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SOME REMARKS ON $\mathcal{T}$-SEMIGROUPS

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In this paper we deal with $\mathcal{T}$-semigroups, i.e. semigroups S having the property that every transitive representation of S by partial transformations is a representation by one-to-one partial transformations. Such semigroups were studied by Schein in [6]. As was shown in [6], Theorem 1, and in [7], Section 2, Proposition 1, a semigroup S is a $\mathcal{T}$-semigroup if and only if every left unitary subsemigroup of S is unitary and strong. In his paper [6], p. 116, Schein writes that it would be desirable to find a simple criterion for a given semigroup to be a $\mathcal{T}$-semigroup. The aim of the present paper is to give such criteria for general semigroups as well as for some particular classes of semigroups.

In Section 1 we give some basic definitions concerning representations of semigroups. Next, in Theorem 1.1 we prove the main result of this paper: a semigroup S is a $\mathcal{T}$-semigroup if and only if every left unitary subsemigroup of S is unitary.

In Section 2 it is proved that a finite semigroup is a $\mathcal{T}$-semigroup if and only if it has no non-trivial right-zero subsemigroups.

Section 3 is devoted to regular semigroups. In particular, it is proved that a regular semigroup is a $\mathcal{T}$-semigroup if and only if it is right inverse and pseudo-invertible (see Corollary 3.1). This result was obtained by Schein ([6], Corollary 5) under the assumption that S is an inverse semigroup.

1. $\mathcal{T}$-semigroups.

We quote here some definitions concerning representations of semigroups. For standard semigroup terminology the reader is referred to [2].

Every partial transformation φ of a set X can be identified with a binary relation φ on X ($(x, y) \in \varphi$ if and only if φ transforms an element x into y) such that for every $x \in X$ the set $x\varphi = \{y \in X \mid (x, y) \in \varphi\}$ contains at most one element. The product of two partial transformations coincides with the product of binary relations, i.e. $(x, y) \in \varphi \circ \psi$ if and only if $(x, z) \in \varphi$ and $(z, y) \in \psi$ for some $z \in X$.

Let $\mathscr{F}_X$ be the semigroup of all partial transformations of X and let $\mathscr{K}_X$ be a subsemigroup of $\mathscr{F}_X$ consisting of all one-to-one partial transformations of X (the operation in these semigroups is the product $\varphi\circ\psi$).

A *representation* of a semigroup S by partial transformations of a set X is any homomorphism P of S into $\mathscr{F}_X$. If P transforms S into $\mathscr{K}_X$, we say that P is a *representation by one-to-one partial transformations.*

The *transitivity relation* τ_P of a representation P is the binary relation

$$\tau_P = \{(x, y) \in X \times X \mid (x, y) \in P(a) \text{ for some } a \in S\}.$$

A representation P is called *symmetric* if τ_P is symmetric (i.e. $\tau_P = \tau_P^{-1}$), and *transitive* if τ_P is universal (i.e. $\tau_P = X \times X$). The *symmetrant* of a representation P is a representation $\tilde{P}$ defined by $\tilde{P}(a) = P(a) \cap \tau_P^{-1}$ for every $a \in S$. The symmetrant $\tilde{P}$ is a symmetric representation.

The *regular representation* of a semigroup S is defined by the formula $P(a) = \varrho_a$, where ϱ_a is a right translation of S (see [2], Section 1.3).

Let S be an arbitrary semigroup and let A and B be subsets of S. Put

$$AB = \bigcup \{ab \mid a \in A,\, b \in B\},$$

$$A^{-1}B = \{a \in S \mid A\{a\} \cap B \neq \emptyset\}, \qquad AB^{-1} = \{a \in S \mid A \cap \{a\}B \neq \emptyset\}.$$

If $A = \{a\}$ or $B = \{b\}$, we shall write Ab, $a^{-1}b$, etc.

A subset H of a semigroup S is called *left* [*right*] *unitary* if $H^{-1}H \subseteq H$ [$HH^{-1} \subseteq H$], and *unitary* if it is both left and right unitary. A subset H of S is called *strong* if, for every $a, b \in S$, $Ha^{-1} \cap Hb^{-1} \neq \emptyset$ implies $Ha^{-1} = Hb^{-1}$.

Now, let P be a representation of a semigroup S by partial transformations of X. Put $H_x = \{a \in S \mid (x, x) \in P(a)\}$ for every $x \in X$. Every such set H_x is called a *stabilizer* of S relative to a representation P (see [7], p. 24). It is known that every stabilizer is a left unitary subsemigroup of S (see [7], Section 2, Theorem 2). Moreover, if P is a representation by one-to-one partial transformations, then every stabilizer of S relative to P is a unitary strong subsemigroup of S (see [7], Section 2, Proposition 1 and Theorem 3). In view of these facts the following lemma seems to be interesting.

LEMMA 1.1. *Let P be a representation of a semigroup S by partial transformations of a set X. Then the following conditions are equivalent:*

(i) *the symmetrant $\tilde{P}$ is a representation by one-to-one partial transformations;*

(ii) *every stabilizer of S relative to P is a unitary strong subsemigroup of S;*

(iii) *every stabilizer of S relative to P is a unitary subsemigroup of S.*

Proof. The implication (i) $\Rightarrow$ (ii) follows from the fact that the stabilizers relative to P are exactly the stabilizers relative to $\tilde{P}$ and from the remarks preceding the lemma. The implication (ii) $\Rightarrow$ (iii) is trivial.

For the proof of the implication (iii) $\Rightarrow$ (i) let us take an arbitrary $a \in S$ and suppose that every stabilizer of S relative to P is a unitary subsemigroup of S. We will prove that $\tilde{P}(a)$ is a one-to-one partial transformation of X. Suppose that $(x, z) \in \tilde{P}(a)$ and $(y, z) \in \tilde{P}(a)$ for some $x, y, z \in X$. Since $\tilde{P}$ is symmetric, we have $(z, x) \in \tilde{P}(b)$ and $(z, y) \in \tilde{P}(c)$ for some $b, c \in S$. Hence

$$(x, x) \in \tilde{P}(a)\circ\tilde{P}(b) = \tilde{P}(ab),$$

$$(x, y) \in \tilde{P}(a)\circ\tilde{P}(c) = \tilde{P}(ac), \quad \text{and} \quad (y, x) \in \tilde{P}(a)\circ\tilde{P}(b) = \tilde{P}(ab),$$

which implies $(x, x) \in \tilde{P}(ac)\circ\tilde{P}(ab) = \tilde{P}(acab)$. Therefore, $ab \in H_x$ and $acab \in H_x$. Since the stabilizer H_x is unitary, $ac \in H_x$, i.e. $(x, x) \in \tilde{P}(ac)$. On the other hand, $(x, y) \in \tilde{P}(ac)$, which implies $x = y$ and completes the proof.

A semigroup S is called a *$\mathscr{T}$-semigroup* ([6], p. 113) if every transitive representation of S by partial transformations is a representation by one-to-one partial transformations.

In the following theorem the equivalence of conditions (i)-(iv) is known and it was proved in [6], Theorem 1. They are stated here for easy references in the next parts of the paper.

THEOREM 1.1. *Let S be a semigroup. The following conditions are equivalent:*

(i) *S is a $\mathscr{T}$-semigroup;*

(ii) *every symmetric representation of S by partial transformations is a representation by one-to-one partial transformations;*

(iii) *the symmetrant of every representation of S by partial transformations is a representation by one-to-one partial transformations;*

(iv) *every left unitary subsemigroup of S is unitary and strong;*

(v) *every left unitary subsemigroup of S is unitary.*

Proof. Since a symmetric representation is a sum of transitive and null representations (see [5]), we have (i) $\Rightarrow$ (ii). Clearly, (ii) $\Rightarrow$ (iii). Since the stabilizers relative to P are exactly the stabilizers relative to $\tilde{P}$, the implication (iii) $\Rightarrow$ (iv) follows from [7], Section 2, Theorems 2 and 3 and Proposition 1. Next, (iv) $\Rightarrow$ (v) and (v) $\Rightarrow$ (i) by Lemma 1.1, which completes the proof.

From Theorem 1.1 we infer immediately that every abelian semigroup is a $\mathscr{T}$-semigroup, which was proved in [6], Corollary on p. 115.

The intersection of any family of left unitary subsemigroups of S is a left unitary subsemigroup. Denote by $\langle H\rangle$ the smallest left unitary subsemigroup of S containing a subset $H \subseteq S$ and let $\langle a\rangle = \langle\{a\}\rangle$.

COROLLARY 1.1. *A semigroup S is a $\mathcal{T}$-semigroup if and only if for every $a, b \in S$ we have $a \in \langle ab, b\rangle$.*

Proof. Suppose that for every $a, b \in S$ we have $a \in \langle ab, b\rangle$ and let H be a left unitary subsemigroup of S. If $ab, b \in H$ for some $a, b \in S$, then $\langle ab, b\rangle \subseteq H$ and, by the assumption, $a \in H$. Therefore, H is unitary and, by Theorem 1.1, S is a $\mathcal{T}$-semigroup, which completes the proof.

We shall need the following lemma which was proved in [7], Section 2, Lemma 3, for the case $S = S^1$ and is obviously true in the general case.

LEMMA 1.2. *$b \in \langle a\rangle$ if and only if $a^m b = a^n$ for some natural m and n.*

By Corollary 1.1 a semigroup S is a $\mathcal{T}$-semigroup if and only if every left unitary subsemigroup of S generated by two elements is unitary. Let us call a semigroup S a *$\mathcal{T}_1$-semigroup* if every left unitary subsemigroup generated by one element is unitary. The following example shows that there exist $\mathcal{T}_1$-semigroups which are not $\mathcal{T}$-semigroups.

Example. Let S be a free semigroup with n free generators ($n > 1$). For $a, b \in S$ ($a \neq b$) let H be the smallest subsemigroup of S generated by ab and b. It is easy to see that H is left unitary but not unitary. On the other hand, using Lemma 1.2 we see that every right cancellative semigroup is a $\mathcal{T}_1$-semigroup. Therefore, S, being cancellative, is a $\mathcal{T}_1$-semigroup.

We shall need the following

LEMMA 1.3. *Let S be a $\mathcal{T}_1$-semigroup. Then for every $a, b \in S$ with $ba = a$ there exists a natural number n such that $a^n b = a^n$.*

Proof. If $ba = a$, then $a, ba \in \langle a\rangle$. Since S is a $\mathcal{T}_1$-semigroup, we have $b \in \langle a\rangle$. By Lemma 1.2 there exist natural numbers m and n such that $a^m b = a^n$. Hence $a^{n+1} = (a^m b)a = a^{m+1}$. Therefore $a^{n+1} b = a^{m+1} b = a^{n+1}$, which completes the proof.

COROLLARY 1.2. *A left simple semigroup is a $\mathcal{T}$-semigroup if and only if it is a left group.*

Proof. Obviously, every left group is a $\mathcal{T}$-semigroup, which follows, e.g., from Theorem 1.1 and from [2], Exercise 10 in Section 10.2. Conversely, let S be a left simple semigroup. If the set of idempotents E of S is empty, then $a^{-1}a = \emptyset$ for every $a \in S$ (see [2], Lemma 8.3). On the other hand, since S is left simple, we have $aa^{-1} \neq \emptyset$ for every $a \in S$. Therefore, if S is a $\mathcal{T}$-semigroup, then, by Lemma 1.3, we have $E \neq \emptyset$, i.e. S is a left group.

2. Finite $\mathcal{T}$-semigroups. Observe that a semigroup S has no non-trivial right-zero subsemigroups if and only if $ee^{-1} \subseteq e^{-1}e$ for every $e \in E$. Hence it follows from Lemma 1.3 that every $\mathcal{T}$-semigroup has no non-trivial right-zero subsemigroups.

We need the following lemma which can be also of independent interest.

LEMMA 2.1. *Let S be a semigroup having no non-trivial right-zero subsemigroups. Suppose that H is a left unitary subsemigroup of S having the completely simple kernel K. Then H is a unitary subsemigroup of S.*

Proof. It follows from the assumption that the completely simple kernel K of H is a left group. Now, suppose that $ab, b \in H$ for some $a, b \in S$. If $k_1 \in K$, then $bk_1 \in K$. Let $k_2 \in bk_1K$ be such that $bk_1k_2 = e$, where $e = e^2$ is the identity of the group bk_1K. Put $k = k_1k_2$. Then $bk = e$, so $abk = ae \in K$. Since K is left simple, there exists $x \in K$ such that $xae = e$, which means that $xa \in ee^{-1}$. Hence, by the observation given above, we obtain $xa \in e^{-1}e$. Thus ex, $(ex)a \in H$ and, since H is left unitary, it follows that $a \in H$, which completes the proof.

THEOREM 2.1. *A finite semigroup is a $\mathscr{T}$-semigroup if and only if it has no non-trivial right-zero subsemigroups.*

Proof. Let S be a finite semigroup having no non-trivial right-zero subsemigroups and let H be a left unitary subsemigroup of S. Since H is finite, it has the completely simple kernel K. By Lemma 2.1, H is unitary and, by Theorem 1.1, S is a $\mathscr{T}$-semigroup.

3. Regular $\mathscr{T}$-semigroups. Let S be an arbitrary semigroup and let $V(a) = \{b \in S \mid aba = a \text{ and } bab = b\}$ denote the set of all inverses of an element $a \in S$. A semigroup S is called *regular* if $V(a) \neq \emptyset$ for every $a \in S$, and S is called *right inverse* if it is regular and for any pair of inverses $a', a'' \in V(a)$ we have $aa' = aa''$. In [1], Theorem 3, it is proved that S is right inverse if and only if each $\mathscr{R}$-class of S contains a unique idempotent. An *orthodox semigroup* is defined as a regular semigroup in which the idempotents E form a subsemigroup. It is known that a regular semigroup S is orthodox if and only if $V(b)V(a) \subseteq V(ab)$ for every $a, b \in S$ (see [3], Chapter VI, Theorem 1.1). Moreover, every right inverse semigroup is orthodox (see [1], Theorem 7 (a)). These facts will be basic in the sequel and will be used in our further considerations.

A semigroup S is called *inverse* if for every $a \in S$ the set $V(a)$ contains exactly one element. It is well known that the symmetrant of the regular representation of an inverse semigroup is a representation by one-to-one partial transformations (see, e.g., the proof of the Vagner-Preston representation theorem in [3], p. 135). The following lemma generalizes this fact.

LEMMA 3.1. *If S is a regular semigroup, then the following statements are equivalent:*

(i) *the symmetrant of the regular representation of S is a representation by one-to-one partial transformations;*

(ii) *S is right inverse;*

(iii) *S has no non-trivial right-zero subsemigroups;*

(iv) *$efe = ef$ for every two idempotents $e, f \in S$.*

In particular, a regular $\mathcal{T}$-semigroup is right inverse.

Proof. The implication (iv) $\Rightarrow$ (iii) is obvious. Next, the equivalence (ii) $\Leftrightarrow$ (iii) follows from the fact that for any pair of idempotents $e, f \in S$ we have $e\mathcal{R}f$ if and only if $\{e, f\}$ is a right-zero semigroup.

(i) $\Rightarrow$ (iii). By Lemma 1.1 a semigroup S satisfies (i) if and only if for every $a, b, c \in S$ such that $abc = a$ and $ac = a$ we have $ab = a$. Obviously, every such semigroup satisfies (iii).

(ii) $\Rightarrow$ (iv). If S is right inverse, then it is orthodox and, by the above-proved implication (ii) $\Rightarrow$ (iii), it has no non-trivial right-zero subsemigroups. Now, if $e, f \in E$, then $efe = ef$; otherwise $\{efe, ef\}$ would be a non-trivial right-zero semigroup.

(ii) $\Rightarrow$ (i). Let S be right inverse. First, we shall prove that for every $a, b \in S$ and for every $\bar{b} \in V(b)$ we have

$$b \in H_a \Rightarrow \bar{b} \in H_a, \tag{1}$$

where $H_a = \{c \in S \mid ac = a\}$. Obviously, it suffices to check that (1) holds for every $a \in E$. However, if $ab = a$ and $a \in E$, then, by the above-proved implication (ii) $\Rightarrow$ (iv), $a = ab = ab\bar{b}b = ab\bar{b}ab = ab\bar{b}a = ab\bar{b} = a\bar{b}$ i.e. $\bar{b} \in H_a$.

Now, let $c, bc \in H_a$ for some $a, b, c \in S$. Since S is orthodox, we infer from (1) that $\bar{c}, \bar{c}\bar{b} \in H_a$ for arbitrary $\bar{b} \in V(b)$, $\bar{c} \in V(c)$. Hence $\bar{b} \in H_a$ and, by (1), $b \in H_a$. Therefore, by Lemma 1.1, S satisfies (i). This completes the proof.

Observe that the symmetrant of the regular representation of a right inverse semigroup is obtained by the restriction of ϱ_a to $Saa' = Sa'$, where a' is an arbitrary fixed element of $V(a)$ (see also Theorem 6 in [8]; in [8] right inverse semigroups are called *left inverse*).

The following theorem was proved in [6], Corollary 3, under the assumption that S is orthodox. The general case can be easily deduced from Lemma 3.1 and that corollary. However, using Theorem 1.1 we are able to give a simple and direct proof of this theorem.

THEOREM 3.1. *A regular semigroup S is a $\mathcal{T}$-semigroup if and only if for every $a \in S$ and every $\bar{a} \in V(a)$ there exists a natural number n such that $a^n a\bar{a} = a^n$.*

Proof. Suppose that S is a regular $\mathcal{T}$-semigroup and let $a \in S$ and $\bar{a} \in V(a)$. Hence $(a\bar{a})a = a$ and, by Lemma 1.3, there exists a natural number n such that $a^n(a\bar{a}) = a^n$.

Conversely, assume that for every $a \in S$ and every $\bar{a} \in V(a)$ there exists a natural number n such that $a^n a\bar{a} = a^n$. Hence, by Lemma 1.2, for every $a \in S$ we have

$$V(a) \subseteq \langle a \rangle. \tag{2}$$

Furthermore, if $\{e, f\} \subseteq S$ is a right-zero semigroup, then $f \in V(e)$. Hence, by assumption, $ef = e$. Therefore, S has no non-trivial right-zero subsemigroups. By Lemma 3.1, S is right inverse, and hence orthodox. Now, let H be a left unitary subsemigroup of S and let $ab, b \in H$. We infer from (2) that $\bar{b}\bar{a}, \bar{b} \in H$. Hence $\bar{a} \in H$ and, by (2), $a \in H$. Therefore H is unitary and, by Theorem 1.1, S is a $\mathcal{T}$-semigroup. This completes the proof.

An element a of a semigroup S is called *pseudo-invertible* if there is an element $\bar{a} \in S$ such that $a\bar{a} = \bar{a}a$, $a^n a\bar{a} = a^n$ for some natural number n, and $\bar{a}^2 a = \bar{a}$. In [4], Theorem 1, it is proved that an element $a \in S$ is pseudo-invertible if and only if some power of a lies in a subgroup of S. A semigroup S is called *pseudo-invertible* if every element of S is pseudo-invertible.

The following theorem was proved in [6], Corollary 5, under the assumption that S is an inverse semigroup.

THEOREM 3.2. *If S is a right inverse semigroup, then the following conditions are equivalent:*

(i) *S is a $\mathcal{T}$-semigroup;*

(ii) *for every $a \in S$ the descending chain of principal right ideals $aS \supseteq a^2S \supseteq \ldots \supseteq a^nS \supseteq \ldots$ is finite;*

(iii) *S is pseudo-invertible.*

Proof. The implication (i) $\Rightarrow$ (ii) follows easily from Theorem 3.1.

(ii) $\Rightarrow$ (i). Let S be a right inverse semigroup satisfying (ii). Thus, for a fixed $a \in S$ there exists a natural number n such that $a^{n+1}S = a^nS$. Therefore $a^{n+1}\mathcal{R}a^n$. Moreover, since S is orthodox, we have $a^n\mathcal{R}a^n\bar{a}^n$ and $a^{n+1}\mathcal{R}a^{n+1}\bar{a}^{n+1}$ for an arbitrary $\bar{a} \in V(a)$. Hence $a^n\bar{a}^n\mathcal{R}a^{n+1}\bar{a}^{n+1}$. Since each $\mathcal{R}$-class in a right inverse semigroup has a unique idempotent, we obtain

$$a^n\bar{a}^n = a^{n+1}\bar{a}^{n+1} \quad \text{for every } \bar{a} \in V(a). \tag{3}$$

Now, from (3) and from Lemma 3.1 (iv) we obtain

$$\begin{aligned} a^n &= a^n\bar{a}^n a^n = a^{n+1}\bar{a}^{n+1}a^n = a^n(a\bar{a})(\bar{a}^n a^n) = a^n(a\bar{a})(\bar{a}^n a^n)(a\bar{a}) \\ &= a^{n+1}\bar{a}^{n+1}a^n(a\bar{a}) = a^n\bar{a}^n a^n(a\bar{a}) = a^n(a\bar{a}). \end{aligned}$$

By Theorem 3.1, S is a $\mathcal{T}$-semigroup.

(i) $\Rightarrow$ (iii). Let $a \in S$, $\bar{a} \in V(a)$, and suppose that S is a $\mathcal{T}$-semigroup. By Theorem 3.1 there exists a natural number n such that

$$a^n a\bar{a} = a^n \tag{4}$$

and

$$\bar{a}^n \bar{a} a = \bar{a}^n. \tag{5}$$

From (4) we obtain

$$a^n(a^n \bar{a}^n) = a^{n-1}(a^n a\bar{a})\bar{a}^{n-1} = a^{n-1} a^n \bar{a}^{n-1}$$
$$= a^{n-2}(a^n a\bar{a})\bar{a}^{n-2} = a^{n-2} a^n \bar{a}^{n-2} = \ldots = aa^n \bar{a} = a^n a\bar{a} = a^n.$$

Hence we have

$$a^n a^n \bar{a}^n = a^n. \tag{6}$$

Analogously, (5) implies

$$\bar{a}^n \bar{a}^n a^n = \bar{a}^n. \tag{7}$$

Put $b = a^n$, $\bar{b} = a^n \bar{a}^n \bar{a}^n$. Using (6) and (7) we obtain $b^2\bar{b} = b$, $\bar{b}^2 b = \bar{b}$, and $b\bar{b} = \bar{b}b$. Therefore, $b = a^n$ is pseudo-invertible, and so is a.

(iii) $\Rightarrow$ (i). Suppose that S is pseudo-invertible and let $a \in S$. Hence there is a natural number n such that $a^n \in G_e$, where G_e is a subgroup of S with the identity e. Obviously, $e\mathcal{R}a^n$. Moreover, since S is orthodox, we have $a^n \mathcal{R} a^n \bar{a}^n$. Hence $e\mathcal{R}a^n \bar{a}^n$ and, since S is right inverse, $a^n \bar{a}^n = e$. Using Lemma 3.1 (iv) we obtain

$$a^n a\bar{a} = a^n e a\bar{a} = a^n e(a\bar{a})e = a^n e(a\bar{a})(a^n \bar{a}^n) = a^n e(a\bar{a}a)a^{n-1}\bar{a}^n$$
$$= a^n e a^n \bar{a}^n = a^n.$$

By Theorem 3.1, S is a $\mathcal{T}$-semigroup. This completes the proof.

By Lemma 3.1 a regular $\mathcal{T}$-semigroup is right inverse. Hence

COROLLARY 3.1. *A regular semigroup is a $\mathcal{T}$-semigroup if and only if it is right inverse and satisfies one of the equivalent conditions in Theorem 3.2.*

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