128**)

PROBLEM 11.5. Relate to the Q_K^* -independence (or K -independence, or Q -independence) the general algebraic dependence relations considered by V. Dlab in [118*] and [119*]. (**P 1129**)

PROBLEM 11.6. Insert, in the general algebraic scheme of Q_K^* -dependence, various special dependence relations considered in group theory (see, e.g., [287], compare with Grätzer's problem P 607 of [194*], and also see [345], [405], [449], [462], [671], [641], [628], [791]), in the theory of semigroups (e.g., [276], [324], [787]), in the theory of semilattices (e.g., [411]) and in ring theory ([46], [83], [87], [172], [204], [232], [355], [356], [515], [516], [689], [758] and [825]). (**P 1130**)

PROBLEM 11.7. Insert in the general scheme of Q_K^* -independence the following notions: k -independence in Euclidean spaces (in the sense of K. Borsuk, see: [49], [349], [350], [404], [410], [545]), independence of real numbers in the sense of Turán ([135], [136], [264], [367], [595a]), independence in Riesz spaces and independence of measures [54], stochastic independence of functions (in the sense of Steinhaus, see [246],

or in the sense of Kolmogoroff, [258], p. 50; in general, these two definitions are not equivalent, see e.g. [396*]). (P 1131)

PROBLEM 11.8. Let B_μ be the field of subsets of a set U which are measurable with respect to some measure μ such that $\mu(U) = 1$. Considering B_μ as a Boolean ring¹ (with the unity U) insert in the general scheme of Q_K^2 -independence the notion of stochastic independence (see [302*], [142*], and also for related topics [443], [366] and [338*]). (P 1132)

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Besides the references in Section 1, we can mention some recent papers on general theory of matroids: [550], [554], [559], [560], [563], [572]-[574], [596], [608], [609], [614], [620]-[624], [626], [627], [632]-[635], [639], [644], [660], [662]-[664], [666]-[669] and [683]. There are also remarkable papers on applications of the matroid theory to the network theory: [561], [562], [571], [602], [603], [647], [648], [651], [652], [656]-[659], [661], [672], [732], [790], [830], and on applications to the theory of optimization: [210], [569], [570], [577], [589]-[591], [604], [611], [613], [625], [685]-[687], [710], [728], [730], [750] and [761].

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The bibliography consists of three alphabetically arranged parts: [1]-[548], [549]-[687] and [688]-[834] (added in proof). With the exceptions listed below we use the standard abbreviations of "Mathematical Reviews". In the case where such an abbreviation does not exist we did the best to give the full data. An asterisk denotes items especially connected with the work of Professor Edward Marczewski.

The exceptions are:

AL — Алгебра и логика,

AU — Algebra Universalis,

AUC — Acta Facultatis Rerum Naturalium Universitatis Comenianae. Mathematica (Bratislava),

BAMS, NAMS, PAMS and TAMS — Bulletin, Notices, Proceedings and Transactions of the American Mathematical Society, respectively,

BAPS — Bulletin de l'Académie Polonaise des Sciences, Série des sciences mathématiques, astronomiques et physiques,

CM — Colloquium Mathematicum,

CMUC — Commentationes Mathematicae Universitatis Carolinae,

DAN — Доклады Академии наук СССР,

FM — Fundamenta Mathematicae,

IAN — Известия Академии наук СССР, серия математическая,

JCTh-A — Journal of Combinatorial Theory, Series A,
 JCTh-B — Journal of Combinatorial Theory, Series B,
 UZap — Ученые записки Ленинградского государственного педагогического
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ADDED IN PROOF

We have to mention some papers on matroids: [705], [723]-[725], [729], [733], [736], [743], [744], [763], [764], [783], [784], [795], [799], [823], [824] and [831]. For infinite matroids see also: [42*], [62], [93*], [115*], [222], [558], [598], [619], [721], [737]-[739], [797], [798] and [829]. For the linear representation problem see: [690], [698], [826]-[828]. It is worth to add that recently P. Vámos has obtained [827] the negative answer to Rado's problem P 533 of [394] (and also to some Whitney's problem of [538]) concerning the existence of an intrinsic characterization of linear matroids. Let us note a few papers concerning the geometric and lattice theoretical approach of the independence theory: [550], [626], [654], [665], [666], [692], [709], [713], [726], [753]-[756], [762], [770], [771], [782], [794], [804], [805], [807], [809], [818]-[822] (see also [792] and [769]). We want to pay attention to the open problems concerning the theory of matroids (matroid lattices) in [706], [748] and [826]. Let us remark that some part of the convexity theory is very near to considerations of \mathcal{C} -independence (see [310*], [623], [691], [694]-[696], [708], [722], [759], [766] and [803]). Note also papers concerning the notions of freedom: [499*], [688], [712], [734], [767], [768], [793], [801], [802].

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