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SOME PROBLEMS ON SUSLIN SPACES AND TOPOLOGICAL ALGEBRAS

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We give in the sequel some problems with a little motivation.

Problem 1. We have recently observed that any completely regular Suslin space (continuous surjective image of a Polish space) can be represented as follows:

If $(X, \mathcal{O})$ is completely regular Suslin, then there exists a separable metrizable space $(Y, \mathcal{P})$ such that X is homeomorphic to a subset of the continuous real-valued functions $C(Y)$ (with topology of compact convergence) and such that this subset is closed in $\mathcal{F}(Y)$ – the space of all real-valued functions with topology of pointwise convergence.

It seems likely that if Y can be chosen to be Suslin, then X is σ-compact. If $C(X)$ is Suslin, then Y may be chosen to be Suslin (note that if Y is Suslin, it may even be chosen to be the space of irrationals). Suppose that $(X, \mathcal{O})$ is completely regular and that $C(X)$ is Suslin. If we could only show that X were also Suslin, we would be able to show that X were σ-compact (non-trivial argument). We know that X has an embedding into $C(Y)$ as before where Y is Suslin. This motivates: Let X be a closed subset of $\mathcal{F}(Y)$ (space of real-valued functions with topology of pointwise convergence), where Y is a Suslin metrizable space, and suppose that X is contained in $C(Y)$. Is then X necessarily Suslin or may be even σ-compact? (**P 1080**)

Problem 2. It has recently been shown that the classical separable descriptive topology and set theory may be applied with some success to automatic continuity theory (see [1] and [3]). Let B be a separable complete metrizable topological vector space. Almost nothing is known about the descriptive properties of discontinuous linear functionals on B (it does not seem to make any difference if we assume that B is a Banach space). It seems to be unknown whether the hyperplane corresponding to such a functional may be of first category or universally measurable, or whether it may be the complement of an analytic (Suslin) set (it follows from [2] that the hyperplane cannot be analytic or Borel measurable). Information in this direction would most probably be very useful for automatic continuity theory.

It is now a classical problem in automatic continuity theory whether or not any multiplicative linear functional on a complete metrizable linear topological algebra (over the reals or complex numbers) is necessarily continuous (see, e.g., [4] and [5]). It is of course enough to study separable algebras. We now consider a commutative separable complete metrizable linear topological algebra A with unit e and discuss some questions which might be of use in this problem. Since the set I of invertible elements is the injective continuous image by projection onto the first coordinate of $\{(a, b) \in A^2 \mid ab = e\}$ which is closed, the set I is Borel measurable. It seems to be unknown whether I may be of any Baire class. (**P 1081**)

Let J be a maximal ideal which is dense. It would be interesting to know whether J may be nice from a descriptive point of view, e.g., whether J may be analytic or Borel (it can be shown by an argument similar to the above argument that if J is analytic, J is necessarily Borel). Even for the algebra of holomorphic functions on the complex plane with the usual topology of compact convergence this question seems to be completely open. (**P 1082**)

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