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INVERSE INVARIANCE OF METRIZABILITY FOR ORDERED SPACES

BY

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In this paper we examine the inverse invariance of metrizability and weight under continuous mappings defined on topological ordered spaces.

Let K be a topological ordered space and $f\colon K \twoheadrightarrow Y$ a continuous mapping of K onto Y. We ask under what conditions on f:

1. the metrizability of Y implies the metrizability of K?
2. the weight of K is not greater than the weight of Y?

We prove some theorems presenting conditions for positive answers to 1 and 2 and give examples showing the essentiality of assumptions.

Let us recall that the problem of inverse invariance of metrizability and weight under continuous mappings has been investigated for example in [2], [3], [5] and [8]-[13].

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1. Definitions. All undefined terms and notions are as in [4]. The symbol $f\colon X \twoheadrightarrow Y$ denotes a continuous mapping of X onto Y. All spaces are assumed to be *infinite Hausdorff*.

1.1. A family $\mathfrak{N}$ of subsets of X is called a *net* in X if, for every $x \in X$ and its neighbourhood U, there exists an $N \in \mathfrak{N}$ such that $x \in N \subset U$.

1.2. We adopt the following symbols:

$w(X)$ — the *weight* of X,

$nw(X)$ — the *net weight* of X, i.e. the minimal cardinality of a net in X,

$|A|$ — the *cardinality* of A,

$X \in MTR$ means that X *is metrizable*,

$L(X)$ — the *Lindelöf number* of X; i.e. $\min\{\mathfrak{m} \in \mathrm{Card}\colon$ every open covering of X admits a subcovering of cardinality not greater than $\mathfrak{m}\}$,

$I = [0, 1]$.

1.3. A space K is called a *topological ordered space* if K is linearly ordered and is equipped with the topology induced by this order. Every topological ordered space is hereditarily collectionwise normal [14]. *In what follows, the space denoted by the symbol K is assumed to be a topological ordered space.*

1.4. A set $A \subset K$ is *convex* if, for every $x, y \in A$, the interval $[x, y]$ is contained in A. If $A \subset B \subset K$, then A is called a *convex component of B* if A is a maximal convex subset of B. Convex components of any set B are disjoint and closed in B. If B is open, then its convex components are also open.

1.5. A mapping $f\colon K \twoheadrightarrow Y$ is said to be *zero-dimensional in the sense of ordering* (abr. *order-zero-dimensional*) [9] if, for every $y \in Y$, convex components of $f^{-1}(y)$ are one-point sets.

1.6. A pair of *non-isolated* points $\{x^-, x^+\}$ is called a *proper jump in K* if $x^-, x^+ \in K$, $x^- < x^+$ and $(x^-, x^+) = \emptyset$. The points x^-, x^+ are called *proper jump points* in K. The set consisting of all proper jump points in K is denoted by $\mathscr{J}(K)$.

For $f\colon K \twoheadrightarrow Y$ the symbol $\mathscr{J}(K, f)$ denotes the set of all proper jump points corresponding to those proper jumps $\{x^-, x^+\}$ which satisfy $f(x^-) = f(x^+)$. Thus $\mathscr{J}(K, f) \subset \mathscr{J}(K)$.

1.7. A space K is *order-dense* if for every two distinct points of K there exists a point in K lying between them.

1.8. A mapping $f\colon X \twoheadrightarrow Y$ is *quasi-open* if $\operatorname{Int} f(U) \neq \emptyset$ for every non-empty, open set $U \subset X$.

1.9. A subspace $A \subset X$ is *σ-discrete in X* if $A = \bigcup_{n=1}^{\infty} F_n$, where F_n are closed and discrete.

1.10. Let us recall that if $f\colon X \twoheadrightarrow Y$ is perfect, then $w(X) \geqslant w(Y)$ and the metrizability of X implies the metrizability of Y (cf. [4], Problem 4.S).

2. Inverse invariance of metrizability. The following lemma is a particular case of Theorem 5.3 from [6], but its proof given here is much simpler and avoids introducing superfluous notions.

LEMMA 2.1. *A topological ordered space is metrizable if and only if it has a σ-discrete net.*

Proof. Assume that

$$\mathfrak{F} = \bigcup_{n=1}^{\infty} \mathfrak{F}_n,$$

where the families $\mathfrak{F}_n$ are discrete and consist of closed sets, is a net in K.

For every $k = 1, 2, \ldots$ let

$$\mathfrak{K}_k = \bigcup_{n=1}^{k} \mathfrak{F}_n$$

and for $F \in \mathfrak{K}_k$ let

$$V_k(F) = K \setminus \bigcup \{F' \in \mathfrak{K}_k \colon F' \cap F = \emptyset\}.$$

As the families $\mathfrak{K}_k$ are locally finite, the sets $V_k(F)$ are open and $F \subset V_k(F)$. Let $U_k(F)$ be the union of all convex components of $V_k(F)$ intersecting F.

Since K is collectionwise normal (see 1.3), for every $n = 1, 2, \ldots$ and $k \geqslant n$ we can find a discrete family $\mathfrak{B}(n, k) = \{B_{n,k}(F) \colon F \in \mathfrak{F}_n\}$ of open sets such that $F \subset B_{n,k}(F) \subset U_k(F)$.

It is easy to check that the family

$$\mathfrak{B} = \bigcup_{n=1}^{\infty} \bigcup_{k=n}^{\infty} \mathfrak{B}(n, k)$$

forms a σ-discrete base in K, and hence K is metrizable. The inverse implication is obvious.

The following theorem can be easily deduced from Lemma 2.1 and the result of Proizvolov [11]:

THEOREM 2.2 ([8], Proposition 4.13(c)). *Let $f \colon K \twoheadrightarrow Y$ be finite-to-one and open. If $Y \in MTR$, then $K \in MTR$.*

LEMMA 2.3. *Let Y be metrizable. If $f \colon K \twoheadrightarrow Y$ satisfies the conditions*

(i) *$\mathscr{J}(K, f)$ is σ-discrete in K,*

(ii) *for every $a, b \in K$, if $(a, b) \neq \emptyset$ and $f([a, b)) = \{y\}$ or $f((a, b]) = \{y\}$, then y is an isolated point and $f^{-1}(y) \in MTR$;*

then $K \in MTR$.

Proof. We show that K has a σ-discrete net. Let

$$Y_0 = \{y \in Y \colon y \text{ is an isolated point and } f^{-1}(y) \in MTR\}.$$

Since Y_0 is a σ-discrete subset of Y, there exists in K a σ-discrete family $\mathfrak{A}$, which forms a net in $f^{-1}(Y_0)$.

Put $\mathfrak{C} = \{\{x\} \colon x \in \mathscr{J}(K, f)\}$. According to (i) the family $\mathfrak{C}$ is σ-discrete in K. For every $n = 1, 2, \ldots$ let $\mathfrak{U}_n$ be a σ-discrete covering of Y consisting of non-void, open sets with diameters less than $1/2n$.

From every

$$U \in \bigcup_{n=1}^{\infty} \mathfrak{U}_n$$

let us choose an element $y(U)$. We have $\overline{U} \subset B(y(U), 1/n)$, where the symbol $B(y, r)$ denotes the ball in Y with the radius r and the centre at the point y.

Let $\mathfrak{W}_U = \{W(U, s)\}_{s \in S_U}$ be the family of all convex components of the set $f^{-1}(B(y(U), 1/n))$ and put:

$$\mathfrak{B}_U = \{W(U, s) \cap f^{-1}(U)\}_{s \in S_U}; \quad \mathfrak{B}_n = \bigcup \{\mathfrak{B}_U \colon U \in \mathfrak{U}_n\}; \quad \mathfrak{B} = \bigcup_{n=1}^{\infty} \mathfrak{B}_n.$$

As for every U the family $\mathfrak{B}_U$ is discrete in K and $\mathfrak{U}_n$ is a σ-discrete family in Y, the family $\mathfrak{B}$ is σ-discrete in K.

It suffices to show that the family $\mathfrak{F} = \mathfrak{A} \cup \mathfrak{B} \cup \mathfrak{C}$ forms a net in K.

Let $x \in (a, b) \subset K$.

1. If $x \in f^{-1}(Y_0)$, then there exists $A \in \mathfrak{A}$ such that $x \in A \subset (a, b)$, for $\mathfrak{A}$ is a net in $f^{-1}(Y_0)$.

2. If $x \in \mathscr{J}(K, f)$, then the set $\{x\}$ belongs to $\mathfrak{F}$.

3. If $x \notin f^{-1}(Y_0) \cup \mathscr{J}(K, f)$, then there exist $c, d \in K$ such that $a \leqslant c < x < d \leqslant b$ and $f(c) \neq f(x) \neq f(d)$.

Suppose for example that $f([a, x]) = \{f(x)\}$; then according to (ii) we would have $(a, x) = \emptyset$ and one of the points a, x should be isolated, say a. Then there would exist a predecessor a' of a satisfying $f((a', x]) = \{f(x)\}$, and we would have $f(x) \in Y_0$, which is impossible.

Choose n such that $f(c), f(d) \notin B(f(x), 2/n)$ and a $U \in \mathfrak{U}_n$ containing $f(x)$. We have

$$f(x) \in U \subset B\left(y(U), \frac{1}{n}\right) \subset B\left(f(x), \frac{2}{n}\right)$$

and therefore the convex component $W(U, s)$ of x lies in the interval (c, d) and $x \in W(U, s) \cap f^{-1}(U) \in \mathfrak{B}$.

We have shown that K has a σ-discrete net which, together with Lemma 2.1, completes the proof.

LEMMA 2.4. *If K is metrizable, then the set $\mathscr{J}(K)$ is σ-discrete.*

Proof. Let $\{\{x_a^-, x_a^+\}\}_{a \in A}$ be the family of all proper jumps and $\mathfrak{B}$ a σ-discrete base in K. For every $a \in A$ there exist $B_a^-, B_a^+ \in \mathfrak{B}$ such that $x_a^- \in B_a^- \subset (-\infty, x_a^-]$ and $x_a^+ \in B_a^+ \subset [x_a^+, +\infty)$. As different B_a^- (B_a^+) correspond to different a, therefore $\mathscr{J}(K)$ is σ-discrete in K.

Lemmas 2.3 and 2.4 imply the following theorems:

THEOREM 2.5. *Let $Y \in MTR$ and $f\colon K \twoheadrightarrow Y$; assume that*

(a) *f is quasi-open and inverse images of isolated points are metrizable; or*

(b) *inverse images of points by f are nowhere dense.*

Then, $K \in MTR$ if and only if the set $\mathscr{J}(K, f)$ is σ-discrete in K.

THEOREM 2.6. *Let $f\colon K \twoheadrightarrow Y$ be order-zero-dimensional. If $Y \in MTR$, then $K \in MTR$.*

COROLLARY 2.7. *Let K be order-dense and $f\colon K \twoheadrightarrow Y$ open with metrizable or compact inverse images of points.*

If $Y \in MTR$, then $K \in MTR$.

Proof. If inverse images of points are metrizable, then Corollary 2.7 follows immediately from Theorem 2.5(a). In the case of compact inverse images of points one can easily observe that Y has to be dense-in-itself and once more Theorem 2.5(a) can be applied. We recall that all spaces are assumed to be infinite.

COROLLARY 2.8. *Let K be order-dense and $f: K \twoheadrightarrow Y$ have nowhere dense inverse images of points.*

If $Y \in MTR$, then $K \in MTR$.

Remark 2.9. All assumptions in 2.5-2.8 are essential. The projection of the square $I \times I$, with the lexicographic order topology, onto I shows the necessity of assumptions (a) and (b) in Theorem 2.5, of openness of f in Corollary 2.7, and of nowhere density of inverse images of points in Corollary 2.8.

The projection of the "double arrow" of Alexandroff (see [1], p. 97, or [4], Exercise 3.9.C) onto the interval I shows that the assumptions of σ-discretness of $\mathcal{J}(K,f)$ in Theorem 2.5 and of order-density of K in Corollary 2.8 are essential.

The conditions imposed on inverse images of points in Corollary 2.7 cannot be omitted. Let X be the square $I \times I$ with the lexicographic order topology without the end points and $K = N \times X$, where N is the space of natural numbers. Consider K with the topology induced by the lexicographic order. The projection $f: K = N \times X \twoheadrightarrow N$ is open and continuous, and K is an order-dense, non-metrizable space.

The example given in section 4 shows the essentiality of the assumption of order-density of K in Corollary 2.7.

3. Inverse invariance of weight.

LEMMA 3.1. *For every topological ordered space K we have $w(K) = nw(K)$.*

Proof. It suffices to show that $w(K) \leqslant nw(K)$. Let $\mathfrak{F}$ be a net in K of cardinality $nw(K)$. We may assume that $\mathfrak{F}$ consists of closed sets. For every triple of disjoint sets $F_1, F_2, F_3 \in \mathfrak{F}$ let $B(F_1, F_2, F_3)$ denote the union of all convex components of $K \setminus (F_2 \cup F_3)$ intersecting F_1. It is easy to see that the family

$$\mathfrak{B} = \{B(F_1, F_2, F_3): F_1, F_2, F_3 \text{ are disjoint and belong to } \mathfrak{F}\}$$

forms a base in K and has cardinality $nw(K)$.

THEOREM 3.2. *If $f: K \twoheadrightarrow Y$ is finite-to-one and open, then $w(K) = w(Y)$.*

Proof. We have only to show that $w(K) \leqslant w(Y)$. For $k = 1, 2, \ldots$ let $Y_k = \{y \in Y: |f^{-1}(y)| = k\}$. According to [11] we have $w\big(f^{-1}(Y_k)\big)$

$\leqslant w(Y)$ and therefore $nw(K) \leqslant w(Y)$. Lemma 3.1 implies that $w(K) = nw(K) \leqslant w(Y)$, which completes the proof.

LEMMA 3.3. *If $f\colon X \twoheadrightarrow Y$ is closed, then*

$$L(X) \leqslant \max\{L(Y), \sup_{y \in Y} L(f^{-1}(y))\}.$$

The validity of Lemma 3.3 is obvious.

LEMMA 3.4. *If $f\colon K \twoheadrightarrow Y$ is quasi-open, then for every open $U \subset Y$ the family of all convex components of $f^{-1}(U)$ is of cardinality not greater than* $\max\{w(Y), \sup_{y \in Y} L(f^{-1}(y))\}$.

Proof. Let $\mathfrak{W}$ be the family of all convex components of $f^{-1}(U)$ and

$$\mathfrak{m} = \max\{w(Y), \sup_{y \in Y} L(f^{-1}(y))\}.$$

According to 1.4, elements of $\mathfrak{W}$ are open. For every $y \in Y$ we have

$$|\{W \in \mathfrak{W}\colon W \cap f^{-1}(y) \neq \emptyset\}| \leqslant L(f^{-1}(y)) \leqslant \mathfrak{m}.$$

Therefore the family $\mathfrak{V} = \{\mathrm{Int} f(W)\colon W \in \mathfrak{W}\}$ is pointwise of cardinality not greater than $\mathfrak{m}$. It follows that $|\mathfrak{V}| \leqslant \mathfrak{m}$ and hence $|\mathfrak{W}| \leqslant \mathfrak{m}$.

The proof of the following lemma is similar to the proof of Lemma 2.4:

LEMMA 3.5. *For every topological ordered space K we have $|\mathscr{J}(K)| \leqslant w(K)$.*

LEMMA 3.6. *Let $f\colon K \twoheadrightarrow Y$ satisfy the conditions:*

(i) $|\mathscr{J}(K, f)| \leqslant w(Y)$;

(ii) *for every $a, b \in K$, if $(a, b) \neq \emptyset$ and $f([a, b)) = \{y\}$ or $f((a, b]) = \{y\}$, then y is an isolated point and $w(f^{-1}(y)) \leqslant w(Y)$;*

(iii) *for every open $U \subset Y$ the family of all convex components of $f^{-1}(U)$ is of cardinality not greater than $w(Y)$.*

Then $w(K) \leqslant w(Y)$.

Proof. By Lemma 3.1 we have only to show that $nw(K) \leqslant w(Y)$. Let

$$Y_0 = \{y \in Y\colon y \text{ is an isolated point and } w(f^{-1}(y)) \leqslant w(Y)\}.$$

As $|Y_0| = w(Y_0) \leqslant w(Y)$, we can find in $f^{-1}(Y_0)$ a net $\mathfrak{A}$ of cardinality not greater than $w(Y)$.

Let $\mathfrak{C} = \{\{x\}\colon x \in \mathscr{J}(K, f)\}$ and $\mathfrak{G}$ be a base in Y such that $|\mathfrak{G}| = w(Y)$. According to (iii), for every $U \in \mathfrak{G}$, the family $\mathfrak{W}_U$ of all convex components of $f^{-1}(U)$ is of cardinality not greater than $w(Y)$. Put

$$\mathfrak{B} = \bigcup_{U \in \mathfrak{G}} \mathfrak{W}_U.$$

To show that the family $\mathfrak{F} = \mathfrak{A} \cup \mathfrak{B} \cup \mathfrak{C}$ forms a net in K we argue similarly to the proof of Lemma 2.3.

Lemmas 3.3-3.6 imply:

THEOREM 3.7. *If* $f\colon K \twoheadrightarrow Y$ *is*

(a) *quasi-open,* $w\big(f^{-1}(y)\big) \leqslant w(Y)$ *for every isolated* $y \in Y$ *and* $L\big(f^{-1}(y)\big) \leqslant w(Y)$ *for every* $y \in Y$; *or*

(b) *closed and for every* $y \in Y$ *we have* $\mathrm{Int}\big(f^{-1}(y)\big) = \emptyset$ *and* $L\big(f^{-1}(y)\big) \leqslant w(Y)$;

then, $w(K) \leqslant w(Y)$ *if and only if* $|\mathscr{I}(K,f)| \leqslant w(Y)$.

THEOREM 3.8 (cf. [9]). *If* $f\colon K \twoheadrightarrow Y$ *is perfect, order-zero-dimensional, then* $w(K) = w(Y)$.

COROLLARY 3.9. *Let* K *be order-dense. If* $f\colon K \twoheadrightarrow Y$ *is open and* $w\big(f^{-1}(y)\big) \leqslant w(Y)$ *for every* $y \in Y$, *then* $w(K) = w(Y)$.

COROLLARY 3.10. *Let* K *be order-dense. If* $f\colon K \twoheadrightarrow Y$ *is open and has compact inverse images of points, then* $w(K) = w(Y)$.

COROLLARY 3.11. *Let* K *be order-dense. If* $f\colon K \twoheadrightarrow Y$ *is perfect and has nowhere dense inverse images of points, then* $w(K) = w(Y)$.

Remark 3.12. All assumptions in 3.7-3.11 are essential (cf. Remark 2.9). The necessity of the assumption of order-density of K in Corollaries 3.9 and 3.10 follows from the example given in section 4. The assumption of perfectness of f in Theorem 3.8 and Corollary 3.11 cannot be omitted as can be shown by the projection of the space $[0,1] \times (0,1)$, with the lexicographic order topology, onto $(0,1)$.

4. Non-metrizable inverse images of metric spaces. Since every countable subset of a topological ordered space is metrizable, Corollaries 2.7, 2.8, 3.9, 3.11 and the Baire theorem imply the following

COROLLARY 4.1. *Let* K *be order-dense. If* $f\colon K \twoheadrightarrow Y$ *is open* (*or perfect*) *and countable-to-one, then*:

(i) *if* $Y \in MTR$, *then* $K \in MTR$;

(ii) $w(K) = w(Y)$.

The assumption of order-density of K is essential as the following theorem shows:

THEOREM 4.2. *There exists a compact, perfectly normal and non-metrizable topological ordered space* K *and an open, perfect, countable-to-one mapping* $f\colon K \twoheadrightarrow C$ *of* K *onto the Cantor set* C.

LEMMA 4.3. *Let* L *be a compact topological ordered space and* $g\colon L \twoheadrightarrow Y$ *an open, countable-to-one mapping of* L *onto a metric separable space* Y *satisfying the following conditions*:

(i) *there exists an uncountable subset* P *of* L *such that for every* $x \in P$ *and* $z \in L$, *if* $x \neq z$, *then there exists a* $p \in P$ *lying between* x *and* z;

(ii) *for every* $x, z \in P$, *if* $x < z$, *then the set* $g([x,z])$ *is open in* Y.

Under these conditions, there exists a compact, perfectly normal and non-metrizable topological ordered space K and an open, countable-to-one mapping $f\colon K \twoheadrightarrow Y$.

Proof. Consider $K = L\times\{0\}\cup P\times\{1\}\subset L\times\{0,1\}$ with the order induced by the lexicographic order in $L\times\{0,1\}$. Introduce in K the order topology. It is easily seen that the projection h of K onto L is continuous and perfect. Therefore the space K is compact.

As $w(K)\geqslant|\mathscr{J}(K)|>\aleph_0$ (cf. Lemma 3.5), the space K is non-metrizable. The mapping $f = g\circ h\colon K\twoheadrightarrow Y$ is perfect and countable-to-one. Using conditions (i) and (ii) one can show that f is open. It is easy to notice that K is separable. By Theorem 2.2 from [7], K is perfectly normal, which completes the proof.

To prove Theorem 4.2 it suffices to give an example satisfying the conditions of Lemma 4.3 with $Y = C$. Such an example has been given by R. Pol.

Example 4.4. (R. Pol). Let C be the Cantor set in the interval I, i.e.,

$$C = \left\{x\in I\colon x = \sum_{n=1}^{\infty}\frac{x_n}{3^n},\ \text{where } x_n \text{ equals 0 or 2 for } n = 1, 2, \ldots\right\}.$$

For $k = 0, 1, 2, \ldots$ put

$$F_k = \left\{x\in I\colon x = \sum_{n=1}^{\infty}\frac{x_n}{3^n},\ \text{where } x_n \text{ equals 0, 1, or 2 and } x_n\neq 1 \text{ for } n = 1, 2, \ldots, k\right\}.$$

Therefore,

$$F_k = \bigcup_{m=1}^{2^k}[u(k,m), v(k,m)]$$

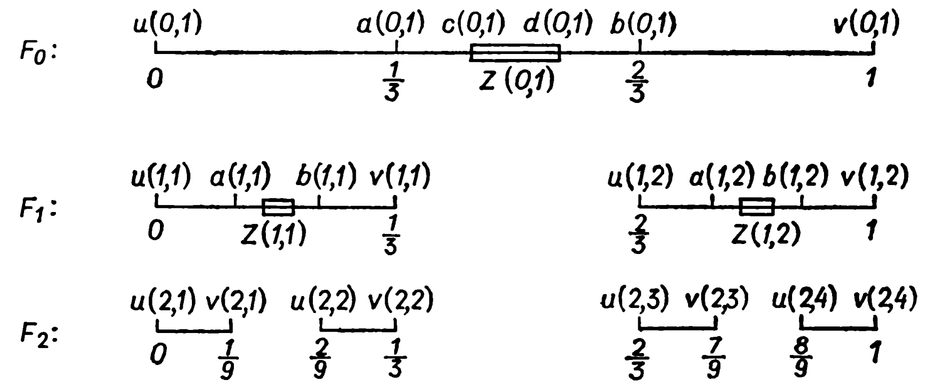
and the intervals $[u(k,m), v(k,m)]$ are components of F_k.

Put

$$a(k,m) = u(k,m)+\frac{1}{3^{k+1}},\qquad b(k,m) = v(k,m)-\frac{1}{3^{k+1}},$$

$$c(k,m) = a(k,m)+\frac{1}{3^{k+2}},\qquad d(k,m) = b(k,m)-\frac{1}{3^{k+2}}.$$

Let $Z(k,m)$ be the Cantor set constructed in an analoguous way in the interval $[c(k,m), d(k,m)]$; $S(k,m) = C\cap[u(k,m), v(k,m)]$, and $g_{k,m}\colon Z(k,m)\twoheadrightarrow S(k,m)\subset C$ be a homeomorphism of $Z(k,m)$ onto $S(k,m)$.



Since

$$L = C \cup \bigcup_{k=0}^{\infty} \bigcup_{m=1}^{2^k} Z(k, m)$$

is a compact subspace of I, its topology coincides with the topology induced by the natural order of I.

Let us define the mapping $g\colon L \twoheadrightarrow C = Y$ by

$$g(x) = \begin{cases} x & \text{if } x \in C, \\ g_{k,m}(x) & \text{if } x \in Z(k, m). \end{cases}$$

As the family

$$\{[u(k, m), v(k, m)] \cap C\}_{k=0, m=1}^{\infty, 2^k}$$

forms a base in C and

$$[u(k, m), v(k, m)] \cap L = g^{-1}\big([u(k, m), v(k, m)] \cap C\big),$$

therefore g is continuous. Similarly, it is easy to prove that g is open and countable-to-one. The set

$$P = C \setminus \bigcup_{k=0}^{\infty} \bigcup_{m=1}^{2^k} \{a(k, m), b(k, m)\}$$

satisfies condition (i) of Lemma 4.3. It suffices to show that condition (ii) is also satisfied.

Assume that $x, z \in P$, $x < z$. Let k_0 be the largest $k = 0, 1, 2, \ldots$ such that $[x, z] \subset F_k$. Choose m_0 such that

$$[x, z] \subset H = [u(k_0, m_0), v(k_0, m_0)].$$

Then $[x, z] \supset Z(k_0, m_0)$ and we obtain

$$H \cap C = g\big(Z(k_0, m_0)\big) \subset g([x, z] \cap L) \subset g(H \cap L) = H \cap C,$$

whence

$$g([x, z] \cap L) = [u(k_0, m_0), v(k_0, m_0)] \cap C$$

and this set is open in C.

REFERENCES

[1] П. С. Александров и П. С. Урысон, *Мемуар о компактных топологических пространствах*, Москва 1971.

[2] А. В. Архангельский, *Признак существования бикомпактного элемента в непрерывном разбиении. Теорема об инвариантности веса при открыто-замкнутых конечно-кратных отображениях*, Доклады Академии наук СССР 166 (1966), p. 1263-1266.

[3] — *Теорема о метризуемости прообраза метрического пространства при открыто-замкнутом конечно-кратном отображении; примеры и нерешенные задачи*, ibidem 170 (1966), p. 754-762.

[4] R. Engelking, *Outline of general topology*, Amsterdam 1968.

[5] — and A. Lelek, *Metrizability and weight of inverses under confluent mappings*, Colloquium Mathematicum 21 (1970), p. 239-246.

[6] D. J. Lutzer, *On generalized ordered spaces*, Dissertationes Mathematicae 89 (1971), p. 1-36.

[7] — and H. R. Bennet, *Separability, the countable chain condition and the Lindelöf property in linearly ordered spaces*, Proceedings of the American Mathematical Society 23 (1969), p. 664-667.

[8] V. I. Mancuso, *Inverse images and first countability*, General Topology and Applications 2 (1972), p. 29-44.

[9] S. Mardešić and P. Papić, *Continuous images of ordered compacta, the Suslin property and diadic compacta*, Glasnik Matematičko-Fizički a Astronomski 17 (1962), p. 3-22.

[10] В. В. Произволов, *О разбивающих отображениях локально связных пространств*, Доклады Академии наук СССР 159 (1964), p. 516-518.

[11] — *О конечно-кратных открытых отображениях*, ibidem 166 (1966), p. 38-40.

[12] T. Przymusiński, *Metrizability of inverse images of metric spaces under open, perfect and zero-dimensional mappings*, Colloquium Mathematicum 24 (1972), p. 175-180.

[13] — *Non-metrizable inverse images of metric spaces under open, perfect, countable-to-one mappings*, Bulletin de l'Académie Polonaise des Sciences, Série des sciences mathématiques, astronomiques et physiques, 20 (1972), p. 47-49.

[14] L. A. Steen, *A direct proof that the interval topology is collectionwise normal*, Proceedings of the American Mathematical Society 24 (1970), p. 727-728.

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