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ON CARTESIAN POWERS OF 2-POLYHEDRA

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1. Introduction. In paper [3] we have proved that if K and L are compact connected 2-polyhedra and their Cartesian squares $K^2 = K \times K$ and $L^2 = L \times L$ are homeomorphic, then K and L are homeomorphic. In the present paper we consider the question which is a natural generalization of the question considered there. Is it true that K and L are homeomorphic if their Cartesian powers K^k and L^k are homeomorphic? This question was posed by J. Mycielski in his letters to R. Engelking and me.

We prove that the answer is "yes" for some classes of 2-polyhedra, but in the remaining cases the problem is still open.

2. Results. The problem is the easiest when K is a compact 2-manifold with boundary. This fact was proved by Fox [2] in 1947 for the case $k = 2$.

Let us start with a definition.

DEFINITION 2.1. Let M be a compact 2-manifold with boundary $\partial M \neq \emptyset$. We define the number

$$\sigma(M) = \operatorname{rank} H_1(M) - \operatorname{rank} H_1(\partial M) + 1.$$

THEOREM 2.1. *If M, N are compact connected 2-manifolds with boundary and $M^k \approx N^k$ for some natural number k, then $M \approx N$.*

Proof. If the surfaces M and N are both orientable or both nonorientable, $\operatorname{rank} H_1(M) = \operatorname{rank} H_1(N)$ and $\sigma(M) = \sigma(N)$, then $M \approx N$.

The first condition is obvious. By Künneth's formula, if

$$H_1(M) \approx Z^m, \quad H_1(N) \approx Z^n,$$

then

$$H_k(M^k) \approx Z^{m^k} \quad \text{and} \quad H_k(N^k) \approx Z^{n^k}.$$

Since $M^k \approx N^k$, we have $m^k = n^k$ and $m = n$.

Now, we consider the map

$$i_*\colon H_k(M^k) \to H_k(M^k, \partial M^k),$$

induced by the inclusion of the pair $(M, \emptyset)$. The image of this map is generated by generators $\zeta_1 \otimes \ldots \otimes \zeta_k$ such that $j_*(\zeta_i) \neq 0$ for every $i = 1, \ldots, k$, where

$$j_*\colon H_1(M) \to H_1(M, \partial M)$$

is given by inclusion. There are exactly $\sigma(M)$ such generators. So there are $(\sigma(M))^k$ generators $\zeta_1 \otimes \ldots \otimes \zeta_k$ such that

$$i_*(\zeta_1 \otimes \ldots \otimes \zeta_k) \neq 0.$$

Since $M^k \approx N^k$, we have $(\sigma(M))^k = (\sigma(N))^k$ and $\sigma(M) = \sigma(N)$.

Further on in this paper we use the method similar to that used in [3]. We obtain the affirmative answer for all 2-polyhedra except for the class considered in Section 6 of [3].

First we prove a lemma.

LEMMA 2.1. *Suppose that D_1K and D_1L are nowhere dense subpolyhedra of compact connected 2-polyhedra K and L, respectively. We define $D_0K = K$, $D_0L = L$ and*

$$X_i = \bigcup \{D_{i_1}K \times \ldots \times D_{i_k}K\colon\ i_j = 0, 1;\ i_1 + \ldots + i_k = i\},$$

$$X_i' = \bigcup \{D_{i_1}L \times \ldots \times D_{i_k}L\colon\ i_j = 0, 1;\ i_1 + \ldots + i_k = i\}.$$

If $F\colon K^k \to L^k$ is a homeomorphism such that $F(X_i) = X_i'$ for $i = 0, 1, \ldots, k$, then

$$F(K \times D_1K \times \ldots \times D_1K) = D_1L \times \ldots \times D_1L \times L \times D_1L \times \ldots \times D_1L.$$

Proof. Let us observe that

$$F(X_{k-1} \backslash X_k) = X'_{k-1} \backslash X'_k.$$

A point belongs to $X_{k-1} \backslash X_k$ iff one of its coordinates belongs to $K \backslash D_1K$. So, we can consider a component $A \times Z_2 \times \ldots \times Z_k$ of this set, where

$$A \in \square(K \backslash D_1K), \qquad Z_i \in \square D_1K$$

($\square$ denotes the set of components). Let

$F(A \times Z_2 \times \ldots \times Z_k) = A' \times Z_2' \times \ldots \times Z_k'$, where $A' \in \square(L \backslash D_1L)$, $Z_i' \in \square D_1L$.

Let B be another component of $K \backslash D_1K$ such that $\bar{A} \cap \bar{B} \neq \emptyset$. We will prove that

$$F(B \times Z_2 \times \ldots \times Z_k) = B' \times Z_2' \times \ldots \times Z_k'.$$

Suppose that

$$F(B \times Z_2 \times \ldots \times Z_k) = Z_1'' \times \ldots \times B'' \times \ldots \times Z_k'',$$

where $B'' \in \square(L \backslash D_1L)$, $Z_i'' \in \square D_1L$. Since

$$\overline{A' \times Z_2' \times \ldots \times Z_k'} \cap \overline{Z_1'' \times \ldots \times B'' \times \ldots \times Z_k''} \neq \emptyset,$$

there exist points $x' \in \bar{A}' \cap Z_1''$ and $y' \in Z_i' \cap \bar{B}''$, and $Z_j' = Z_j''$ for the remaining coordinates. Let $a' \in A'$, $b' \in B''$. There exist arcs

$$a'x' \subset A' \cup \{x'\} \quad \text{and} \quad b'y' \subset B'' \cup \{y'\}.$$

Let us notice that

$$V' = (a'x') \times Z_2' \times \ldots \times (b'y') \times \ldots \times Z_k' \subset A' \times Z_2' \times \ldots \times B'' \times \ldots \times Z_k'$$

((xy) denotes the interior of the arc xy). Hence

$$V' \cap X_{k-1}' = \emptyset \quad \text{and} \quad V' \subset X_{k-2}'.$$

Let

$$\bar{a} = F^{-1}(a', z_2', \ldots, y', \ldots, z_k') \quad \text{and} \quad \bar{b} = F^{-1}(x', z_2', \ldots, b', \ldots, z_k'),$$

where $z_i' \in Z_i'$. Hence

$$\bar{a} \in A \times Z_2 \times \ldots \times Z_k, \quad \bar{b} \in B \times Z_2 \times \ldots \times Z_k$$

and there exists an arc $\bar{a}\bar{b}$ such that

$$(\bar{a}\bar{b}) \subset F^{-1}(V') \subset F^{-1}(X_{k-2}' \backslash X_{k-1}') = X_{k-2} \backslash X_{k-1}.$$

But the interior of any arc $(\bar{a}\bar{b}) \subset X_{k-2}$ and the set X_{k-1} are not disjoint, because every component of the set $X_{k-2} \backslash X_{k-1}$ is of the form

$$Z_1 \times \ldots \times C \times \ldots \times D \times \ldots \times Z_k, \quad \text{where } C, D \in \square(K \backslash D_1 K).$$

Hence

$$F(B \times Z_2 \times \ldots \times Z_k) = B' \times Z_2' \times \ldots \times Z_k'.$$

Since K is connected and $D_1 K$ is nowhere dense, we have

$$F(K \times Z_2 \times \ldots \times Z_k) = L \times Z_2' \times \ldots \times Z_k'$$

and

$$F(K \times D_1 K \times \ldots \times D_1 K) = L \times D_1 L \times \ldots \times D_1 L.$$

We use this lemma in the proof of the following

THEOREM 2.2. *Suppose K and L are compact connected 2-polyhedra, $F\colon K^k \to L^k$ is a homeomorphism and K has local cut points. Then K and L are homeomorphic.*

Proof. First we consider the case where there exists a point x of K such that $\dim_x K = 1$ (where $\dim_x K$ denotes the local dimension of the space K at the point x). We define

$$K_0 = \{x \in K\colon \dim_x K = 2\}, \quad K_1 = \overline{\{x \in K\colon \dim_x K = 1\}}$$

and analogously L_0 and L_1. Next we define

$$Y_i = \bigcup \{K_{i_1} \times \ldots \times K_{i_k}\colon i_1 + \ldots + i_k = i\},$$
$$Y_i' = \bigcup \{L_{i_1} \times \ldots \times L_{i_k}\colon i_1 + \ldots + i_k = i\}.$$

Let us observe that

$$Y_i = \overline{\{x \in K^k\colon \dim_x K^k = 2k-i\}} \quad \text{and} \quad Y'_i = \overline{\{x \in L^k\colon \dim_x L^k = 2k-i\}},$$

so $F(Y_i) = Y'_i$.

Now we write

$$D_1 K = K_0 \cap K_1, \quad D_0 K = K$$

and

$$X_i = \bigcup \{D_{i_1} K \times \ldots \times D_{i_k} K\colon i_1 + \ldots + i_k = i\}.$$

The set $D_1 K$ is finite. Therefore, if we prove that

$$F(X_i) = X'_i = \bigcup \{D_{i_1} L \times \ldots \times D_{i_k} L\colon i_1 + \ldots + i_k = i\},$$

we obtain $K \approx L$ by Lemma 2.1.

Since $F(Y_i) = Y'_i$, it is enough to notice that

$$X_i = \bigcup_{j=0}^{k-i} \bigcap_{l=0}^{i} Y_{j+l} \quad \text{and} \quad X'_i = \bigcup_{j=0}^{k-i} \bigcap_{l=0}^{i} Y'_{j+l}.$$

We will prove the first equality.

Let $\bar{x} \in X_i$. We can assume that

$$\begin{aligned}
\bar{x} &\in \underbrace{D_1 K \times \ldots \times D_1 K}_{i \text{ times}} \times \underbrace{D_0 K \times \ldots \times D_0 K}_{k-i \text{ times}} \\
&= (K_0 \cap K_1) \times \ldots \times (K_0 \cap K_1) \times K \times \ldots \times K \\
&= \bigcap_{j=0}^{i} \{K_{i_1} \times \ldots \times K_{i_i} \times K \times \ldots \times K\colon i_p = 0, 1;\ i_1 + \ldots + i_i = j\} \\
&= \bigcap_{j=0}^{i} \{K_{i_1} \times \ldots \times K_{i_i} \times (K_0 \cup K_1) \times \ldots \times (K_0 \cup K_1)\colon \\
&\qquad\qquad i_p = 0, 1;\ i_1 + \ldots + i_i = j\} \\
&= \bigcup_{l=0}^{k-i} \bigcap_{j=0}^{i} \{K_{i_1} \times \ldots \times K_{i_i} \times K_{i_{i+1}} \times \ldots \times K_{i_k}\colon i_p = 0, 1;\ i_1 + \ldots + i_i = j, \\
&\qquad\qquad i_{i+1} + \ldots + i_k = l\} \\
&\subset \bigcup_{l=0}^{k-i} \bigcap_{j=0}^{i} Y_{l+j}.
\end{aligned}$$

Let $\bar{x} \in Y_l \cap \ldots \cap Y_{l+i}$. If $\bar{x} \in Y_l$, then l coordinates of $\bar{x}$ belong to K_1. If $\bar{x} \in Y_{l+i}$, then $l+i$ coordinates of $\bar{x}$ belong to K_1. Hence at least i coordinates of $\bar{x}$ belong to $D_1 K = K_0 \cap K_1$. Thus $\bar{x} \in X_i$. This concludes the first part of the proof.

Now, we consider the case $\dim_x K = 2$ for every $x \in K$. The set of local cut points $D_1 K$ is finite. Let us observe that the assumptions of Lemma 2.1 hold.

The set

$$X_1 = \bigcup \{D_{i_1}K \times \ldots \times D_{i_k}K\colon\ i_p = 0, 1;\ i_1 + \ldots + i_k = 1\}$$

consists of those points of the space K^k at which the space is locally cut by a set of dimension $2k-2$. Similarly, the set

$$X_i = \bigcup \{D_{i_1}K \times \ldots \times D_{i_k}K\colon\ i_p = 0, 1;\ i_1 + \ldots + i_k = i\}$$

consists of those points of the set X_{i-1} at which X_{i-1} is locally cut by a set of dimension $2k-2i$.

Analogous formulas hold for $X_i' \subset L^k$.

Hence $F(X_i) = X_i'$ and by Lemma 2.1 we obtain

$$F(K \times D_1 K \times \ldots \times D_1 K) = L \times D_1 L \times \ldots \times D_1 L.$$

So $K \approx L$.

Further on we consider the polyhedra without local cut points only. We define some subsets of non-Euclidean points of a polyhedron X in the same way as in [3].

DEFINITION 2.2. If X is a k-polyhedron, then we define inductively the sets $n_i X$ for $i = 0, 1, \ldots, k$.

(i) $n_0 X = X$.

(ii) $n_i X$ denotes the subset of $n_{i-1} X$ consisting of points which have no neighborhood homeomorphic to R^{k-i+1} or R_+^{k-i+1} in the set $n_{i-1}X$.

Remark. It is easy to see that every set $n_i X$ is a polyhedron and $\dim n_i X \leqslant k-i$.

LEMMA 2.2. *If K is a 2-polyhedron without local cut points, then*

$$n_i(K^k) = \bigcup \{n_{i_1}K \times \ldots \times n_{i_k}K\colon\ i_p = 0, 1, 2;\ i_1 + \ldots + i_k = i\}.$$

Proof. Let us observe that if $x \in n_1 K$, then each neighborhood of x in K contains a subset homeomorphic to $T \times I$ (where $T \approx$ cone $\{1, 2, 3\}$ and I is an arc). If $x \in n_2 K$, then each neighborhood of x in $n_1 K$ contains a triod T.

We prove inductively:

1° If

$$x \in \bigcup \{n_{i_1}K \times \ldots \times n_{i_k}K\colon\ i_j = 0, 1, 2;\ i_1 + \ldots + i_k = 1\},$$

then x belongs to one of the components of the union (say $x \in n_1 K \times K \times \ldots \times K$). Then each neighborhood of x in K^k contains a set homeomorphic to $(T \times I) \times I^2 \times \ldots \times I^2$, which is not embeddable in R^{2k}. So $x \in n_1(K^k)$. The inverse inclusion is obvious.

2° Suppose that our formula is true for $i \leqslant m$. Let

$$x \in \bigcup \{n_{i_1}K \times \ldots \times n_{i_k}K\colon\ i_j = 0, 1, 2;\ i_1 + \ldots + i_k = m+1\}.$$

We can assume that

$$x \in \underbrace{n_2 K \times \ldots \times n_2 K}_{p\,\text{times}} \times \underbrace{n_1 K \times \ldots \times n_1 K}_{r\,\text{times}} \times K \times \ldots \times K \qquad (2p + r = m+1).$$

If $r \neq 0$, then each neighborhood of x in $n_m(K^k)$ contains a subset homeomorphic to

$$\{z_1\} \times \ldots \times \{z_p\} \times \underbrace{I \times \ldots \times I}_{r-1 \text{ times}} \times (T \times I) \times I^2 \times \ldots \times I^2,$$

which is not embeddable in R^{2k-m}. If $r = 0$, then

$$x \in \underbrace{n_2K \times \ldots \times n_2K}_{p-1 \text{ times}} \times n_1K \times K \times \ldots \times K \subset n_m(K^k)$$

(because $n_2K \subset n_1K$) and each neighborhood of x in $n_m(K^k)$ contains a subset homeomorphic to

$$\{z_1\} \times \ldots \times \{z_{p-1}\} \times T \times I^2 \times \ldots \times I^2,$$

which is not embeddable in R^{2k-m}. Hence $x \in n_{m+1}(K^k)$.

The inverse inclusion is obvious.

Now we prove a theorem.

THEOREM 2.3. *If K, L are compact connected 2-polyhedra, $n_2K \neq \emptyset$ and $F\colon K^k \to L^k$ is a homeomorphism, then $K \approx L$.*

Proof. The theorem is not simply a consequence of Lemmas 2.1 and 2.2 because the condition proved in Lemma 2.2 is different from the assumption of Lemma 2.1.

Let us observe that $n_{2k-2}(K^k) \backslash n_{2k-1}(K^k)$ consists of sets of the form

$$(K \backslash n_1K) \times n_2K \times \ldots \times n_2K \quad \text{or} \quad (n_1K \backslash n_2K) \times (n_1K \backslash n_2K) \times n_2K \times \ldots \times n_2K$$

and sets obtained by permutation of the factors of these sets.

Let us consider the component $A \times \{x_2\} \times \ldots \times \{x_k\}$ of the set

$$n_{2k-2}(K^k) \backslash n_{2k-1}(K^k),$$

where $A \in \square(K \backslash n_1K)$ and $x_i \in n_2K$, $i = 2, \ldots, k$. The two cases are possible:

$$F(A \times \{x_2\} \times \ldots \times \{x_k\}) = A' \times \{x'_2\} \times \ldots \times \{x'_k\},$$

where $A' \in \square(L \backslash n_1L)$ and $x'_i \in n_2L$, $i = 2, \ldots, k$, or

$$F(A \times \{x_2\} \times \ldots \times \{x_k\}) = U' \times V' \times \{x'_3\} \times \ldots \times \{x'_k\},$$

where $U', V' \in \square(n_1L \backslash n_2L)$ and $x'_i \in n_2L$.

The final part of the proof is analogous to the proof of Lemma 3.4 in [3].

LEMMA 2.3. *Let K be a compact connected 2-polyhedron. If $n_2K = \emptyset$ and $F\colon K^k \to L^k$ is a homeomorphism, then*

$$F(K \times n_1K \times \ldots \times n_1K) = n_1L \times \ldots \times n_1L \times L \times n_1L \times \ldots \times n_1L.$$

Proof. If $n_2K = \emptyset$, then

$$n_i(K^k) = \bigcup \{n_{i_1}K \times \ldots \times n_{i_k}K\colon\ i_j = 0, 1;\ i_1 + \ldots + i_k = i\}$$

by Lemma 2.2. Hence the assumptions of Lemma 2.1 hold and we obtain the assertion.

THEOREM 2.4. *If K is a compact connected 2-polyhedron and there exists a point $x \in n_1 K$ such that its regular neighborhood is not homeomorphic to the set* $\mathrm{cone}\{1, \ldots, m\} \times I$, *and* $K^k \approx L^k$, *then* $K \approx L$.

Proof. We can assume that K does not have local cut points and $n_2 K = \emptyset$. Hence, if $F\colon K^k \to L^k$ is a homeomorphism, then

$$F(K \times n_1 K \times \ldots \times n_1 K) = L \times n_1 L \times \ldots \times n_1 L$$

by Lemma 2.3. Let us observe that

$$F((K \backslash n_1 K) \times n_1 K \times \ldots \times n_1 K) = (L \backslash n_1 L) \times n_1 L \times \ldots \times n_1 L.$$

The set of points of $n_1 K$ such that their regular neighborhoods are not homeomorphic to the set $\mathrm{cone}\{1, \ldots, m\} \times I$ for a natural number m is denoted by DK. This set is finite. The corresponding subset of $n_1 L$ is denoted by DL.

If $x \in (K \backslash n_1 K) \times (n_1 K \backslash DK) \times \ldots \times (n_1 K \backslash DK)$, then its regular neighborhood in K^k is homeomorphic to

$$I^2 \times \mathrm{cone}\{1, \ldots, m_2\} \times I \times \ldots \times \mathrm{cone}\{1, \ldots, m_k\} \times I.$$

In 1938 Borsuk proved [1] that the decomposition into a Cartesian product of 1-polyhedra is unique. If there exists a coordinate x_i of the point $x = (x_1, x_2, \ldots, x_k)$ such that $x_i \in DK$, then its regular neighborhood in K is not a product of 1-polyhedra, so the regular neighborhood of x in K^k is not homeomorphic to the above set. So

$$F((K \backslash n_1 K) \times (n_1 K \backslash DK) \times \ldots \times (n_1 K \backslash DK))$$
$$= (L \backslash n_1 L) \times (n_1 L \backslash DL) \times \ldots \times (n_1 L \backslash DL).$$

Let $A \in \square(K \backslash n_1 K)$, $I, J \in \square(n_1 K \backslash DK)$ and $X = \bar{I} \cap \bar{J} \neq \emptyset$ (X is finite). Then

$$F(A \times I \times \ldots \times I) = A' \times I'_2 \times \ldots \times I'_k,$$

$$F(A \times J \times \ldots \times J) = A'' \times J'_2 \times \ldots \times J'_k,$$

where $A', A'' \in \square(L \backslash n_1 L)$ and $I'_i, J'_i \in \square(n_1 L \backslash DL)$. Let us consider the sets

$$\overline{(A \times I \times \ldots \times I)} \cap \overline{(A \times J \times \ldots \times J)} = \bar{A} \times X \times \ldots \times X$$

and

$$F(\bar{A} \times X \times \ldots \times X) = (\bar{A}' \cap \bar{A}'') \times (\bar{I}'_2 \cap \bar{J}'_2) \times \ldots \times (\bar{I}'_k \cap \bar{J}'_k).$$

Since the first set has dimension 2, the second set has also dimension 2. If $A' \neq A''$, then $\bar{A}' \cap \bar{A}'' \subset n_1 L$. This is impossible because

$$F(A \times X \times \ldots \times X) \subset (L \backslash n_1 L) \times n_1 L \times \ldots \times n_1 L.$$

So $A' = A''$ and $\bar{I}'_i \cap \bar{J}'_i = X'_i$ are finite. The polyhedron K is connected. Hence

$$F(B \times X \times \ldots \times X) = B' \times X'_2 \times \ldots \times X'_k \quad \text{for every } B \in \square(K \backslash n_1 K),$$

where $B' \in \square(L \backslash n_1 L)$. The set $n_1 K$ is nowhere dense in K, so

$$F(K \times X \times \ldots \times X) = L \times X'_2 \times \ldots \times X'_k \quad \text{and} \quad K \approx L.$$

The next part of the paper is analogous to Section 4 of [3]. For every component A of the set $K \backslash n_1 K$ we define a 2-manifold $M(A)$ in exactly the same way as in Section 4 of [3].

If $G\colon K \times S^1 \times \ldots \times S^1 \to L \times S^1 \times \ldots \times S^1$ is a homeomorphism (K is a compact 2-polyhedron), then for every $A \in \square(K \backslash n_1 K)$ there exists a homeomorphism

$$G_A\colon M(A) \times S^1 \times \ldots \times S^1 \to M(A') \times S^1 \times \ldots \times S^1,$$

where $A' \in \square(L \backslash n_1 L)$, such that

$$(g_{A'} \times \mathrm{id}_{S^1 \times \ldots \times S^1}) \circ G_A = G|_{\bar{A} \times S^1 \times \ldots \times S^1} \circ (g_A \times \mathrm{id}_{S^1 \times \ldots \times S^1}),$$

where $g_A\colon M(A) \to \bar{A}$ and $g_{A'}\colon M(A') \to \bar{A}'$ are defined exactly in the same way as in Section 4 of [3].

The homeomorphism G_A is given by the formula

$$G_A([\{x_n\}], t_2, \ldots, t_k)$$
$$= \big([\{P_1 G(x_n, t_2, \ldots, t_k)\}], P_2(\lim G(x_n, t_2, \ldots, t_k)), \ldots, P_k(\lim G(x_n, t_2, \ldots, t_k))\big),$$

where P_i is the projection on the i-th factor and the rest of the notation remains unchanged (Section 4 of [3]).

Now we can prove the following

THEOREM 2.5. *If K is a compact connected 2-polyhedron, the set $n_1 K$ contains a simple closed curve and $K^k \approx L^k$, then $K \approx L$.*

Proof. We can assume that K has no local cut points, $n_2 K = \emptyset$ and for every point $x \in n_1 K$ its regular neighborhood in K is homeomorphic to cone$\{1, \ldots, m\} \times I$. From Lemma 2.3 we know that

$$F(K \times n_1 K \times \ldots \times n_1 K) = L \times n_1 L \times \ldots \times n_1 L.$$

Since at least one of the components of $n_1 K$ is homeomorphic to S^1, we can assume that

$$F(K \times S^1 \times \ldots \times S^1) = L \times S^1 \times \ldots \times S^1.$$

The final part of the proof is analogous to the proof of Proposition 4.2 of [3].

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