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INVERSE LIMITS AND HYPERSPACES

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1. Introduction. Throughout the paper X is assumed to be a Hausdorff space. By a hyperspace of X we shall mean any family $\mathfrak{H}(X)$ consisting of non-empty compact subsets of X provided with a topology. Of particular importance will be the hyperspace $\mathrm{Comp}(X)$ consisting of all non-empty compact subsets of X, the hyperspace $C(X)$ consisting of all non-empty and connected subsets (i.e., of all subcontinua) of X, and in the case of X being metric, the hyperspace $\mathrm{Conv}(X)$ consisting of all compact convex subsets of X — all three hyperspaces provided with the Vietoris topology.

A base of the Vietoris topology in a hyperspace $\mathfrak{H}(X)$ consists of all sets of the form

$$\langle U_1, \ldots, U_n; \mathfrak{H}(X)\rangle = \{A \in \mathfrak{H}(X)\colon A \subset \bigcup_{i=1}^{n} U_i, A \cap U_i \neq 0 \text{ for } i = 1, 2, \ldots, n\},$$

where $U_1, \ldots, U_n$ are open in X and n is any integer.

As is well known, if X is a metric space with a metric ϱ, then a hyperspace $\mathfrak{H}(X)$ with the Vietoris topology can be metrized by the Hausdorff distance

$$\varrho^1(A, B) = \max[\sup_{a\in A} \varrho(a, B), \sup_{b \in B} \varrho(A, b)].$$

An *inverse system* is a family $\boldsymbol{X} = \{X_\alpha, \pi_\beta^\alpha, \Sigma\}$, where Σ is a directed set, X_α is a topological space for each $\alpha \in \Sigma$, and $\pi_\beta^\alpha\colon X_\alpha \to X_\beta$ is a continuous mapping for $\alpha \geqslant \beta$. Moreover, $\pi_\alpha^\alpha = \mathrm{id}_{X_\alpha}$ and if $\alpha \geqslant \gamma \geqslant \beta$, then $\pi_\beta^\gamma \cdot \pi_\gamma^\alpha = \pi_\beta^\alpha$.

A subset of the Cartesian product $\mathop{\times}_{\alpha \in \Sigma} X_\alpha$ consisting of all threads of the inverse system $\boldsymbol{X}$, i.e. of all elements $\{x_\alpha\}_{\alpha\in\Sigma}$ such that $\pi_\beta^\alpha(x_\alpha) = x_\beta$ for $\alpha \geqslant \beta$, and provided with the product topology, is called the *limit of inverse system* $\boldsymbol{X}$ (or, shortly, (*inverse*) *limit of* $\boldsymbol{X}$) and is denoted by $\lim\{X_\alpha, \pi_\beta^\alpha, \Sigma\}$ (or, shortly, $\lim \boldsymbol{X}$). Mapping $\pi_\beta\colon \lim \boldsymbol{X} \to X_\beta$ defined by $\pi_\beta(\{x_\alpha\}) = x_\beta$ is called a *projection*.

If Σ is the set N of natural numbers, then an inverse system $\{X_n, f_k^n, N\}$ can be written simply in the form

$$X_1 \longleftarrow \ldots \longleftarrow X_n \xleftarrow{f_n} X_{n+1} \longleftarrow \ldots$$

or, quite short, $\{X_n, f_n\}$.

Recall that if $f\colon X \to Y$ is a mapping and $A \subset X$, then $f[A]$ denotes the set $\{f(a)\colon a \in A\}$, and if $B \subset Y$, then $f^{-1}[B]$ is the set $\{f^{-1}(b)\colon b \in B\}$.

Segal [9] has proved that if a metric continuum X is the limit of an inverse system $\{X_n, f_n\}$ of metric continua X_n and continuous mappings f_n, then the hyperspace $C(X)$ with the Vietoris topology is the limit of the inverse system $\{C(X_n), \tilde{f}_n\}$ of hyperspaces $C(X_n)$ with the Vietoris topology and continuous mappings $\tilde{f}_n\colon C(X_{n+1}) \to C(X_n)$ defined by $\tilde{f}_n(A) = f_n[A]$, $A \in C(X_{n+1})$.

Recently, Sirota [10] has shown that if a compact Hausdorff space X is the limit of an inverse system $\{X_a, \pi_\beta^a, \Sigma\}$ of compact Hausdorff spaces X_a and continuous mappings $\pi_\beta^a\colon X_a \to X_\beta$, then the hyperspace $\mathrm{Comp}(X)$ with the Vietoris topology is the limit of the inverse system $\{\mathrm{Comp}(X_a), \tilde{\pi}_\beta^a, \Sigma\}$ of hyperspaces $\mathrm{Comp}(X_a)$ with the Vietoris topology and continuous mappings $\tilde{\pi}_\beta^a\colon \mathrm{Comp}(X_a) \to \mathrm{Comp}(X_\beta)$ defined by $\tilde{\pi}_\beta^a(A) = \pi_\beta^a[A]$, $A \in \mathrm{Comp}(X_a)$.

Both these results are particular cases of the more general Theorem 2 which is the main result of the present note.

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Notions and notation not defined in the note come from [4] and [5].

2. Convexity of inverse limits. In this section we shall find a necessary and sufficient condition for convexity of the limit of an inverse system of metric continua. This result will be then applied in Section 4 to yield a corollary from Theorem 2.

Let $\{X_n, f_n\}$ be an inverse system of metric spaces (X_n, ϱ_n). If all spaces (X_n, ϱ_n) are bounded, then the limit $\lim\{X_n, f_n\}$ can be metrized by the formula

$$\varrho(\{x_n\}, \{y_n\}) = \sum_{n=1}^{\infty} a_n \cdot \varrho_n(x_n, y_n), \tag{1}$$

where $a_n > 0$ for each $n = 1, 2, \ldots$, and the series $\sum_{n=1}^{\infty} a_n \cdot \delta_n(X_n)$ is convergent, δ_n denoting diameter in the space (X_n, ϱ_n) (cf. [4], p. 178).

A metric space (X, ϱ) is said to be *convex* if for any two points a and b of it there exists a point $c \in X$ which lies between a and b (i.e., such that $\varrho(a, c) + \varrho(c, b) = \varrho(a, b)$) and is distinct from both a and b. If (X, ϱ) is complete, then this is equivalent to say: for any two points a and b of X there exists an arc in X between a and b isometric to the real segment $[0, \varrho(a, b)]$ (cf. [7], p. 89).

Let $f\colon Z \to X$ be a continuous mapping from a metric continuum Z onto a metric continuum X. We say that f *preserves convexity* if it satisfies the following two conditions:

(i) if $K \subset Z$ is a segment, then $f[K]$ is a segment or a point,

(ii) if $L \subset X$ is a segment or a point, then $f^{-1}[L]$ is convex.

Obviously, if Z is convex, then condition (i) implies convexity of X, and if X is convex, then condition (ii) implies convexity of Z.

THEOREM 1. *Let $\{X_n, f_n\}$ be an inverse system of metric continua (X_n, ϱ_n) and mappings $f_n\colon X_{n+1} \to X_n$ preserving convexity. Then the limit*

$$X = \lim\{X_n, f_n\} \tag{2}$$

with the metric (1) *is convex if and only if each continuum (X_n, ϱ_n) is convex.*

Moreover, each projection $\pi_k\colon X \to X_k$ preserves convexity.

Proof. I. Assume first that all spaces (X_n, ϱ_n) are convex (in fact, since all mappings f_n satisfy (ii) by hypothesis, it suffices to assume in this case convexity of (X_1, ϱ_1) only).

Take two arbitrary threads, $a = (a_1, a_2, \ldots)$ and $b = (b_1, b_2, \ldots)$, of the limit (2). In order to prove its convexity it suffices to find a thread $x = (x_1, x_2, \ldots)$ which lies between a and b with respect to the metric (1) and is distinct from both a and b.

Let k be a natural number such that $a_k \neq b_k$. Since the space (X_k, ϱ_k) is convex by hypothesis, there exists a segment $a_k b_k$ joining a_k to b_k in X_k. Let x_k be a point of this segment distinct from both a_k and b_k.

If $k > 1$, set

$$x_{k-1} = f_{k-1}(x_k),\; x_{k-2} = f_{k-2}(x_{k-1}), \ldots, x_1 = f_1(x_2).$$

Thus, for $i = 2, 3, \ldots, k$, there is

$$x_i \in f_{i-1}^{-1}(x_{i-1}) \tag{3}$$

and, by virtue of (i), for $i = 1, 2, \ldots, k$, we have

$$\varrho(a_i, x_i) + \varrho(x_i, b_i) = \varrho(a_i, b_i). \tag{4}$$

Now proceed by induction. Assume that for some $n \geqslant k$ we have already defined a sequence $x_1, x_2, \ldots, x_n$ such that (3) holds if $2 \leqslant i \leqslant n$ and (4) holds if $1 \leqslant i \leqslant n$. In view of (ii), the set $f_n^{-1}[a_n b_n]$ is convex. Since x_n cuts $a_n b_n$ between a_n and b_n, and f_n is continuous by hypothesis, $f_n^{-1}(x_n)$ cuts $f_n^{-1}[a_n b_n]$ between any point of $f_n^{-1}(a_n)$ and any point of $f_n^{-1}(b_n)$. In particular, $f_n^{-1}(x_n)$ cuts each segment in $f_n^{-1}[a_n b_n]$ between a_{n+1} and b_{n+1}, and by the convexity of $f_n^{-1}[a_n b_n]$ there is a segment in $f_n^{-1}[a_n b_n]$ between a_{n+1} and b_{n+1}. Consequently, there exists a point $x_{n+1} \in f_n^{-1}(x_n)$ which lies between a_{n+1} and b_{n+1}, i.e. (3) and (4) are satisfied also for $n+1$. Induction is completed.

Hence there exists a thread $x = (x_1, x_2, \ldots)$ such that (4) holds for each $n = 1, 2, \ldots$ Since $a_k \neq x_k \neq b_k$, $a \neq x \neq b$. Multiplying now (4) by a_n and taking a series we receive

$$\sum_{n=1}^{\infty} a_n \cdot \varrho_n(a_n, x_n) + \sum_{n=1}^{\infty} a_n \cdot \varrho_n(x_n, b_n) = \sum_{n=1}^{\infty} a_n \cdot \varrho_n(a_n, b_n),$$

i.e.,

$$\varrho(a, x) + \varrho(x, b) = \varrho(a, b).$$

II. Suppose now that the limit (2) with the metric (1) is convex. To prove convexity of X_k, where $k = 1, 2, \ldots$, take two distinct points $a_k, b_k \in X_k$. Since all mappings f_n are by hypothesis onto and all X_n are continua, hence compact, the limit X contains threads $\{a_n\}$, $\{b_n\}$ through a_k, b_k resp. (cf. [4], theorem 3.2.11). By the convexity of X, there exists a segment S in X between $\{a_n\}$ and $\{b_n\}$. Projection $\pi_k\colon X \to X_k$, being continuous, maps S onto a continuum $\pi_k[S]$ which contains a_k and b_k. We proceed to show that each point of $\pi_k[S]$ lies between a_k and b_k. In fact, if $\{x_n\} \in S$, then

$$\sum_{n=1}^{\infty} a_n \cdot \varrho_n(a_n, x_n) + \sum_{n=1}^{\infty} a_n \cdot \varrho_n(x_n, b_n) = \sum_{n=1}^{\infty} a_n \cdot \varrho_n(a_n, b_n),$$

whence, consequently,

$$\sum_{n=1}^{\infty} a_n \{[\varrho_n(a_n, x_n) + \varrho_n(x_n, b_n)] - \varrho_n(a_n, b_n)\} = 0. \tag{5}$$

And since on the left-hand side of (5) there is a number series such that, for each $n = 1, 2, \ldots$, there is $a_n > 0$ and, by the triangle inequality,

$$\varrho_n(a_n, x_n) + \varrho_n(x_n, b_n) - \varrho_n(a_n, b_n) \geqslant 0,$$

the equality (5) can hold if and only if

$$\varrho_n(a_n, x_n) + \varrho_n(x_n, b_n) = \varrho_n(a_n, b_n) \quad \text{for each } n = 1, 2, \ldots$$

Hence, in fact, each point of $\pi_k[S]$ lies between a_k and b_k. Taking anyone besides a_k and b_k, we have a point which lies between a_k and b_k and is distinct from both. Hence X_k is convex for $k = 1, 2, \ldots$

III. It remains to prove that each projection $\pi_k\colon X \to X_k$ preserves convexity.

Let K be a segment in X. To prove that $\pi_k[K]$ is a segment or a point it suffices to show that for any three point of K such that

$$\varrho(\{a_n\}, \{x_n\}) + \varrho(\{x_n\}, \{b_n\}) = \varrho(\{a_n\}, \{b_n\})$$

there is

$$\varrho_k(a_k, x_k) + \varrho_k(x_k, b_k) = \varrho_k(a_k, b_k).$$

And the proof of this implication goes exactly as in part II above. Hence condition (i) holds.

To check (ii) take $L \subset X_k$ to be a segment or a point. Put

$$A_n = \begin{cases} f_n \cdot f_{n+1} \cdot \ldots \cdot f_{k-1}[L] & \text{if } n < k, \\ L & \text{if } n = k, \\ f_{n-1}^{-1} \cdot f_{n-2}^{-1} \cdot \ldots \cdot f_k^{-1}[L] & \text{if } n > k. \end{cases}$$

Then $\{A_n, f_n|A_{n+1}\}$ is an inverse system of convex continua. Moreover, all mappings $f_n|A_{n+1}\colon A_{n+1} \to A_n$ preserve convexity, because all mappings f_n do. Hence, by virtue of part I, $\lim\{A_n, f_n|A_{n+1}\}$ is convex.

Clearly,

$$\lim\{A_n, f_n|A_{n+1}\} = \pi_k^{-1}[L].$$

Corollary 1. *Let $\{X_n, f_n\}$ be an inverse system of convex continua and mappings $f_n\colon X_{n+1} \to X_n$ preserving convexity. If $\{A_n, f_n|A_{n+1}\}$ is an inverse system of convex subcontinua $A_n \subset X_n$ and mappings $f_n|A_{n+1}\colon A_{n+1} \to A_n$ are onto, then the limit $A = \lim\{A_n, f_n|A_{n+1}\}$ is a convex subcontinuum of the limit $X = \lim\{X_n, f_n\}$.*

Proof. In view of Theorem 1 it suffices to check that each mapping $f_n|A_{n+1}$ preserves convexity. Let $S \subset A_{n+1}$ be a segment. Since f_n preserves convexity by hypothesis, then $f_n[S] = (f_n|A_{n+1})[S]$ is a segment or a point, and so (i) holds. And since, by Theorem 1, projections π_n and π_{n+1} preserve convexity, the set

$$(f_n|A_{n+1})^{-1}[S] = A_{n+1} \cap \pi_{n+1}\pi_n^{-1}[S]$$

is convex, and so also (ii) holds.

3. Main result. Consider the category $\mathfrak{A}$ consisting of all compact Hausdorff spaces and of all continuous mappings between them, and let $\mathfrak{B}$ be a subcategory of $\mathfrak{A}$. A covariant functor $\mathfrak{H}\colon \mathfrak{B} \to \mathfrak{A}$ will be called a *hyperfunctor* of $\mathfrak{B}$ if, for each $(f\colon X \to Y) \in \mathfrak{B}$, $\mathfrak{H}(X)$ is a hyperspace of X, $\mathfrak{H}(Y)$ is a hyperspace of Y, and the induced mapping $\tilde{f} = \mathfrak{H}(f)\colon \mathfrak{H}(X) \to \mathfrak{H}(Y)$ is defined by $\tilde{f}(A) = f[A]$ for each $A \in \mathfrak{H}(X)$.

Of a special value will be hyperfunctors $\mathfrak{H}$ of $\mathfrak{B}$ which satisfy the following condition:

(ω) if $\boldsymbol{X} = \{X_\alpha, \pi_\beta^\alpha, \Sigma\}$ is an inverse system in $\mathfrak{B}$ and if its limit $\lim \boldsymbol{X}$ belongs to $\mathfrak{B}$ together with all projections $\pi_\alpha\colon \lim \boldsymbol{X} \to X_\alpha$, then for each inverse system $\boldsymbol{A} = \{A_\alpha, \pi_\beta^\alpha|A_\alpha, \Sigma\}$ such that $A_\alpha \in \mathfrak{H}(X_\alpha)$ and $\pi_\beta^\alpha|A_\alpha$ is onto A_β there is $\lim \boldsymbol{A} \in \mathfrak{H}(\lim \boldsymbol{X})$.

Theorem 2. *Let*

$$\boldsymbol{X} = \{X_\alpha, \pi_\beta^\alpha, \Sigma\} \tag{6}$$

be an inverse system in $\mathfrak{B}$ and let $\mathfrak{H}$ be a hyperfunctor of $\mathfrak{B}$.

Then

$$\mathfrak{H}(\boldsymbol{X}) = \{\mathfrak{H}(X_a), \tilde{\pi}^a_\beta, \Sigma\} \tag{7}$$

is an inverse system in $\mathfrak{A}$, *and if the limit* $\lim \boldsymbol{X}$ *belongs to* $\mathfrak{B}$ *together with all projections* $\pi_a\colon \lim \boldsymbol{X} \to X_a$, *then there exists a homeomorphism*

$$h\colon \mathfrak{H}(\lim \boldsymbol{X}) \to \lim \mathfrak{H}(\boldsymbol{X}). \tag{8}$$

If, moreover, system (6) *satisfies* (ω), *then the homeomorphism* (8) *is onto, i.e.*

$$\mathfrak{H}(\lim \boldsymbol{X}) \underset{\text{top}}{=} \lim \mathfrak{H}(\boldsymbol{X}).$$

Proof. Consider the diagram

$$\begin{tikzcd}
 & \mathfrak{H}(\lim \mathbf{X}) \arrow[dl, "\tilde{\pi}_\beta"'] \arrow[d, "\tilde{\pi}_\alpha"] \\
\mathfrak{H}(X_\beta) & \mathfrak{H}(X_\alpha) \arrow[l]
\end{tikzcd}$$

obtained by imposing the functor $\mathfrak{H}$ upon the system (6) augmented with its limit. Since $\mathfrak{H}$ is a functor, the diagram commutes and so (7) is an inverse system.

Since $\mathfrak{H}$ is a hyperfunctor, (7) is an inverse system in the category $\mathfrak{A}$. Hence its limit $\lim \mathfrak{H}(\boldsymbol{X})$ does exist (cf. [4], Theorem 3.2.10, p. 115) and so there exists a unique continuous mapping (8) such that the diagram

$$\begin{tikzcd}
 & & \mathfrak{H}(\lim \boldsymbol{X}) \arrow[dll, "\tilde{\pi}_\beta"'] \arrow[dl, "\tilde{\pi}_\alpha"] \arrow[dd, dashed, "h"] \\
\mathfrak{H}(X_\beta) & \mathfrak{H}(X_\alpha) \arrow[l] & \\
 & & \lim \mathfrak{H}(\mathbf{X}) \arrow[ul, "p_\alpha"'] \arrow[ull, "p_\beta"]
\end{tikzcd}$$

commutes, p_a and p_β being projections.

To see what h is like, take $A \in \mathfrak{H}(\lim \boldsymbol{X})$ and put $A_a = \tilde{\pi}_a(A)$ for each $a \in \Sigma$. Since $\tilde{\pi}^a_\beta(A_a) = \tilde{\pi}_\beta(A) = A_\beta$, the set $\{A_a\}_{a\in\Sigma}$ is a thread of the system (7), and since $p_a h(A) = \tilde{\pi}_a(A) = A_a$, there is $h(A) = \{A_a\}_{a\in\Sigma}$.

Now we show that h is one-to-one. Assume, a contrario, that there are two sets $B, C \in \mathfrak{H}(\lim \boldsymbol{X})$ such that $B - C \neq 0$ and

$$B_a = C_a \quad \text{for each } a \in \Sigma. \tag{9}$$

Since $B - C \neq 0$, there is a thread $\{b_a\}_{a\in\Sigma}$ which is in B and not in C. In view of the equality $\tilde{\pi}_a(B) = \pi_a[B]$ we have $b_a \in B_a$ for each $a \in \Sigma$, whence and from the assumption (9) it follows that $b_a \in C_a$ for each $a \in \Sigma$. And by virtue of the last relation there exists, for each $\gamma \in \Sigma$, a thread

$t_\gamma = \{c_a^\gamma\}_{a\in\Sigma} \in C$ such that $c_a^\gamma = b_a$ for all $a \leqslant \gamma$. The set of threads t_γ is a Moore-Smith sequence convergent to $\{b_a\}_{a\in\Sigma}$. In view of the compactness of C, there must be $\{b_a\}_{a\in\Sigma} \in C$. A contradiction.

Hence h, as defined on a compact space $\mathfrak{H}(\lim \boldsymbol{X})$ and being one-to-one and continuous, must be a homeomorphism.

It remains to show that if the system (6) satisfies (ω), h is onto. For that purpose take $A \in \lim \mathfrak{H}(\boldsymbol{X})$. Then $\{p_a(A)\}_{a\in\Sigma}$ is a thread of the inverse system (7) and, consequently, $\boldsymbol{A} = \{p_a(A), \pi_\beta^a | p_a(A), \Sigma\}$ is an inverse system consisting of spaces $p_a(A) \in \mathfrak{H}(X_a)$ and mappings $\pi_\beta^a | p_a(A)$ being onto, and so, by virtue of (ω), its limit $\lim\boldsymbol{A}$ belongs to $\mathfrak{H}(\lim\boldsymbol{X})$. Therefore $p_a(A) = \tilde{\pi}_a(\lim\boldsymbol{A}) = p_a h(\lim\boldsymbol{A})$ for each $a \in \Sigma$ and, consequently, $h(\lim\boldsymbol{A}) = A$.

Thus the proof of Theorem 2 is completed.

4. Applications. In this section we give some examples of hyperfunctors satisfying (ω) and infer relevant corollaries. All hyperspaces are assumed to have Vietoris topology.

1. If $\mathfrak{B} = \mathfrak{A}$, then the functor Comp: $\mathfrak{B} \to \mathfrak{A}$ such that $\mathrm{Comp}(f) = \tilde{f}$ for each morphism f in $\mathfrak{B}$ is a hyperfunctor of $\mathfrak{B}$ satisfying (ω).

In fact, if Y is Hausdorff compact, so is Comp(Y) (cf. [8], Proposition 4.9.2). Now, if f: $Y \to Z$ is a morphism in $\mathfrak{B}$, hence a continuous function from Y into Z, f maps Comp(Y) into Comp Z (cf. [4], Theorem 3.1.8, p. 104) and is continuous (see [8], Proposition 5.10.1). Thus the functor is well defined. And since the limit of an inverse system of compact Hausdorff spaces is a compact Hausdorff space itself (see [4], Theorem 3.2.10, p. 115), the functor satisfies (ω).

COROLLARY 2. *If $\{X_a, \pi_\beta^a, \Sigma\}$ is an inverse system of compact Hausdorff spaces, then*

$$\mathrm{Comp}(\lim\{X_a, \pi_\beta^a, \Sigma\}) = \lim\{\mathrm{Comp}(X_a), \tilde{\pi}_\beta^a, \Sigma\}.$$

As we have said in Section 1, this corollary has been proved by Sirota [10].

2. If $\mathfrak{B}$ is the category of all Hausdorff continua and of all continuous mappings between them, then the functor C: $\mathfrak{B} \to \mathfrak{A}$ such that $C(f) = \tilde{f}$ for each morphism f in $\mathfrak{B}$ is a hyperfunctor satisfying (ω).

In fact, a Hausdorff continuum Y is by the definition a compact (and connected) Hausdorff space, and so is $C(Y)$. Now, if f: $Y \to Z$ is a morphism in $\mathfrak{B}$, hence a continuous function from a Hausdoff continuum Y into a Hausdorff continuum Z, f maps $C(Y)$ into $C(Z)$ (see [4], p. 241) and is continuous (cf. [8], Proposition 5.10.1). Thus the functor is a well defined hyperfunctor. And since the limit of an inverse system of Hausdorff continua is a Hausdorff continuum itself (see [4], Theorem 3.1.5, p. 244), the functor satisfies (ω).

COROLLARY 3. *If* $\{X_a, \pi^a_\beta, \Sigma\}$ *is an inverse system of Hausdorff continua, then*

$$C(\lim\{X_a, \pi^a_\beta, \Sigma\}) = \lim\{C(X_a), \tilde{\pi}^a_\beta, \Sigma\}.$$

This corollary has been proved by Segal [9] in the case of metric continua.

As follows from [1] and [6], each dendroid (i.e., a hereditarily decomposable and hereditarily unicoherent continuum) is a limit of an inverse system of finite dendrites, $X = \lim\{D_n, f_n\}$. Hence $C(X) = \lim\{C(D_n), \tilde{f}_n\}$ and the structure of hyperspaces $C(D_n)$ is known (see [2]).

3. If $\mathfrak{B}$ is the category of all convex continua (or of all convex subcontinua of the Hilbert cube $I^{\aleph_0}$) and of all continuous mappings between them preserving convexity, then Conv is a hyperfunctor of $\mathfrak{B}$.

Indeed, if Y is compact convex, so is Conv (Y) (see [3]). Consequently, if $f\colon Y \to Z$ is continuous, so is $f\colon \operatorname{Conv} Y \to \operatorname{Conv} Z$ (cf. [8], Proposition 5.10.1). Since, moreover, an image of a convex continuum under a mapping preserving convexity is again convex, the functor is well defined. And Corollary 1 implies (ω) in the countable case.

COROLLARY 4. *If* $\{X_n, f_n\}$ *is an inverse system of convex continua* X_n *and mappings* f_n *preserving convexity, then*

$$\operatorname{Conv}(\lim\{X_n, f_n\}) = \lim\{\operatorname{Conv}(X_n), \tilde{f}_n\}.$$

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