COLLOQUIUM MATHEMATICUM

VOL. LII | 1987 | FASC. 2

C^k-ESTIMATES FOR THE CAUCHY–RIEMANN EQUATIONS ON CERTAIN WEAKLY PSEUDOCONVEX DOMAINS

BY

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1. Introduction. It is already a classical result that the $\bar\partial$-equation $\bar\partial u = f$ has a solution on pseudoconvex domains in C^n if the necessary integrability condition $\bar\partial f = 0$ is satisfied. In C^1 such a solution can be written explicitly by means of the Cauchy transform. In 1969 Henkin [2] and Ramirez [6] constructed the analogue of the Cauchy kernel for strictly pseudoconvex domains. This result gave rise to a new approach for solving the $\bar\partial$-equation, developed by Grauert and Lieb [1] and Henkin [3]. During the last ten years several results regarding regularity of certain solutions were obtained by the use of solution operators defined by kernels. It turned out that on a strictly pseudoconvex domain D the equation $\bar\partial u = f$, where $\bar\partial f = 0$, with bounded f has a solution belonging to $\mathrm{Lip}_{1/2}(D)$ (see [4] and [8]). If $f \in C^k(D)$, then there is a k-times differentiable solution with derivatives of order k in $\mathrm{Lip}_{1/2}(D)$ (see [5] and [11]). The absence of strict pseudoconvexity makes the problem more complicated. The recent example of Sibony [10] shows a pseudoconvex domain in C^3 with smooth boundary, and a bounded (0, 1)-form f on D, $\bar\partial f = 0$, such that the equation $\bar\partial u = f$ has no bounded solution on D.

This is the reason to impose certain restrictions on pseudoconvex domains to be considered, so that one may obtain some regularity results for solutions of the $\bar\partial$-equation. It has been known that one can write a certain solution operator for $\bar\partial$ in the case of smooth convex domains. Pseudoconvex domains with real analytic boundary became also a point of interest.

This paper is devoted to a certain class of pseudoconvex domains in C^n. Namely, we define

$$D^{(m)} = \{z\colon\ r(z) < 0\} \quad \text{for } r(z) = \sum_{j=1}^{n} |z_j|^{2m_j} - 1.$$

Such domains were investigated in [8]. The main result of this paper is Theorem 4.1 which states that for $f \in C^k_{0q}(D^{(m)}) \cap C^0_{0q}(\overline{D^{(m)}})$ with $\|f\|_k < \infty$ and $\bar\partial f = 0$ there is a solution u of $\bar\partial u = f$ in $\mathrm{Lip}_{k+\theta}(D^{(m)})$ for $0 < \theta < 1/\max\{2m_j\}$. This result is obtained by constructing a certain solution operator for the $\bar\partial$-equation on domains $D^{(m)}$. One can notice that a similar solution operator can be defined on any convex set with smooth boundary. Moreover, if the boundary is real analytic, one may expect results similar to those presented for the domains $D^{(m)}$. In a particular case of C^2 it is quite easy to obtain such estimates.

2. Preliminaries. The main tool to obtain Hölder estimates is the following classical result:

LEMMA 2.1. *Let $D \subset \boldsymbol{R}^n$ be an open set with smooth (C^∞) boundary and defining function r, and let $0 < \theta < 1$. If $f \in C^1(D) \cap L^\infty(D)$, $A > 0$, and there is a neighbourhood W of $\mathrm{b}D$ such that for every $x \in W \cap D$*

$$|\operatorname{grad} f(x)| \leqslant A\,|r(x)|^{\theta-1},$$

then $f \in C^\theta(D)$. Moreover, there is a constant K which does not depend on f and satisfies the inequality $|f(x)-f(y)| \leqslant KA\,|x-y|^\theta$ for $x, y \in W \cap D$.

LEMMA 2.2. *If $D \subset \boldsymbol{R}^n$ is an open set with smooth boundary and defining function r, then for every neighbourhood U of $\bar{D}$ there are a $\delta_0 < 0$ and a family of linear operators E^δ: $C^0(\bar{D}_\delta) \to C^0_c(U)$, $\delta_0 < \delta \leqslant 0$, such that*

(a) *$E^\delta u|_{\bar{D}_\delta} = u$;*

(b) *for $k = 1, 2, \ldots$, if $u \in C^k(\bar{D}_\delta)$, then $E^\delta u \in C^k_c(U)$, and there are constants C_k independent of δ such that*

$$\|E^\delta u\|_{k,\boldsymbol{R}^n} \leqslant C_k\,\|u\|_{k,D_\delta};$$

(c) *for $u \in C^0(\bar{D})$,*

$$\lim_{\delta \to 0} E^\delta u|_{\bar{D}_\delta} = E^0 u$$

uniformly on $\boldsymbol{R}^n$.

For the proof see [5].

LEMMA 2.3. *Let $0 \leqslant s \leqslant n-2$, $x = (x_1, \ldots, x_{2n-2}) \in \boldsymbol{R}^{2n-2}$, $y, t \in \boldsymbol{R}$, $L > 0$, $M \in \boldsymbol{N}$. For every $A > 0$ and $0 < \theta < 1/M$ there are constants C_i ($i = 1, 2, 3$) independent of L such that*

$$I_1 = \int_0^A \int_0^A \int_{|x| \leqslant A} \frac{dx\,dy\,dt}{|x|^{2s+1}(y+t+L+|x|^M)^3 \prod_{j=1}^{n-s-2} (L+x_{2j-1}^2+x_{2j}^2)} \leqslant C_1 L^{\theta-1},$$

$$I_2 = \int_0^A \int_0^A \int_{|x| \leqslant A} \frac{dx\,dy\,dt}{(y+|x|)^{2s+2}(t+L+|x|^M)^2 \prod_{j=1}^{n-s-2} (L+x_{2j-1}^2+x_{2j}^2)} \leqslant C_2 L^{\theta-1},$$

$$I_3 = \int_0^A \int_0^A \int_{|x| \leqslant A} \frac{dx\,dy\,dt}{(y+t+|x|)^{2s+3}(L+|x|^M) \prod_{j=1}^{n-s-2} (L+x_{2j-1}^2+x_{2j}^2)} \leqslant C_3 L^{\theta-1}.$$

Proof. First we estimate

$$I = \int\limits_{|x| \leqslant A} \frac{dx}{|x|^{2s+1}(L+|x|^M) \prod\limits_{j=1}^{n-s-2} (L+x_{2j-1}^2+x_{2j}^2)}.$$

Let $x' = (x_{2n-2s-3}, \ldots, x_{2n-2}) \in R^{2s+2}$. Then

$$I \leqslant \int\limits_{|x| \leqslant A} \frac{dx}{|x'|^{2s+1}(L+|x'|^M) \prod\limits_{j=1}^{n-s-2} (L+x_{2j-1}^2+x_{2j}^2)}$$

$$\leqslant \int\limits_{|x'| \leqslant A} \frac{dx'}{|x'|^{2s+1}(L+|x'|^M)} \prod_{j=1}^{n-s-2} \int\limits_{-A}^{A}\int\limits_{-A}^{A} \frac{dx_{2j-1}\,dx_{2j}}{L+x_{2j-1}^2+x_{2j}^2}.$$

Notice that for every δ $(0 < \delta < 1)$

$$L+x_{2j-1}^2+x_{2j}^2 \geqslant (x_{2j-1}^2+x_{2j}^2)^{1-\delta} L^\delta.$$

Therefore

$$\int\limits_{-A}^{A}\int\limits_{-A}^{A} \frac{dx_{2j-1}\,dx_{2j}}{L+x_{2j-1}^2+x_{2j}^2} \leqslant \int\limits_{-A}^{A}\int\limits_{-A}^{A} \frac{dx_{2j-1}\,dx_{2j}}{(x_{2j-1}^2+x_{2j}^2)^{1-\delta}} L^{-\delta} \leqslant \text{const}\, L^{-\delta}.$$

Moreover,

$$\int\limits_{|x'| \leqslant A} \frac{dx'}{|x'|^{2s+1}(L+|x'|^M)} = \int\limits_0^A \frac{r^{2s+1}\,dr}{r^{2s+1}(L+r^M)} = \int\limits_0^A \frac{dr}{L+r^M} \leqslant \text{const}\, L^{-1+1/M}.$$

Hence

$$I \leqslant \text{const}\, L^{-\delta(n-s-2)} L^{-1+1/M}.$$

Choosing δ so that $-\delta(n-s-2)+1/M = \theta$ we obtain $I \leqslant \text{const}\, L^{\theta-1}$. Now it is enough to notice that $I_i \leqslant I$ for $i = 1, 2, 3$. In all three cases this is implied by Fubini's Theorem and by the following inequalities:

$$\int\limits_0^A\int\limits_0^A \frac{dy\,dt}{(y+t+L+|x|^M)^3} \lesssim \int\limits_0^A \frac{dt}{(t+L+|x|^M)^2} \lesssim \frac{1}{L+|x|^M},$$

$$\int_0^A\int_0^A \frac{dy\,dt}{(y+|x|)^{2s+2}(t+L+|x|^M)^2} \lesssim \frac{1}{(L+|x|^M)|x|^{2s+1}},$$

$$\int_0^A\int_0^A \frac{dy\,dt}{(t+y+|x|)^{2s+3}} \lesssim \frac{1}{|x|^{2s+1}}.$$

From the proof of Lemma 2.3 we obtain immediately the following

LEMMA 2.4. *For every $A > 0$, $M \in N$, and $L > 0$ there are constants C_i $(i = 1, 2, 3)$ not depending on L such that*

(a)
$$\int_0^A\int_0^A\int_{|x|<A} \frac{dx_1\,dx_2\,dy\,dt}{|x|(y+t+L+|x|^M)^3} \leqslant C_1 L^{-1+1/M},$$

(b)
$$\int_0^A\int_0^A\int_{|x|<A} \frac{dx_1\,dx_2\,dy\,dt}{(y+|x|)^2(t+L+|x|^M)^2} \leqslant C_2 L^{-1+1/M},$$

(c)
$$\int_0^A\int_0^A\int_{|x|<A} \frac{dx_1\,dx_2\,dy\,dt}{(t+y+|x|)^3(L+|x|^M)} \leqslant C_3 L^{-1+1/M}.$$

3. A solution operator for smooth convex domains. Let $D \subset C^n$ be a convex domain with smooth defining function r. Assume there is a $\delta_0 > 0$ such that for $|\delta| \leqslant \delta_0$ domains $\{\xi\colon r(\xi) < \delta\}$ are convex. Notice that in the case where D has real analytic boundary there is a defining function r satisfying the above condition.

For $1 \leqslant j \leqslant n$ put

$$P_j^1(\xi) = \frac{\partial r}{\partial \xi_j}(\xi), \qquad \Phi(\xi, z) = \sum_{j=1}^n \frac{\partial r}{\partial \xi_j}(\xi)(\xi_j - z_j),$$

$$w_j^1(\xi, z) = \frac{P_j^1(\xi)}{\Phi(\xi, z)} \quad \text{on } \{(\xi, z)\colon \Phi(\xi, z) \neq 0\},$$

$$P_j^0(\xi, z) = \bar{\xi}_j - \bar{z}_j, \qquad \Phi^0(\xi, z) = |\xi - z|^2,$$

$$w_j^0(\xi, z) = \frac{P_j^0(\xi, z)}{\Phi^0(\xi, z)} \quad \text{for } \xi \neq z.$$

For $\lambda \in I$ let

$$w_j(\xi, z, \lambda) = (1-\lambda)\, w_j^0(\xi, z) + \lambda w_j^1(\xi, z).$$

Let G be a neighbourhood of $\bar{D}$ such that $\bar{G} \subset \{\xi\colon r(\xi) < \delta_0\}$ and let $K = \bar{G} \setminus D$. Notice that w_j are well defined on $K \times D \times I$.

Let $E\colon C^0(\bar{D}) \to C_c^0(G)$ be a linear operator such that

(a) $Eu|_{\bar{D}} = u$;

(b) if $u \in C^k(\bar{D})$, then $Eu \in C_c^k(G)$, $k = 1, 2, \ldots, \infty$, and there are constants C_k such that $\|Eu\|_{k,C^n} \leqslant C_k \|u\|_{k,D}$.

The existence of such an E follows from Lemma 2.2.

For $q = 0, 1, \ldots, n-1$ and $(\xi, z, \lambda) \in K \times D \times I$ let

$$K_q(\xi, z, \lambda) = (-1)^q \binom{n-1}{q} \det(w, \underbrace{\bar{\partial}_z w, \ldots, \bar{\partial}_z w}_{q}, \underbrace{\bar{\partial}_{\xi\lambda} w, \ldots, \bar{\partial}_{\xi\lambda} w}_{n-q-1}) \wedge d\xi_1 \wedge \ldots \wedge d\xi_n.$$

Let $K_n(\xi, z, \lambda) = K_{-1}(\xi, z, \lambda) = 0$. Put $B_{nq}(\xi, z) = K_q(\xi, z, 0)$; it is a well-defined locally integrable form. Since

$$\sum_{j=1}^{n} w_j(\xi, z, \lambda)(\xi_j - z_j) = 1,$$

it follows from properties of det that $\bar{\partial}_{\xi,\lambda} K_q = (-1)^q \bar{\partial}_z K_{q-1}$ for $0 \leqslant q \leqslant n$.

The following lemma is proved in [8]:

LEMMA 3.1 (the Bochner–Martinelli–Koppelman formula). *For $u \in C_{0q}^1(\bar{D})$ and $z \in D$*

$$u(z) = c_n \Big(\int_{bD} u(\xi) \wedge B_{nq}(\xi, z) - \int_D \bar{\partial} u(\xi) \wedge B_{nq}(\xi, z) - \bar{\partial}_z \int_D u(\xi) \wedge B_{nq-1}(\xi, z) \Big),$$

where

$$c_n = \frac{(-1)^{n(n-1)/2}}{(2\pi i)^n}.$$

Definition 3.1. For $u \in C_{0q}^0(\bar{D})$, $1 \leqslant q \leqslant n$, and $z \in D$ define

$$T_q u(z) = c_n \int_{bD \times I} u(\xi) \wedge K_{q-1}(\xi, z, \lambda) - c_n \int_D u(\xi) \wedge B_{nq-1}(\xi, z).$$

It is well known (see [8]) that T_q is a solution operator for the $\bar{\partial}$-equation; namely, if $u \in C_{0q}^1(\bar{D})$, $1 \leqslant q \leqslant n$, $\bar{\partial} u = 0$, then $\bar{\partial} T_q u = u$ on D.

Definition 3.2. For $1 \leqslant q \leqslant n$, $u \in C_{0q}^0(\bar{D})$, and $z \in D$ define

$$S_q u(z) = c_n \Big[\int_{bD \times I} u(\xi) \wedge K_{q-1}(\xi, z, \lambda) - \int_D u(\xi) \wedge B_{nq-1}(\xi, z) - \int_{K \times \{1\}} Eu(\xi) \wedge K_{q-1}(\xi, z, \lambda) - \bar{\partial}_z \int_{K \times I} Eu(\xi) \wedge K_{q-2}(\xi, z, \lambda) \Big].$$

LEMMA 3.2. *If* $u \in C^1_{0q}(\bar{D})$, $\bar{\partial}u = 0$ *on* D, *then* $\bar{\partial}S_q u(z) = u(z)$ *for* $z \in D$. *Moreover, in this case*

$$S_q u(z) = c_n\Big[-\int_{K\times I}\bar{\partial}Eu(\xi)\wedge K_{q-1}(\xi, z, \lambda)-\int_G Eu(\xi)\wedge B_{nq-1}(\xi, z)\Big].$$

Proof. Notice that

$$S_q u(z) = T_q u(z) - c_n \int_{K\times\{1\}} Eu(\xi)\wedge K_{q-1}(\xi, z, \lambda) - c_n\bar{\partial}_z \int_{K\times I} Eu(\xi)\wedge K_{q-2}(\xi, z, \lambda).$$

Since T_q solves the $\bar{\partial}$-equation, we have

$$\bar{\partial}S_q u(z) = u(z) - \bar{\partial}_z c_n \int_{K\times\{1\}} Eu(\xi)\wedge K_{q-1}(\xi, z, \lambda).$$

The last term is equal to zero, since $K_{q-1}(\xi, z, 1) = 0$ for $q > 1$, and $K_{q-1}(\xi, z, 1)$ is holomorphic in z for $q = 1$. Hence S_q solves the $\bar{\partial}$-equation. Since $\mathrm{b}(K\times I) = \mathrm{b}G\times I - \mathrm{b}D\times I + K\times\{1\} - K\times\{0\}$, by Stokes' Theorem and properties of K_q we obtain

$$\begin{aligned}
&\int_{K\times I}\bar{\partial}Eu(\xi)\wedge K_{q-1}(\xi, z, \lambda) - \bar{\partial}_z\int_{K\times I} Eu(\xi)\wedge K_{q-2}(\xi, z, \lambda)\\
&= \int_{K\times I}\bar{\partial}Eu(\xi)\wedge K_{q-1}(\xi, z, \lambda) + \int_{K\times I} Eu(\xi)\wedge\bar{\partial}_{\xi\lambda}K_{q-1}(\xi, z, \lambda)\\
&= \int_{K\times I}\bar{\partial}_{\xi\lambda}[Eu(\xi)\wedge K_{q-1}(\xi, z, \lambda)] = \int_{\mathrm{b}(K\times I)} Eu(\xi)\wedge K_{q-1}(\xi, z, \lambda)\\
&= \int_{\mathrm{b}G\times I} Eu(\xi)\wedge K_{q-1}(\xi, z, \lambda) - \int_{\mathrm{b}D\times I} Eu(\xi)\wedge K_{q-1}(\xi, z, \lambda) +\\
&\quad + \int_{K\times\{1\}} Eu(\xi)\wedge K_{q-1}(\xi, z, \lambda) - \int_{K\times\{0\}} Eu(\xi)\wedge K_{q-1}(\xi, z, \lambda).
\end{aligned}$$

Since $Eu = 0$ on $\mathrm{b}G$, we have

$$\int_{\mathrm{b}G\times I} Eu(\xi)\wedge K_{q-1}(\xi, z, \lambda) = 0,$$

and the result follows immediately.

To obtain Hölder estimates for derivatives of order k we consider the cases $k = 0$ and $k \geqslant 1$ separately.

In the case $k = 0$ we make use of the form of $S_q u$ given by the definition.

For $k \geqslant 1$, i.e., for $u \in C^k_{0q}(\bar{D})$, $k \geqslant 1$, we use Lemma 3.2. Since it is a classical result that for every θ $(0 < \theta < 1)$

$$\Big\|\int_G Eu(\xi)\wedge B_{nq-1}(\xi, z)\Big\|_{k+\theta} \leqslant \mathrm{const}\,\|u\|_k,$$

it remains to estimate only the term

$$\int_{K\times I} \bar{\partial}Eu(\xi)\wedge K_{q-1}(\xi, z, \lambda).$$

In a next step we write the above integral as a linear combination of terms which are easier to be estimated.

From the definition it follows that K_{q-1} is a linear combination of the terms

$$\lambda^i(1-\lambda)^j \det\big(w^p, \underbrace{\bar{\partial}_z w^0,\ldots, \bar{\partial}_z w^1}_{q-1}, (w^1-w^0)d\lambda, \underbrace{\bar{\partial}_\xi w^0,\ldots, \bar{\partial}_\xi w^1}_{n-q-1}\big) \wedge d\xi_1\wedge\ldots\wedge d\xi_n,$$

$$\lambda^i(1-\lambda)^j \det(w^p, \underbrace{\bar{\partial}_z w^0,\ldots, \bar{\partial}_z w^1}_{q-1}, \underbrace{\bar{\partial}_\xi w^0,\ldots, \bar{\partial}_\xi w^1}_{n-q}) \wedge d\xi_1\wedge\ldots\wedge d\xi_n,$$

where in the first term $i+j = n-1$ and $p = 0, 1$, and in the second one $i+j = n$ and $p = 0, 1$. After multiplying by $\bar{\partial}Eu$ and integrating over λ, integrals involving terms of the second type vanish, and the integral $\int_{K\times I} \bar{\partial}Eu\wedge K_{q-1}$ is a linear combination of the terms

$$\int_K \bar{\partial}Eu \det(w^0, w^1, \bar{\partial}_z w^0,\ldots, \bar{\partial}_z w^1, \bar{\partial}_\xi w^0,\ldots, \bar{\partial}_\xi w^1)\wedge d\xi_1\wedge\ldots\wedge d\xi_n.$$

Notice that for $0\leqslant s\leqslant n-2$

$$\det(w^0, \underbrace{\bar{\partial}_z w^0,\ldots, \bar{\partial}_\xi w^0}_{s}, w^1, \underbrace{\bar{\partial}_z w^1,\ldots, \bar{\partial}_\xi w^1}_{n-s-2}) = \frac{\det(P^0, \bar{\partial}_z P^0,\ldots, \bar{\partial}_\xi P^0, P^1, \bar{\partial}_z P^1,\ldots, \bar{\partial}_\xi P^1)}{(\Phi^0)^{s+1}\Phi^{n-s-1}}.$$

Let

$$\bar{\partial}Eu(\xi) = \sum_{|J|=q+1} f_J(\xi)d\bar{\xi}^J.$$

Then $f_J = 0$ on D and $f_J\in C_c^{k-1}(G)$. If $\{U_j\}_{j=1}^m$ is a finite covering of K and $\{\chi_j\}_{j=1}^m$ are C^∞-functions such that

$$\sum_{j=1}^m \chi_j = 1 \text{ on } K \quad\text{and}\quad \operatorname{supp}\chi_j\subset\subset U_j,$$

then in order to estimate

$$D_z^\alpha \int_{K\times I} \bar{\partial}Eu(\xi)\wedge K_{q-1}(\xi, z, \lambda) \lesssim \|u\|_k |r(z)|^{\theta-1} \quad \text{for } |\alpha|\leqslant k+1$$

it is enough to show that for every j and J

$$\left| D_z^\alpha \int_{U_j} \chi_j f_j \, d\bar{\xi}^J \frac{\det(P^0, \bar{\partial}_z P^0, \ldots, \bar{\partial}_\xi P^0, P^1, \bar{\partial}_z P^1, \ldots, \bar{\partial}_\xi P^1)}{(\Phi^0)^{s+1} \Phi^{n-s-1}} \wedge d\xi_1 \wedge \ldots \wedge d\xi_n \right|$$

$$\lesssim \|\chi_j f_J\|_{k-1} |r(z)|^{\theta-1}, \quad 0 \leqslant s \leqslant n-2.$$

To simplify the notation let $v(\xi) = \chi_j(\xi) f_J(\xi)$. Then $v \in C_c^{k-1}(U_j)$, and $v = 0$ on $U_j \cap D$.

LEMMA 3.3. *There are a finite covering $\{U_j\}$ of K and $a > 0$ such that for every j there is v_j with $|\partial\Phi/\partial\xi_{v_j}| \geqslant a$ on $U_j \times U_j$.*

Proof. Since

$$\frac{\partial\Phi}{\partial\xi_i}(\xi, z) = \sum_{j=1}^n \frac{\partial^2 r}{\partial\xi_i \partial\xi_j}(\xi)(\xi_j - z_j) + \frac{\partial r}{\partial\xi_i}(\xi),$$

we have

$$\frac{\partial\Phi}{\partial\xi_i}(\xi_0, \xi_0) = \frac{\partial r}{\partial\xi_i}(\xi_0) \quad \text{for every } \xi_0 \in K,$$

and therefore

$$\frac{\partial\Phi}{\partial\xi_{v_{\xi_0}}}(\xi_0, \xi_0) \neq 0 \quad \text{for some } v_{\xi_0},$$

hence also in some neighbourhood of (ξ_0, ξ_0). The result follows from compactness of K.

PROPOSITION 3.1. *Assume $|\partial\Phi/\partial\xi_v| \geqslant a$ on $U \times U$. Then for $z \in U$ and $2 \leqslant |\alpha| \leqslant k+1$, $0 \leqslant s \leqslant n-2$ the expression*

$$D_z^\alpha \int_U v(\xi) \frac{(\bar{\xi}_0 - \bar{z}_0) \dfrac{\partial r}{\partial\xi_1}(\xi) \dfrac{\partial^2 r}{\partial\xi_2 \partial\bar{\xi}_2}(\xi) \ldots \dfrac{\partial^2 r}{\partial\xi_{n-s-1} \partial\bar{\xi}_{n-s-1}}(\xi)}{(\Phi^0)^{s+1} \Phi^{n-s-1}} d\xi$$

can be written as a linear combination of the terms

$$\int_U D^\beta v(\xi) \frac{N_j(\xi, z) \psi(\xi)}{|\xi - z|^p \Phi^{n-s-1+m}} \varphi(\xi) \, d\xi,$$

where

$$\psi(\xi) = \begin{cases} \displaystyle\prod_{j=2}^{n-s-1} \frac{\partial^2 r}{\partial\xi_j \partial\bar{\xi}_j}(\xi) & \text{if } v \notin \{2, \ldots, n-s-1\}, \\ \displaystyle\frac{\partial r}{\partial\xi_1}(\xi) \prod_{\substack{j=2 \\ j \neq v}}^{n-s-1} \frac{\partial^2 r}{\partial\xi_j \partial\bar{\xi}_j}(\xi) & \text{if } v \in \{2, \ldots, n-s-1\}, \end{cases}$$

$0 \leqslant |\beta| \leqslant |\alpha|-2$, $\varphi(\xi)$ *is a bounded* C^∞-*function*, $0 \leqslant m \leqslant 2$, *and* $N_j(\xi, z)$*'s are products of j-factors of the form* $(z_i-\xi_i)$ *or* $(\bar{z}_i-\bar{\xi}_i)$ *with*

$$\frac{|N_j(\xi, z)|}{|\xi-z|^p} \leqslant \frac{1}{|z-\xi|^{2s+q+1}}, \qquad (|\alpha|-|\beta|)-(m+q) \geqslant 0.$$

Proof. Induction on $|\alpha|$. Let ∂_i denote either $\partial/\partial z_i$ or $\partial/\partial \bar{z}_i$. Notice that

$$\partial_i\left(\frac{1}{\Phi^q}\right) = A_q \frac{\partial_i \Phi}{\Phi^{q+1}} \quad \text{and} \quad \partial_i \frac{1}{|z-\xi|^q} = B_q \frac{N_1(\xi, z)}{|z-\xi|^{q+2}}.$$

1. Let $|\alpha| = 2$. One can easily calculate that the term

$$\partial_i\,\partial_j \frac{\bar{\xi}_0-\bar{z}_0}{|z-\xi|^{2s+2}\,\Phi^{n-s-1}}$$

is a linear combination of terms of the required form.

2. Assume that the proposition holds for $2 \leqslant |\alpha| \leqslant k+1$ and consider

$$\partial_j \int_U D^\beta v(\xi) \frac{N_i(\xi, z)\psi(\xi)}{|z-\xi|^p\,\Phi^{n-s-1+m}}\varphi(\xi)\,d\xi.$$

Notice that this expression can be written as a linear combination of terms of the forms

$$\int_U D^\beta v(\xi) \frac{N_{i-1}(\xi, z)\psi(\xi)}{|z-\xi|^p\,\Phi^{n-s-1+m}}\varphi(\xi)\,d\xi,$$

$$\int_U D^\beta v(\xi) \frac{N_{i+1}(\xi, z)\psi(\xi)}{|z-\xi|^{p+2}\,\Phi^{n-1-s+m}}\varphi(\xi)\,d\xi,$$

$$\int_U D^\beta v(\xi) \frac{N_i(\xi, z)\psi(\xi)}{|z-\xi|^p\,\Phi^{n-s-1+m+1}}\varphi(\xi)\,d\xi.$$

All terms with the exception of the third one for $m = 2$ are in the required form. Since

$$\frac{1}{\Phi^{n-s+2}} = \frac{\partial}{\partial \xi_\nu}\left(\frac{1}{\Phi^{n-s+1}}\right)\left(\frac{\partial \Phi}{\partial \xi_\nu}\right)^{-1}$$

after integrating by parts and using the fact that v has a compact support contained in U, i.e., all boundary integrals involving v vanish, we obtain

$$\int_U D^\beta v(\xi) \frac{N_i(\xi, z)\psi(\xi)}{|z-\xi|^p\,\Phi^{n-s+2}}\varphi(\xi)\,d\xi$$

$$= \int\limits_U D^\beta v(\xi) \frac{N_i(\xi, z)\psi(\xi)}{|z-\xi|^p} \varphi(\xi) \left(\frac{\partial \Phi}{\partial \xi_\nu}\right)^{-1} \frac{\partial}{\partial \xi_\nu} \left(\frac{1}{\Phi^{n-s+1}}\right) d\xi$$

$$= -\int\limits_U \frac{\partial}{\partial \xi_\nu} \left[\frac{D^\beta v(\xi) N_i(\xi, z) \varphi(\xi) (\partial\Phi/\partial\xi_\nu)^{-1}}{|z-\xi|^p}\right] \psi(\xi) \frac{1}{\Phi^{n-s+1}} d\xi.$$

One can easily see that the resulting integral has the required form, and so the proof is complete.

The following lemma can be shown by standard arguments [3].

LEMMA 3.4. *There is a neighbourhood W of* bD *such that if $z \in W$ and*

$$\frac{\partial r}{\partial \xi_n}(z) \neq 0,$$

then the functions $(x_1, \ldots, x_{2n-2}, y, t)$ form a set of real coordinates in some neighbourhood of z, where

$$y(\xi) = \operatorname{Im} \Phi(\xi, z), \qquad t(\xi) = r(\xi),$$

$$x_{2j}(\xi) = \operatorname{Im}(\xi_j - z_j), \qquad x_{2j-1}(\xi) = \operatorname{Re}(\xi_j - z_j), \qquad j = 1, \ldots, n-1.$$

4. Hölder estimates for derivatives of the solution operator on domains $D^{(m)}$. Let

$$r(z) = \sum_{j=1}^n |z_j|^{2m_j} - 1, \qquad m_j \geqslant 1,$$

$$M = \max\{2m_j \colon j = 1, \ldots, n\}, \qquad D = \{z \colon r(z) < 0\}.$$

Notice that D is convex.

The following inequality is shown in [7]: There is a constant $C > 0$ such that for $|\xi - z| < 1$

(1)
$$r(z) - r(\xi) - 2\operatorname{Re}\left[\sum_{j=1}^n \frac{\partial r}{\partial \xi_j}(\xi)(\xi_j - z_j)\right] \geqslant C\left[\sum_{j=1}^n \frac{\partial^2 r}{\partial \xi_j \partial \bar{\xi}_j}(\xi) |\xi_j - z_j|^2 + |\xi - z|^M\right].$$

Let Φ be defined as in Section 3 and for a neighbourhood G of $\bar{D}$ put $K = \bar{G} \setminus D$.

The following lemma can be obtained immediately from inequality (1).

LEMMA 4.1. *If $z, \xi \in C^n$, $|z - \xi| < 1$, and $r(z) \leqslant r(\xi)$, then*

$$|\Phi(\xi, z)| \gtrsim |\operatorname{Im} \Phi(\xi, z)| + r(\xi) - r(z) + \sum_{j=1}^n \frac{\partial^2 r}{\partial \xi_j \partial \bar{\xi}_j}(\xi) |\xi_j - z_j|^2 + |\xi - z|^M.$$

COROLLARY 4.1. *For every $R > 0$ there is a constant $C > 0$ such that for*

$\xi, z \in B(0, R)$ with $|\xi - z| < 1$ and $r(z) \leqslant r(\xi)$ the following inequality holds:

$$\left|\frac{\dfrac{\partial^2 r}{\partial \xi_j \partial \bar{\xi}_j}(\xi)}{\Phi(\xi, z)}\right| \leqslant C \frac{1}{r(\xi) - r(z) + |\xi_j - z_j|^2}.$$

Proof. Since

$$\frac{\partial^2 r}{\partial \xi_j \partial \bar{\xi}_j} = m_j^2 |\xi_j|^{2(m_j - 1)},$$

from Lemma 4.1 we obtain

$$\left|\frac{\partial^2 r/\partial \xi_j \partial \bar{\xi}_j}{\Phi(\xi, z)}\right| \leqslant \text{const} \frac{m_j^2 |\xi_j|^{2(m_j-1)}}{r(\xi) - r(z) + \sum_{i=1}^{n} m_i^2 |\xi_i|^{2(m_i - 1)} |\xi_i - z_i|^2}$$

$$\lesssim \frac{1}{[r(\xi) - r(z)]/m_j^2 |\xi_j|^{2(m_j - 1)} + |\xi_j - z_j|^2} \lesssim \frac{1}{r(\xi) - r(z) + |\xi_j - z_j|^2}.$$

Remark 4.1. For $z \in B(0, R)$ we have

$$\frac{\partial r}{\partial z_j}(z) = |m_j (z_j \bar{z}_j)^{m_j - 1} \bar{z}_j| \leqslant R m_j^2 (z_j \bar{z}_j)^{m_j - 1} = R \frac{\partial^2 r}{\partial z_j \partial \bar{z}_j}(z).$$

Therefore

$$|\psi(\xi)| \lesssim \frac{\partial^2 r}{\partial \xi_1 \partial \bar{\xi}_1}(\xi) \ldots \frac{\partial^2 r}{\partial \xi_{n-s-2} \partial \bar{\xi}_{n-s-2}}(\xi),$$

where ψ is as in Proposition 3.1.

This remark together with the proof of the next lemma explain why two different forms of ψ in Proposition 3.1 are used. Namely, in both cases ψ can be estimated by the product of $n-s-2$ factors of the form $\partial^2 r/\partial \xi_j \partial \bar{\xi}_j$. In the form of the integral appearing in the assumptions of Proposition 3.1 there are $n-s-2$ such factors, but one of them may disappear when differentiation by parts in the "normal" direction is performed. In this case it is compensated by the additional term $\partial r/\partial \xi_1$.

Now, let $U \subset G$ be an open set such that for every $z \in U$ the functions $r(\xi)$, $\operatorname{Im} \Phi(\xi, z)$, $\operatorname{Im}(\xi_j - z_j)$, $\operatorname{Re}(\xi_j - z_j)$ $(j = 1, \ldots, n,\ j \neq j_0)$ form coordinates on U, and $|\xi - z| < 1$ for $\xi, z \in U$. From Lemma 3.4 it follows that bD can be covered by such sets.

LEMMA 4.2. *If $v \in C_c^{k-1}(U)$, $v = 0$ on $D \cap U$, then for $z \in U \cap D$*

$$\left|\int_U D^\beta v(\xi) \frac{\psi(\xi)}{|z - \xi|^{2s+p+1} \Phi^{n-s-1+m}} d\xi\right| \leqslant \text{const} \|v\|_{k-1} |r(z)|^{\theta - 1},$$

where $0 < \theta < 1/M$, $0 \leqslant m \leqslant 2$, $(k-1-|\beta|)-(p+m) \geqslant -2$, $|\beta| \leqslant k-1$, $0 \leqslant s \leqslant n-2$.

Proof. Notice that $|D^\beta v(\xi)| \leqslant \text{const}\,\|v\|_{k-1}\,|\xi - z|^{k-1-|\beta|}$, since $z \in U \cap D$ and $v = 0$ on $D \cap U$. Therefore

$$\left|D^\beta v(\xi) \frac{\psi(\xi)}{|z-\xi|^{2s+p+1}\,\Phi^{n-s-1+m}}\right| \leqslant \text{const}\,\|v\|_{k-1} \left|\frac{\psi(\xi)}{|z-\xi|^{2s+p+1-(k-1-\beta)}\,\Phi^{n-s-1+m}}\right|$$
$$\leqslant \text{const}\,\|v\|_{k-1} \frac{|\psi(\xi)|}{|z-\xi|^{2s+3-m}\,|\Phi^{n-s-1+m}|}.$$

Thus it remains to estimate the integrals

$$\int\limits_{K \cap U} \frac{|\psi(\xi)|}{|z-\xi|^{2s+m}\,|\Phi^{n-s+2-m}|}\,d\xi, \qquad m = 1, 2, 3, \tag{2}$$

by $\text{const}\,|r(z)|^{\theta-1}$.

On $K \cap U$ we have the following estimates for $z \in D \cap U$:

$$|\psi(\xi)| \lesssim \frac{\partial^2 r}{\partial \xi_1\, \partial \bar{\xi}_1}(\xi) \ldots \frac{\partial^2 r}{\partial \xi_{n-s-2}\, \partial \bar{\xi}_{n-s-2}}(\xi),$$

$$\left|\frac{\psi(\xi)}{\Phi^{n-s-2}}\right| \lesssim \prod_{j=1}^{n-s-2} \frac{1}{r(\xi)-r(z)+|z_j-\xi_j|^2} \leqslant \prod_{j=1}^{n-s-2} \frac{1}{|r(z)|+|z_j-\xi_j|^2},$$

$$\frac{1}{|\Phi|} \lesssim \frac{1}{|\operatorname{Im} \Phi| + |r(z)| + r(\xi) + |z-\xi|^M}.$$

Therefore, integrals of the form (2) can be estimated by

$$\int\limits_{K \cap U} \big[|z-\xi|^{2s+1}\big(|\operatorname{Im} \Phi(\xi, z)| + |r(z)| + r(\xi) + |z-\xi|^M\big)^3 \times \prod_{j=1}^{n-s-2} \big(|r(z)| + |\xi_j - z_j|^2\big)\big]^{-1}\, d\xi, \tag{3}$$

$$\int\limits_{K \cap U} \big[|z-\xi|^{2s+2}\big(|\operatorname{Im} \Phi(\xi, z)| + |r(z)| + r(\xi) + |z-\xi|^M\big)^2 \times \prod_{j=1}^{n-s-2} \big(|r(z)| + |\xi_j - z_j|^2\big)\big]^{-1}\, d\xi, \tag{4}$$

$$\int\limits_{K \cap U} \big[|z-\xi|^{2s+3}\big(|\operatorname{Im} \Phi(\xi, z)| + |r(z)| + r(\xi) + |z-\xi|^M\big) \times \prod_{j=1}^{n-s-2} \big(|r(z)| + |\xi_j - z_j|^2\big)\big]^{-1}\, d\xi. \tag{5}$$

Introducing $y = \operatorname{Im}\Phi(\xi, z)$, $t = r(\xi)$, $x_{2j}(\xi) = \operatorname{Im}(\xi_j - z_j)$, $x_{2j-1}(\xi) = \operatorname{Re}(\xi_j - z_j)$ for $j = 1, \ldots, n-1$ (we can assume $j_0 = n$) as local coordinates on U, we obtain the following estimates of (3)–(5), respectively:

$$(3') \qquad \int_0^A \int_0^A \int_{|x|<A} \frac{dx\,dy\,dt}{|x|^{2s+1}\left(y+t+|r(z)|+|x|^M\right)^3 \prod_{j=1}^{n-s-2} \left(|r(z)|+x_{2j-1}^2+x_{2j}^2\right)},$$

$$(4') \qquad \int_0^A \int_0^A \int_{|x|<A} \frac{dx\,dy\,dt}{\left(y+|x|\right)^{2s+2}\left(t+|r(z)|+|x|^M\right)^2 \prod_{j=1}^{n-s-2} \left(|r(z)|+x_{2j-1}^2+x_{2j}^2\right)},$$

$$(5') \qquad \int_0^A \int_0^A \int_{|x|<A} \frac{dx\,dy\,dt}{\left(y+t+|x|\right)^{2s+3}\left(r(z)+|x|^M\right) \prod_{j=1}^{n-s-2} \left(|r(z)|+x_{2j-1}^2+x_{2j}^2\right)}.$$

By Lemma 2.3 all the above integrals can be estimated by const $|r(z)|^{\theta-1}$, and the proof is complete.

PROPOSITION 4.1. *If*

$$r(z) = \sum_{j=1}^{n} |z_j|^{2m_j} - 1,$$

$$M = \max\{2m_j\colon j = 1, \ldots, n\}, \qquad D = \{z\colon r(z) < 0\},$$

then for every $k \geqslant 1$ and $0 < \theta < 1/M$ there is a constant $C_{\theta k}$ such that, for every $u \in C_{0q}^k(D)$ with $\bar\partial u = 0$,

$$\|S_q u\|_{k+\theta} \leqslant C_{\theta k} \|u\|_k.$$

Proof. From Lemma 4.2 it follows that there is a neighbourhood W of bD such that

$$\|S_q u\|_{k+\theta, W\cap D} \leqslant \text{const} \|u\|_k.$$

The convexity of $D_\delta = \{z\colon r(z) < \delta\}$, $\delta > 0$, implies that $\Phi(\xi, z) \neq 0$ on $K \times (D \setminus W)$; hence $|\Phi(\xi, z)| \geqslant a > 0$ for some $a > 0$ on that compact set. Choosing a smaller $a > 0$ one can also assume that $|\xi - z| > a$ for $(\xi, z) \in K \times (D \setminus W)$. Hence, for $z \in D \setminus W$ the estimate is trivial.

PROPOSITION 4.2. *If $u \in C_{0q}^1(\bar D)$, $\bar\partial u = 0$ on D, $0 < \theta < 1/M$, then there is a constant $C > 0$ such that $\|S_q u\|_\theta \leqslant C\|u\|_\infty$.*

Proof. It is a known result [7] that

$$\Big\| \int_{bD\times I} u(\xi) \wedge K_{q-1}(\xi, z, \lambda) - \int_D u(\xi) \wedge B_{nq-1}(\xi, z) \Big\|_\theta \lesssim \|u\|_\infty.$$

Hence it is enough to show that for $q > 1$

$$\Big|D_z \bar{\partial}_z \int_{K\times I} Eu(\xi) \wedge K_{q-2}(\xi, z, \lambda)\Big| \lesssim \|Eu\|_\infty |r(z)|^{\theta-1}.$$

To see this it is enough to find a covering $\{U_j\}_{j=1}^m$ of bD such that for $v \in C_0^1(U_j)$ and $z \in U_j$ the following estimate holds:

$$\left|D_z^2 \int_{K\cap U_j} v(\xi) \frac{(\bar{\xi}_0 - \bar{z}_0)\dfrac{\partial r}{\partial \xi_1}(\xi)\dfrac{\partial^2 r}{\partial \xi_2 \partial \bar{\xi}_2}(\xi) \dots \dfrac{\partial^2 r}{\partial \xi_{n-s-1} \partial \bar{\xi}_{n-s-1}}(\xi)}{(\Phi^0)^{s+1} \Phi^{n-s-1}} d\xi\right|$$
$$\lesssim \|v\|_\infty |r(z)|^{\theta-1}.$$

After direct calculations one can easily see that this follows from Lemmas 2.3 and 3.4.

Let G be a neighbourhood of D. For $\delta \leqslant 0$ let $D_\delta = \{z\colon r(z) < \delta\}$ and let $E^\delta\colon C^0(\bar{D}_\delta) \to C_c^0(G)$ be a family of linear operators satisfying conditions of Lemma 2.2. For each $\delta < 0$ and $u \in C_{0q}^0(\bar{D}) \cap C_{0q}^1(D)$ with $\bar{\partial} u = 0$ define

$$S_q^\delta u(z) = c_n\Big[-\int_{K_\delta\times I} \bar{\partial} E^\delta u(\xi) \wedge K_{q-1}(\xi, z, \lambda) - \int_G E^\delta u(\xi) \wedge B_{nq-1}(\xi, z)\Big],$$

where $K_\delta = \bar{G} \setminus D_\delta$.

Notice that S_q^δ solves the $\bar{\partial}$-equation on D_δ and the Hölder estimates obtained for S_q also hold for operators S_q^δ. Moreover, basically from Lemma 4.1 it follows that all constants appearing in estimates can be chosen independently of δ for $\delta_0 < \delta < 0$.

LEMMA 4.3. *If $u \in C_{0q}^0(\bar{D}) \cap C_{0q}^1(D)$, $\bar{\partial} u = 0$ on D, then $S_q^\delta u(z)$ converges to $S_q u(z)$ for every $z \in D$. If in addition $u \in C_{0q}^k(D)$ for $k \geqslant 1$, then the same holds for all derivatives of $S_q^\delta u(z)$ of order not greater than k.*

Proof. Observe that

$$S_q^\delta u(z) = \begin{cases} T_q^\delta u(z) - c_n \displaystyle\int_{K_\delta\times\{1\}} E^\delta u(\xi) \wedge K_{q-1}(\xi, z, \lambda) & \text{for } q = 1,\\ T_q^\delta u(z) - c_n \bar{\partial}_z \displaystyle\int_{K_\delta\times I} E^\delta u(\xi) \wedge K_{q-2}(\xi, z, \lambda) & \text{for } q > 1.\end{cases}$$

Let $F \subset D$ be a compact set. Then $F \subset D_\eta$ for some $\eta < 0$, and for $\eta < \delta < 0$ we have

$$\Big\|\int_{D_\delta\setminus D_\eta} u \wedge B_{nq-1}\Big\|_{C^k(F)} \lesssim \|u\|_\infty \operatorname{vol}(D_\delta \setminus D_\eta).$$

Moreover, by Stokes' Theorem,

$$\int_{bD_\delta\times I} u \wedge K_{q-1} - \int_{bD_\eta\times I} u \wedge K_{q-1}$$
$$= \int_{(D_\delta\setminus D_\eta)\times I} d(u \wedge K_{q-1}) - \int_{(D_\delta\setminus D_\eta)\times\{1\}} u \wedge K_{q-1} - \int_{(D_\delta\setminus D_\eta)\times\{0\}} u \wedge K_{q-1}.$$

Therefore, taking the C^k-norm on F, we obtain

$$\|T_q^\delta u - T_q^\eta u\|_{C^k(F)} \lesssim (\|\bar{\partial} u\|_\infty + \|u\|_\infty)\,\mathrm{vol}(D_\delta \setminus D_\eta).$$

Since $E^\delta u$ converges uniformly to Eu, this implies the statement for $S_q^\delta u$. In fact, for fixed $z \in D$ there is a neighbourhood W of z such that $W \subset D_\delta$ for $\delta > \delta_0$. Therefore, for $\delta > \delta_0$ both $\Phi_\delta^0(\xi, z)$ and $\Phi_\delta(\xi, z)$ are bounded away from zero; hence they are convergent uniformly in ξ on K_δ.

THEOREM 4.1. *For $k \geqslant 1$ and $0 < \theta < 1/M$ there are constants $C_{k\theta} > 0$ such that if $u \in C_{0q}^0(\bar{D}) \cap C_{0q}^k(D)$ and $\bar{\partial} u = 0$ on D, then $\bar{\partial} S_q u = u$ and*

$$\|S_q u\|_{k+\theta} \leqslant C_{k\theta} \|u\|_k.$$

Proof. Let $|\alpha| = k$ and let v and v^δ be coefficients of $D^\alpha S_q u$ and $D^\alpha S_q^\delta u$, respectively. Fix $z, w \in D$ and $\varepsilon > 0$. Let $\delta < 0$ be so small that

$$|v(z) - v^\delta(z)| < \varepsilon \quad \text{and} \quad |v(w) - v^\delta(w)| < \varepsilon.$$

Then, by Proposition 4.2,

$$|v^\delta(w) - v^\delta(z)| \lesssim \|u\|_k |z - w|^\theta,$$

and therefore

$$|v(z) - v(w)| \leqslant |v^\delta(z) - v^\delta(w)| + 2\varepsilon \lesssim \|u\|_k |z - w|^\theta + 2\varepsilon.$$

Hence $\|S_q u\|_{k+\theta} \leqslant C_{k\theta} \|u\|_k$ for some $C_{k\theta}$.

Remark 4.2. In the case $n = 2$ one can obtain sharper results. Namely, applying in estimates Lemma 2.4 instead of Lemma 2.3 one can obtain Hölder estimates of order $k + 1/M$. This is the best possible value of θ (see [7]).

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Reçu par la Rédaction le 20.4.1982;
en version modifiée le 25.10.1982