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A REMARK ON (p, q)-ABSOLUTELY SUMMING OPERATORS IN l^p-SPACES

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The purpose of this note* is to give a simple proof to Kwapień's generalization of Grothendieck's inequality, which was proved in [1] by using a complex interpolation method.

THEOREM. *Let $1 \leqslant p \leqslant 2$ and let N be an arbitrary positive integer. Let $(a_{mn})_{m,n=1}^N$ be a finite matrix of complex numbers such that*

$$\Big|\sum_{m,n} a_{mn} t_m s_n\Big| \leqslant 1$$

whenever $t_m, s_n \in C$, $|t_m|$, $|s_n| \leqslant 1$ for $m, n = 1, \ldots, N$. Let $(x_m)_{m=1}^N$ be an arbitrary sequence of elements in the unit ball of l^p. Then

$$(1) \qquad \Big(\sum_n \Big\|\sum_m a_{mn} x_m\Big\|_p^{r(p)}\Big)^{1/r(p)} \leqslant \mathcal{O}^{2/p-1} \mathcal{G}^{2-2/p},$$

where $r(p) \geqslant 2p/(3p-2)$, and $\mathcal{O}$ and $\mathcal{G}$ are universal constants.

Proof. Write the left-hand side of (1) as

$$(2) \qquad \Big(\sum_n \Big(\sum_k \Big|\sum_m a_{mn} x_m(k)\Big|^{2-p} \Big|\sum_m a_{mn} x_m(k)\Big|^{2p-2}\Big)^{2/(3p-2)}\Big)^{(3p-2)/2p}.$$

In (2) we apply Hölder's inequality to $\sum_k$ with the exponents $1/(2-p)$ and $1/(p-1)$ to the first and second factors of the summand, respectively. Thus we infer that (1) is bounded by

$$\Big[\sum_n \Big(\sum_k \Big|\sum_m a_{mn} x_m(k)\Big|\Big)^{2(2-p)/(3p-2)} \Big(\sum_k \Big|\sum_m a_{mn} x_m(k)\Big|^2\Big)^{2(p-1)/(3p-2)}\Big]^{(3p-2)/2p}.$$

Next, we apply Hölder's inequality in the preceding line to $\sum_n$ with the exponents $(3p-2)/(2-p)$ and $(3p-2)/(4p-4)$ to the first and second

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factors of the summand, respectively. Therefore, we infer that (1) is bounded by

$$(3)\qquad \Big[\sum_n\Big(\sum_k\Big|\sum_m a_{mn}x_m(k)\Big|\Big)^2\Big]^{1/p-1/2}\Big[\sum_n\Big(\sum_k\Big|\sum_m a_{mn}x_m(k)\Big|^2\Big)^{1/2}\Big]^{2-2/p}.$$

The first factor in (3) is bounded by $\mathcal{O}^{2/p-1}$ and the second factor in (3) is bounded by $\mathcal{G}^{2-2/p}$, where $\mathcal{O}$ is the universal constant in the "general Orlicz inequality" and $\mathcal{G}$ is the universal constant in the "general Littlewood inequality" (see (2.10) and (2.11) in [2]).

Remarks. 1. The constant in (1) is an improvement over the constant $\mathcal{O}^{2/p-1}\mathcal{G}^{2/p}$ that was obtained by Kwapień (see Remark 1 on p. 333 of [1]).

2. As is shown on p. 332 of [1], the inequality in (1) is sharp with respect to $r(p)$. This can be seen also as follows:

Let N be a fixed positive integer, and put

$$(4)\qquad a_{mn} = \frac{1}{N^{3/2}}\exp(2\pi mni/N), \qquad \text{where } m, n = 1, \ldots, N.$$

A routine verification yields

$$\Big|\sum_{m,n=1}^{N} a_{mn}t_m s_n\Big| \leqslant 1$$

for all (t_m), (s_n) in the unit ball of l^∞. Let $x_m = e_m$ be the m-th basic vector l^p, $e_m(n) = \delta_{mn}$, and compute

$$(5)\qquad \Big(\sum_{n=1}^{N}\Big\|\sum_{m=1}^{N} a_{mn}e_m(n)\Big\|_p^{r(p)}\Big)^{1/r(p)} = N^{1/r(p)+1/p-3/2}.$$

But the right-hand side of (5) is an unbounded function of N unless $r(p) \geqslant 2p/(3p-2)$.

Note that the matrix given in (4) appears in a similar context on p. 172 of Littlewood's classical paper [3] on bounded bilinear forms.

3. Kwapień notes in [1] that the interpolation method that he uses to prove the theorem above can be adapted to establish a similar result in the case $2 < p < \infty$. We were unable to modify our simple argument and obtain Kwapień's result in this case.

REFERENCES

[1] S. Kwapień, *Some remarks on (p, q)-absolutely summing operators in l^p-spaces*, Studia Mathematica 29 (1968), p. 327-337.

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[3] J. E. Littlewood, *On bounded bilinear forms in an infinite number of variables*, Quarterly Journal of Mathematics 1 (1930), p. 164-174.

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