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ON MODELS OF FINITE DIRECT PRODUCTS OF THEORIES

BY

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0. Introduction. Let A and B be structures for the language L, and let T_1 and T_2 denote $\mathrm{Th}(A)$ and $\mathrm{Th}(B)$, respectively. In this paper it will be shown that the models of $T_1 \times T_2 = \mathrm{Th}(A \times B)$ can always be elementarily embedded in the direct product of models of T_1 and T_2. Also, a relationship between the forcing companion of $T_1 \times T_2$, T_1, and T_2 will be given; and as an application, the model companion of the finite direct product of the theories of linear orderings will be found. Furthermore, given an arbitrary theory T, a characterization of precisely when T is the direct power of another theory is given.

Many of the results contained here are in the author's dissertation [21]. Our notation will be as in [4] with the exception that we use $|A|$ to denote the universe of the structure A, and we assume familiarity with [16].

1. Preliminaries. In this section we introduce some basic definitions and ideas which will be used throughout the remaining parts of this paper.

Let $x_0, \ldots, x_n, \ldots$ be a list of variables for the language $L(\bar{x})$ and $x_0, y_0, \ldots, x_n, y_n, \ldots$ be a list of variables for the language $L(\bar{x}, \bar{y})$. Then Weinstein in [23] associates with each formula φ of $L(\bar{x})$ a formula φ' of $L(\bar{x}, \bar{y})$ and uses φ' to characterize those sentences which are preserved under direct power. Here φ' will be used to develop a connection between the relations in T_1 and $T_1 \times T_1$.

DEFINITION 1.1. Let φ be a formula of $L(\bar{x})$. Then a formula φ' of $L(\bar{x}, \bar{y})$ is defined inductively as follows:

(1) For atomic $R(t_1, \ldots, t_n)$ or $t_1 = t_2$, φ' is

$$R(t_1, \ldots, t_n) \wedge R(t'_1, \ldots, t'_n) \quad \text{or} \quad t_1 = t_2 \wedge t'_1 = t'_2,$$

respectively, where t'_i for each i is the resulting of replacing each variable x_j in t_i with y_j.

(2) If φ is $\sim \varphi_1$, then φ' is $\sim \varphi'_1$.

(3) If φ is $(\varphi_1 \rightarrow \varphi_2)$, then φ' is $(\varphi'_1 \rightarrow \varphi'_2)$.

(4) If φ is $\forall x_i \varphi_1$, then φ' is $\forall x_i \forall y_i \varphi'_1$.

The following lemma states that satisfaction in $A \times A$ is effectively reducible to satisfaction in A by means of φ'.

LEMMA 1.1. *Let* $\varphi(x_1, \ldots, x_n)$ *be a formula of* $L(\bar{x})$ *and* $(a_{01}, a_{11}), \ldots, (a_{0n}, a_{1n})$ *be in* $|A \times A|$. *Then*

$$A \times A \models \varphi[(a_{01}, a_{11}), \ldots, (a_{0n}, a_{1n})] \quad \textit{iff} \quad A \models \varphi'[a_{01}, a_{11}, \ldots, a_{0n}, a_{1n}].$$

Proof. By induction on the complexity of φ.

COROLLARY 1.2. *Let* φ *be a sentence of* $L(\bar{x})$. *Then* φ *is true in* $A \times A$ *iff* φ' *is true in* A.

The following definition and result are due to Galvin [7].

DEFINITION 1.2. Let $x_1, \ldots, x_n$ be distinct variables. An *autonomous system* is a triple $\langle S, \pi, \varrho \rangle$ satisfying the following conditions:

(1) S is a finite set of formulas with free variables among $x_1, \ldots, x_n$.

(2) π is a binary operation over S.

(3) ϱ is a function over $S \times S$ whose values are sentences in the language of Boolean algebras.

(4) For each structure A and elements $a_1, \ldots, a_n \in |A|$ there is exactly one $\varphi \in S$ such that

$$A \models \varphi[a_1, \ldots, a_n].$$

(5) If $\varphi, \psi \in S$ and

$$A \models \varphi[a_1, \ldots, a_n] \quad \text{and} \quad B \models \psi[b_1, \ldots, b_n],$$

then

$$A \times B \models \pi(\varphi, \psi)[(a_1, b_1), \ldots, (a_n, b_n)].$$

(6) Let $\varphi, \psi \in S$ and $f_1, \ldots, f_n \in |A|^I$ be such that, for each $i \in I$,

$$A \models \varphi[f_1(i), \ldots, f_n(i)];$$

then

$$A_D^I \models \psi[f_1/D, \ldots, f_n/D] \quad \text{iff} \quad 2_D^I \models \varrho(\varphi, \psi).$$

LEMMA 1.3. *For each formula* φ *with the free variables* $x_1, \ldots, x_n$ *there is an effective procedure by which one can find an autonomous system* $\langle S, \pi, \varrho \rangle$ *and a subset* S_1 *of* S *such that*

$$\vdash \varphi \leftrightarrow \bigvee \{\varphi \colon \varphi \in S_1\}.$$

2. Elementary embedding. Waszkiewicz and Węglorz prove in [22] that if A and B are countable saturated structures, then $A \times B$ is also a countable saturated structure. Also, in [24] it is shown, as a corollary, that if A and B are countable saturated models of T_1 and T_2, respectively, then any countable model of $T_1 \times T_2$ can be elementarily embedded in $A \times B$. The next two results will generalize this latter result and state that for any model M of $T_1 \times T_2$ there exist two models A_1 and B_1 of T_1 and T_2, respectively, such that M is elementarily embedded into $A_1 \times B_1$.

THEOREM 2.1. *If M is a model of $T_1 \times T_1$, then there exists a model A of T_1 such that M is elementarily embedded into $A_1 \times A_1$.*

Proof. Let Γ_M be the elementary diagram of M and suppose for a contradiction that Γ'_M is not consistent with T_1. Then there exists a finite set of sentences $\{\varphi_1, \ldots, \varphi_n\}$ of Γ_M such that $T_1 \vdash \sim\psi'$, where ψ is $\varphi_1 \wedge \ldots \wedge \varphi_n$. Assume $\bar{m}_1, \ldots, \bar{m}_s$ are the only new constants in ψ; then $\bar{m}_{01}, \bar{m}_{11}, \ldots, \bar{m}_{0s}, \bar{m}_{1s}$ are the new constants in ψ'. Let $x_1, y_1, \ldots, x_s, y_s$ be appropriate variables substituted in ψ' for $\bar{m}_{01}, \bar{m}_{11}, \ldots, \bar{m}_{0s}, \bar{m}_{1s}$, respectively. Then, since $\bar{m}_{01}, \ldots, \bar{m}_{1s}$ do not occur in the sentences of T_1, we have

$$T_1 \vdash \forall x_1 \ \forall y_1 \ldots \forall x_s \ \forall y_s \ \sim\psi\begin{pmatrix}\bar{m}_{01}, \ldots, \bar{m}_{1s}\\ x_1, \ldots, y_s\end{pmatrix}.$$

Hence this new sentence is true in any model A of T_1 and, by Corollary 1.2, $\forall x_1 \ldots \forall x_s \ \sim\psi$ is true in any model of $T_1 \times T_1$, so it is true in M. Let $m_1, \ldots, m_s$ be the interpretation of $\bar{m}_1, \ldots, \bar{m}_s$ in M. Then M satisfies $\sim\psi$ with $m_1, \ldots, m_s$, but this means that $\sim\varphi_i\begin{pmatrix}x_1, \ldots, x_s\\ \bar{m}_1, \ldots, \bar{m}_s\end{pmatrix}$ for some i is in the elementary diagram Γ_M, and this is a contradiction to the fact that $\varphi_i\begin{pmatrix}x_1, \ldots, x_s\\ \bar{m}_1, \ldots, \bar{m}_s\end{pmatrix}$ in Γ_M. This shows that Γ_M is consistent with T_1. Let A be a model of $\Gamma'_M \cup T_1$ and define

$$f\colon |M| \to |A_1 \times A_1|$$

by

$$f(m) = (\bar{m}_0^{A_1}, \bar{m}_1^{A_1}).$$

Suppose $\varphi(x_1, \ldots, x_n)$ is satisfied with $m_1, \ldots, m_n$ in M, since $\varphi\begin{pmatrix}x_1, \ldots, x_n\\ \bar{m}_1, \ldots, \bar{m}_n\end{pmatrix}$ is in Γ_M. Then, clearly, $\varphi'(x_1, y_1, \ldots, x_n, y_n)$ is satisfied in A_1 with $\bar{m}_{01}^{A_1}, \bar{m}_{11}^{A_1}, \ldots, \bar{m}_{0n}^{A_1}, \bar{m}_{1n}^{A_1}$. This shows by Lemma 1.1 that φ is satisfied in $A_1 \times A_1$ with $f(m_1), \ldots, f(m_n)$. Hence f is an elementary embedding of M into $A_1 \times A_1$.

The next result generalizes Theorem 2.1 to direct product, and its proof uses Waszkiewicz and Węglorz's ideas in [22].

THEOREM 2.2. *If M is a model of $T_1 \times T_2$, then there exist models A_1 and B_1 of T_1 and T_2, respectively, such that M is elementarily embedded into $A_1 \times B_1$.*

Proof. Let Γ_M be the elementary diagram of M and let φ be an element of Γ_M. Suppose $\bar{m}_1, \ldots, \bar{m}_n$ are the only new constants in φ. Then replace each of these new constants with a new appropriate variable $x_{\bar{m}_1}, \ldots, x_{\bar{m}_n}$. Let Σ denote the set of all formulas obtained from those by replacing the new constants from $|M|$ occurring in each formula φ with appropriate variables x_m for $m \in |M|$, and let L denote this extended language by variables. Since M is a model of Γ_M, Σ is a maximal consistent set of formulas and induces an ultrafilter in the Boolean algebra of formulas of language L. For each element φ of L, let S_φ denote the autonomous set for φ and form

$$S = \prod_{\varphi \in L} S_\varphi .$$

Let P be any ultrafilter in the Boolean algebra of formulas of L and let

$$S_\varphi = \{\psi_1, \ldots, \psi_n\}.$$

Suppose ψ_i for $i = 1, \ldots, n$ do not belong to P. Then $\sim\psi_i$ for all $i = 1, \ldots, n$ belongs to P. Since S_φ is an autonomous set,

$$\sim(\psi_1 \vee \ldots \vee \psi_n) = 0,$$

and therefore $0 \in P$, which is a contradiction to the fact that P is an ultrafilter. This shows that $P \cap S_\varphi$ is not empty, and thus P can be thought of as an element of S by defining $P(\varphi) = \psi$ iff $S_\varphi \cap P = \{\psi\}$. The fact that the image ψ is uniquely determined follows from the assertion that, in an autonomous set S_φ, $\psi_i \wedge \psi_j = 0$ for $i \neq j$. Let $Z = S \times S$ and for each ultrafilter P define

$$Z_\varphi^P = \{(\psi, \gamma) \text{ in } S_\varphi \times S_\varphi \colon \psi \times \gamma = P(\varphi)\} \quad \text{and} \quad Z^P = \prod_{\varphi \in L} Z_\varphi^P .$$

Define the discrete topology on S_φ and, by using Tychonoff's theorem, Z is compact. The following observations are made in order to complete the rest of the proof:

(1) Z^P is closed.

In order to show (1), take $x \in Z - Z^P$ and suppose x is a limit point of Z^P. Since x does not belong to Z^P, there exists at least one of its coordinates, say φ_i, such that $\psi \times \gamma \neq P(\varphi_i)$. Let

$$W = \prod_{\varphi \in L} O_\varphi, \quad \text{where } O_{\varphi_i} = \{(\psi, \gamma)\} \text{ and } O_\varphi = S_\varphi \times S_\varphi \text{ for } \varphi \neq \varphi_i .$$

Then W is open, contains x, and $W \cap Z^P$ is empty. Hence x cannot be a limit point of Z^P. Thus Z^P is closed.

(2) For any ultrafilter P which is obtained from a model M of $T_1 \times T_2$ as above, there exist ultrafilters Q_1, Q_2 in the Boolean algebras of formulas of L induced by T_1 and T_2, respectively, such that (Q_1, Q_2) is in Z^P.

In order to show (2), let $T_{\varphi_1,\ldots,\varphi_k}$ be the closed subset of Z^P consisting of all those (f, g) for which

$$T_1 \cup \{f(\varphi_1), \ldots, f(\varphi_k)\} \quad \text{and} \quad T_2 \cup \{g(\varphi_1), \ldots, g(\varphi_k)\}$$

are consistent. Since $\{P(\varphi_1), \ldots, P(\varphi_k)\}$ is a consistent set in $T_1 \times T_1$ and $A \times B$ is a model of $T_1 \times T_2$ for models A and B of T_1 and T_2, respectively, then there exist $(a_1, b_1), \ldots, (a_n, b_n)$ in $|A \times B|$ such that

$$A \times B \models P(\varphi_i)[(a_1, b_1), \ldots, (a_n, b_n)] \quad \text{for } i = 1, \ldots, n.$$

This shows that

$$A \models \psi_i[a_1, \ldots, a_n] \quad \text{and} \quad B \models \gamma_i[b_1, \ldots, b_n]$$

for some $\psi_i, \gamma_i \in S_{\varphi_i}$ and $\psi_i \times \gamma_i = P(\varphi_i)$. Hence $T_{\varphi_1,\ldots,\varphi_k}$ is not empty. By a similar argument to (1), one can show that $T_{\varphi_1,\ldots,\varphi_k}$ is closed in Z^P. Also, it is clear that

$$T_{\varphi_1,\ldots,\varphi_k} \subset T_{\varphi_1,\ldots,\varphi_{k-1}},$$

and this produces a chain of closed sets with the finite intersection property. Since Z^P is compact, the intersection over this chain is not empty, and let (f, g) be an element of this intersection. Every finite subset of

$$T_1 \cup \{f(\varphi)\colon \varphi \in L\} \quad \text{or} \quad T_2 \cup \{g(\varphi)\colon \varphi \in L\}$$

is consistent, since for any $\varphi_1, \ldots, \varphi_k$ we have $(f, g) \in T_{\varphi_1,\ldots,\varphi_k}$. By the Compactness Theorem, the sets

$$T_1 \cup \{f(\varphi)\colon \varphi \in L\} \quad \text{and} \quad T_2 \cup \{g(\varphi)\colon \varphi \in L\}$$

are consistent.

(3) $T_1 \cup \{f(\varphi)\colon \varphi \in L\}$ is complete.

In order to show (3), let φ be a formula of L and $S_1 \subset S_\varphi$ as in Lemma 1.3 such that

(1) $$\vdash \varphi \leftrightarrow \bigvee \{\gamma\colon \gamma \in S_1\}.$$

Since $f(\varphi) \in S_\varphi$, either $f(\varphi) \in S_1$ or $f(\varphi) \in S_\varphi - S_1$.

Case 1. $f(\varphi) \in S_1$.

Take A to be any model of $T_1 \cup \{f(\varphi)\colon \varphi \in L\}$, which means that A is a model of T_1 and there is a sequence S of elements of $|A|$ such that, for all φ, $A \models f(\varphi)[S]$. Since $\vdash f(\varphi) \to \varphi$ by (1), it follows that $A \models \varphi[S]$. Therefore,

$$T \cup \{f(\varphi)\colon \varphi \in L\} \models \varphi.$$

Case 2. $f(\varphi) \in S_\varphi - S_1$.

The analogous argument as in Case 1 shows that

$$T \cup \{f(\varphi)\colon \varphi \in L\} \vdash \sim \varphi.$$

Let (A_1, S_1) and (B_1, S_2) be any models of

$$\mathrm{Th}(A) \cup \{f(\varphi)\colon\ \varphi \in L\} \quad \text{and} \quad \mathrm{Th}(B) \cup \{g(\varphi)\colon\ \varphi \in L\},$$

respectively. Then define a function $h\colon |M| \to |A_1 \times B_1|$ by

$$h(m) = (S_1(x_{\bar{m}}),\ S_2(x_{\bar{m}})).$$

Suppose $\varphi(x_1, \ldots, x_n)$ is a formula of L and $m_1, \ldots, m_n$ a sequence of elements in M such that

$$M \models \varphi[m_1, \ldots, m_n].$$

Thus

$$\varphi\begin{pmatrix} x_1, \ldots, x_n \\ x_{\bar{m}_1}, \ldots, x_{\bar{m}_n} \end{pmatrix} \in P.$$

Clearly,

$$A_1 \models f(\varphi)[S_1(x_{\bar{m}_1}), \ldots, S_1(x_{\bar{m}_n})] \quad \text{and} \quad B_1 \models g(\varphi)[S_2(x_{\bar{m}_1}), \ldots, S_2(x_{\bar{m}_n})].$$

By Lemma 1.3, since $f(\varphi) \times g(\varphi) = P(\varphi)$, we have

$$A_1 \times B_1 \models \varphi[h(m_1), \ldots, h(m_n)],$$

and hence M is elementarily embedded in $A_1 \times B_1$.

3. Model completeness and direct product. In this section the concept of forcing companion T^F of the theory T is studied and it is shown that the model companion of $T_1 \times T_2$ is always the same as the model companion of $T_1^F \times T_1^F$. This is used to show that the model companion of a direct product of two infinite linear orderings is always $(Q \times Q)^F$, where Q is the theory of rational numbers with its usual ordering. Also, another proof of preservation of ω-categoricity under direct power is given. Finally, for a given theory T, a characterization of exactly when T is the direct power of another theory is given.

One can be motivated by the fact that frequently for T model complete, $T \times T$ is model complete; however, there are some simple counterexamples to this conjecture [11]. The next result does show this phenomenon occurring in some generality in the opposite direction.

LEMMA 3.1. *Let T be a complete theory with model companion T^*. If $T \times T$ is model complete, then there exists a structure B such that* $\mathrm{Th}(B)$ *is model complete and* $\mathrm{Th}(B \times B) = T \times T$.

Proof. Since $T \times T$ can be taken to be Π_2, it follows that $(T \times T)'$ is Π_2. Also, since T^* is the model companion of T and is model complete, by a theorem of [16] we have $(T \times T)' \subset T^*$. Let B be any model of T^*. Then $\mathrm{Th}(B \times B) = T \times T$.

Saracino in [20] proves that ω-categorical theories have ω-categorical model companions. This and Lemma 3.1 will give the following corollary:

COROLLARY 3.2. *Let T be an ω-categorical complete theory. If $T\times T$ is model complete, then there exists a structure B such that* $\mathrm{Th}(B)$ *is model complete, ω-categorical, and* $\mathrm{Th}(B\times B) = T\times T$.

The next result is concerned with the relationship between the forcing companions of T_1 and T_2 and the forcing companion of $T_1\times T_2$.

THEOREM 3.3. *Let* $T_1 = \mathrm{Th}(A)$ *and* $T_2 = \mathrm{Th}(B)$; *then*

$$(T_1\times T_2)^F = (T_1^F\times T_2^F)^F.$$

Proof. Let M be a model of $T_1\times T_2$. Then by Theorem 2.2 there are models A_1 and B_1 of T_1 and T_2, respectively, such that M is elementarily embedded into $A_1\times B_1$. Since T_1 and T_2 are mutually model consistent with T_1^F and T_2^F, respectively, A_1 and B_1 can be embedded into A' and B'. This shows that $T_1\times T_2$ is model consistent with $T_1^F\times T_2^F$. An analogous argument shows that $T_1^F\times T_2^F$ is model consistent with $T_1\times T_2$. Thus the result is proved.

Let T be the theory of any infinite linear ordering. Then it is observed in [20] that the model companion of T is the theory of rational numbers Q with its usual ordering. The following application of 3.3 produces the model companion of the direct product of two infinite linear orderings.

APPLICATION 3.4. Let T_1 and T_2 be the theories of two infinite linear orderings. Then the model companion of $T_1\times T_2$ is the theory of any subset of $Q\times Q$ which is dense in $Q\times Q$, without first or last element, and is such that whenever x and y are different elements of the subset, every coordinate of x is different from the corresponding coordinate of y.

Proof. By Lemma 3.3, $(T_1\times T_2)^F = (T_1^F\times T_2^F)^F$. But $T_1^F = T_2^F = \mathrm{Th}(Q)$. Therefore, $(T_1\times T_2)^F$ is $(\mathrm{Th}(Q\times Q))^F$ and in [13] the model companion of $\mathrm{Th}(Q\times Q)$ is shown to be the theory of the desired subset of $Q\times Q$.

The result of Lemma 1.1 will supply an easy proof of preservation of ω-categoricity under direct powers.

LEMMA 3.5. *If T_1 is ω-categorical, then $T_1\times T_1$ is also ω-categorical.*

Proof. By Lemma 1.1, for any two formulas φ, ψ of $L(\bar{x})$,

$$(2)\qquad T_1\times T_1\vdash \varphi\leftrightarrow\psi \quad\text{iff}\quad T_1\vdash \varphi'\leftrightarrow\psi'.$$

Suppose $T_1\times T_1$ is not ω-categorical. Then by a theorem of Ryll-Nardzewski (see [19]) there is a natural number n such that it is possible to find an infinite number of inequivalent formulas with at most $x_1, \ldots, x_n$ as their free variables under $T_1\times T_1$. Let $\varphi_1, \ldots, \varphi_k, \ldots$ be this infinite list and suppose, for some i, j, $i\neq j$ and $T_1\vdash \varphi_i'\leftrightarrow\varphi_j'$. This by (2) implies

$$T_1\times T_1\vdash \varphi_i\leftrightarrow\varphi_j.$$

Thus $\varphi'_1, \ldots, \varphi'_k, \ldots$ is a list of an infinite number of inequivalent formulas under T_1 with at most $x_1, y_1, \ldots, x_n, y_n$ as their free variables, and this is a contradiction to the ω-categoricity of T.

The next result characterizes when T is a theory of some direct power of order 2 of some structure.

LEMMA 3.6. *Let T be a complete theory. Then $T = \mathrm{Th}(A \times A)$ for some A iff $T' = \{\varphi' : \varphi \in T\}$ is consistent.*

Proof. ($\Rightarrow$) Suppose $T = \mathrm{Th}(A \times A)$. Then by Corollary 1.2 for any $\varphi' \in T'$, φ' is true in A iff φ is true in $A \times A$. Hence all elements of T' are true in A, and thus T' is consistent.

($\Leftarrow$) Suppose T' is consistent, and let A be any model of T'. Since, for each $\varphi \in T$, $\varphi' \in T'$ and, by Lemma 1.1, it follows that $A \times A$ is a model of T. Finally, since T is complete, $T = \mathrm{Th}(A \times A)$.

It should be noted that one can find many theories which satisfy the hypothesis of Lemma 3.6. One such example is in [24], which shows that there are complete theories T such that T' has several models which are not elementarily equivalent.

4. Final remarks. From the examples which have been looked at throughout this study it is observed that if A is n-model complete, then $A \times A$ is $(n+1)$-model complete. It seems unlikely that there is an example such that A is n-model complete and $A \times A$ is not k-model complete for all k. This motivates the following question:

QUESTION. Is there any ω-categorical theory which for all n is not Π_n-axiomatizable? **(P 1362)**

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