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ON THE PARADOX OF TWO R_1, R_2 DOMAINS IN SCHWARZSCHILD EXTERIOR SOLUTION

BY

A. ZIĘBA (KRAKÓW)

1. The classic Schwarzschild form

$$ds^2 = \left(1-\frac{r_g}{r}\right)dt^2 - r^2(d\varphi^2+\sin^2\varphi d\vartheta^2) - \left(1-\frac{r_g}{r}\right)^{-1}dr^2 \tag{1}$$

characterizes the gravitation field either of a mass point or of an extended mass of a centrally-symmetric distribution in surrounding empty space. Form (1) has a singularity at the sphere $r = r_g$. The trajectories of free particles lie either completely in T-region ($r < r_g$) or in the R-region ($r > r_g$). Moreover, two different kinds of the T-region are to be distinguished according to the time direction. In the regions of the first kind the only possible motions are those from the central singularity $r = 0$ to the singularity $r = r_g$, the situation being opposite in the regions of the second kind.

Kruskal [4] and Novikov [5]-[7] found a generalization of the Schwarzschild geometry (1) in which motions from $r = 0$ to $r = r_g$ and in the opposite direction are both allowed, and in which the sphere $r = r_g$ appears not to be singular. The geodetics pass in a regular manner from regions T to R and from R to T.

The paradox of the Schwarzschild-Kruskal-Novikov (SKN) geometry is the occurrence of two regions R_1 and R_2 (the union of whose is R), Euclidean at infinity and isolated from each other: no geodetics pass from R_1 to R_2 and from R_2 to R_1. This would mean that the central singularity has seemingly two exterior regions, identical and mutually non-commutative. The discussion of this paradox is well known (cf., e.g., [1]-[3], [8] and [9]). In this paper is presented an attempt at a simple explanation of this paradox.

2. A uniform distribution of dust-like (i.e., interacting only through the gravitation) matter filling the whole space leads, as is well known, to the so-called Friedmann models of the Universe. There exists a class of such models, let us choose, for example, spherical models. The metric in such a model is of the form

$$ds^2 = d\tau^2 - R^2(\tau)(d\Phi^2 + \sin^2\Phi d\Theta^2 + \sin^2\Phi \sin^2\Theta d\psi^2), \tag{2}$$

where the function $R(\tau)$ is given by the equation

$$\left(\frac{dR}{d\tau}\right)^2 = \frac{R_0 - R}{R}.$$

If $\tau = \text{const}$, such a model is a 3-dimensional hypersphere of radius $R(\tau)$ which varies in time, increasing from 0 to a maximal value, and then decreasing again to 0.

In the theory of gravitational collapse and semi-closed universes there are considered combined Schwarzschild-Friedmann models, in which one part F of the space is filled by spherically-symmetric dust-like matter, and the other part SKN of the space is empty. In the region SKN geometry is valid, whereas in the region F it is of type (2). At the boundary common to regions F and SKN the geometries patch together smoothly having in the whole space, in the part filled with matter as well as in the empty part, a regular metric form.

3. Let us denote by B the set which is the union od the two identical copies of 3-dimensional hemispheres F_1 and F_2 tangent each other at their centres ω and situated as shown in Fig. 1. Assume that B is filled with a uniformly distributed dust-like matter. The Einstein equations equip B with a space-time Friedmann metric.

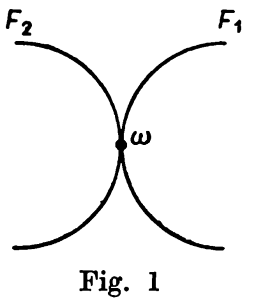

Fig. 1

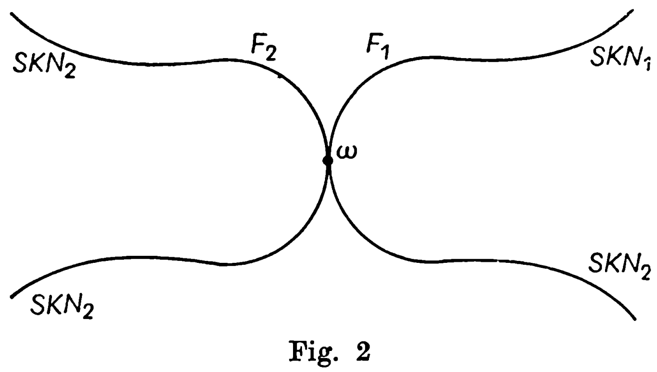

Fig. 2

Due to the centrally-symmetric distribution of the matter in B the extention of its Friedmann's metric to the surrounding empty complement is identical with SKN metric. In our case, in the surrounding empty space we obtain the whole SKN geometry whereas in the theory of gravitational collapse and semi-closed universes there appears only a part of SKN.

In our case each F_i has its is own exterior region SKN_i (Fig. 2) and its own region R_i. The R is now equal to $R_1 \cup R_2$.

Let us finally descuss the idealized case of a material point. The corresponding geometry is obtained from the one just described by taking away the regions F_i and by patching SKN_1 and SKN_2 together. It is clear that this patching is regular.

It seems that the possibility of a regular patching of SKN metric with Friedmann metric on B explains the apparent paradox of the SKN geometry and, in particular, the occurrence of two R_1 and R_2 regions in this geometry.

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