COLLOQUIUM MATHEMATICUM

VOL. XXIV 1972 FASC. 2

Stefan Zubrzycki

(26. III. 1927–18. XII. 1968)

BY

B. KOPOCIŃSKI AND J. ŁUKASZEWICZ

Stefan Zubrzycki was born in Zawichost in Poland, in the neighbourhood of Sandomierz. In 1946 he entered the Wrocław University and in 1950 obtained master's degree in mathematics having written a diploma work on a mathematical treatment of the relief of an area as an erosion factor (see [8] (1)).

While as a student, he has become an active member of two seminars: on applications of mathematics under the guidance of H. Steinhaus and on ergodic theory and stochastic processes under the guidance of E. Marczewski and H. Steinhaus.

In December 1948, still a student, Zubrzycki was appointed an assistant lecturer in mathematics at the Wrocław University. Since 1949 he worked in the State Institute of Mathematics (later: the Institute of Mathematics of the Polish Academy of Sciences) where he passed all steps of scientific career up to the post of professor in mathematics. His doctor's degree he obtained in 1954 in the Institute on the ground of the dissertation [5] on some inequalities between moments of equivalent random variables (according to the definition by de Finetti). In 1965 Zubrzycki was appointed to the post of associate professor and head of the Department of Biological, Economical and Technical Applications in the Institute of Mathematics of the Polish Academy of Sciences. He also read lectures in mathematics at the Department of Mathematics, Physics and Chemistry of the Wrocław University and at the Wrocław School of Economics where he was leading the Chair of Mathematics (1958-1963). During the academic year 1964/65 he was a visiting professor at the University of Washington in Seattle (Washington, U. S. A.).

Stefan Zubrzycki was a disciple and later an associate of Professor Hugo Steinhaus. As such, he was one of the founders of the Wrocław centre for applications of mathematics. He is the author of 40 scientific

(1) Bibliography of Stefan Zubrzycki is published in this fascicle on p. 281-283. Numbers in square brackets refer to the papers in the bibliography.

publications — all of them concerning probability theory, mathematical statistics and various applications of mathematics. In his papers on applications one can clearly see both the profound understanding of practical problems and essential contributions to the mathematical theory and methods. It is difficult to decide what is more important: his mathematical results or practical solutions.

In a review which follows we shall fix our attention to some of the results obtained by Stefan Zubrzycki.

The problem of estimation gangue parameters. Far reaching results Stefan Zubrzycki obtained in the domain of estimation gangue parameters. He started research in this field in 1952 when on the initiative of Professor Hugo Steinhaus the Institute of Mathematics of the Polish Academy of Sciences established the co-operation with the Committee of Geological Sciences in the Academy. As a result of his research Zubrzycki published a series of papers ([11], [14], [15], [20], [21], [27], [30]) of both the theoretical and practical importance. To construct a mathematical model of a gangue he proposed to assume that at any point p of a given region D the gangue parameter in question, say the thickness of a lode, is a random variable $y(p)$. All random variables $y(p), p \in D$, form a plane stochastic process, which is assumed to be stationary, isotropic and continuous, i. e., having a constant expected value $E\big(y(p)\big) = m$, a constant variance $D^2\big(y(p)\big) = s^2$, and a coefficient of correlation $R\big(y(p), y(q)\big)$ between two random variables $y(p), y(q)$ which depends only on the distance $d = d(p, q)$ of the points p, q,

$$R\big(y(p), y(q)\big) = f(d),$$

where the covariance function $f(d)$ is continuous at 0.

To estimate the amount of ore, or some other resources in the region D, one has to estimate the integral

$$V(D) = \iint_D y(p)\,dp \tag{1}$$

or, which is equivalent, to estimate the average thickness of the lode

$$y = \frac{1}{|D|} V(D), \tag{2}$$

where $|D|$ stands for the area of the region D.

The values (1) and (2) are estimated by means of the test borings performed at some points of D. Test borings provide a set of observations $y(p_i)$, where p_i, $i = 1, 2, \ldots, n$, are points belonging to D. It is obvious that for the defined values the error of estimation depends on the choice of an estimator, on the arrangement of observation point in D, and on the gangue covariance function which represents the variability of the gangue parameter.

In an extensive paper [11], Stefan Zubrzycki studies, for a given set of observation points, the efficiency of various estimators in the class of linear functions of the performed measurements. He also discusses the problem of estimating the gangue covariance function on the ground of observations distorted by random errors. Using the measurements of zinc content in the Upper Silesia gangues, Zubrzycki determines the empirical covariance functions and gives their approximates by some theoretical ones. Of all those the best fitting was achieved for the exponential covariance function.

Making an assumption that the arithmetic mean of the measurements is used for an estimator of the mean thickness of the lode, he looks for the optimum arrangement of the observation points in the region D. Since in practice the covariance function of a stochastic process may be determined only with a limited accuracy, it seems natural to put the question as to whether in some classes of covariance functions one can compare the efficiency of the main sampling techniques: random, stratified and systematic sampling. Similar questions had been studied by many authors but only with respect to stochastic processes on a straight line. In this case, a rather general result is known: for stochastic processes with convex covariance functions the systematic sampling is the most efficient in the class of all arrangements which guarantee the expected number of observations in any given interval to be proportional to the length of the interval. For stochastic processes on the plane, Zubrzycki has proved [14] that the random sampling is less efficient than the stratified one and that it is not possible to decide as to whether the stratified samplings are better than the systematic ones even under the severe restrictions imposed upon the covariance function and the shape of the strata.

The effect of the pattern of regular networks of observations on the efficiency of sampling has been examined in the paper [20] published together with Dalenius and Hajek. To avoid the influence of the border of region D on the shape of the network of points at which observations are to be made, the authors consider the limiting properties of networks on the region extended to the whole plane. They compare networks $N = (p_1, p_2, \ldots)$ having fixed densities $g(N)$ defined by the formula

$$g(N) = \lim_{R\to\infty} \frac{n_R}{\pi R^2},$$

where n_R is the number of points of the network N inside the circle $K(0, R)$ with the centre 0 and the radius R. The efficiency of a given network is defined as the limiting variance of the estimator

$$s^2(N) = \lim_{R\to\infty} n_R D^2(\bar{y}_R),$$

where $\bar{y}_R$ is the mean of observations inside the circle $K(0, R)$.

The counter-examples shown in [20] prove that there are no generally optimum networks of observations. The pattern of an optimum network depends not only on the covariance function of the process but also on the density of the network. A conjecture was formulated that for the processes with exponential covariance function the network of regular triangles is the optimum one.

The series of papers on estimating gangue parameters was completed with the paper [27] on a probabilistic model of variability on the plane. Stefan Zubrzycki presents in it a description of sedimentary deposits with the help of a shifting integration process and gives the weight-function of an isotropic stationary process on the plane with an exponential covariance function.

Various biological applications. The first two papers ([1] and [2]) originated from the co-operation with anthropologists on the methods of classification of excavated sculls. Stefan Zubrzycki is the co-author of Wrocław taxonomy, a simple method of classification a collection of individuals and graphical representation of the similarities between the examined individuals. This method has found broad applications in various biological researches. One of the most interesting results of Wrocław taxonomy was the rejection of a hypothesis on star chains. Zubrzycki has proved [3] that there is no justification for a conjecture set forth by some astronomers on a specific tendency of stars to form chain-like configurations in the sky. Zubrzycki returned to problems in taxonomy in the paper [36] on the methods of discrimination by means of linear indices. This paper contains also a critical discusion of known methods of discrimination on the plane.

Paper [19], written together with two biologists, contains an analysis of blood groups data to test some genetical hypotheses on a chromosomal linkage of genes determining the Rh group system. A publication by Hubert Szczotka and Stefan Zubrzycki [23] is a report on the statistical check-up performed on empirical data collected in the anthropological survey of the population of Poland during the years 1955-1959. In this immense undertaking the two co-authors were responsible for the elimination of clerical, random and systematic errors. Another paper [28] contains practical advices concerning methods and estimating the precision of measurement of the moisture content in seed.

Two papers ([26] and [40]) concern the method of estimation the number of animals in a given population on the ground of data obtained by catching, marking, and catching anew. In [26] one can find a new proof of the theorem by Czen Pin (2) on minimax estimators of the popu-

(2) Czen Pin, *O minimaksowym estymatorze liczebności populacji* [*On the minimax estimate of the population size*; in Polish], Zastosowania Matematyki 6 (1962), p. 137-148.

lation size and an extension of former results by a discusion on the admissibility of minimax estimators.

The last paper by Stefan Zubrzycki [40], under a somewhat enigmatic title *Large sample analysis of a problem in ecology or the hunting for snails*, deals with a method of estimation the average number of individuals for a unit of the area of habitat, if the research is carried on in a limited area and one has to take into account both the mobility of snails and their ability of taking cover in the litter. The research on ecological problems of estimating the population size gave rise to the more general statistical study in the theory of estimation. In [31] Stefan Zubrzycki discusses the problem of estimation of the parameter $\beta = 1/\lambda$ in gamma distribution of the form

$$g(x \mid \lambda) = \frac{\lambda^s}{\Gamma(s)} x^{s-1} e^{-\lambda x}, \qquad x > 0,$$

given that the parameter λ fulfils the restriction $0 < \lambda \leqslant \lambda_0 < \infty$, and the problem of estimation of the parameter λ if $0 \leqslant \lambda_0 < \lambda < \infty$. Assuming the function $L(\beta, \beta') = (\beta - \beta')^2/\beta^2$ be a loss caused by the estimation β' of β in the first of the problems above, Zubrzycki proves that 1° the estimators of the form

$$\hat{\beta}(x) = \frac{x}{s+1} + b, \qquad \text{where } 0 \leqslant b \leqslant \frac{2}{\lambda_0(s+1)},$$

are the minimax ones; 2° they are the only minimax estimators in the class of linear estimators; 3° among them only the estimators for which

$$\frac{1}{\lambda_0(s+1)} \leqslant b \leqslant \frac{2}{\lambda_0(s+1)}$$

are admissible in the class of linear estimators. Similar results have been obtained in the second problem.

Quality control. In two papers ([6] and [7]) Stefan Zubrzycki deals with the efficiency of the methods of testing flowmeters. Considering both the systematic and the random error in readings of a flowmeter, he proposes some improvements on the methods used in practice. Another problem in quality control has been discussed in papers [12] and [13] written together with Hugo Steinhaus. Studying the problem of comparison between two Poisson processes, the authors extend previous observations ([3]) on the relation between the probability and the credibility.

([3]) J. Oderfeld, *On the dual aspect of sampling plans*, Colloquium Mathematicum 2 (1951), p. 89-97; K. Sarkadi, *On the rule of dualism concerning the Bayes probability limits of the fraction defective*, Alkalmazött Matematikai Intézetémek Közleményei II, Budapest 1953, p. 275-285.

Paper [25] refers to a series of works published in Applicationes Mathematicae (Zastosowania Matematyki) in 1959 on the best arrangement of a given number of observations inside a pack of a formless commodity to get the most precise estimate for the average value of the contents of a pack. For this purpose various assumptions had been made on the available information concerning the distribution of the attribute in question inside the solid figure filled by the commodity. Stefan Zubrzycki has systematized these questions and, for a wide class of functions describing distribution of the attribute, he has found minimax formulas for approximate integration which provide a solution of the problem.

Teaching, editorial, and organizing activities. Since 1948, the year of the foundation by Hugo Steinhaus of the seminar in applications of mathematics in the Wrocław Branch of the Institute of Mathematics of the Polish Academy of Sciences, Stefan Zubrzycki was one of its most active participants, and for the last three years of his life one of its co-chairmen. This seminar was (and still is) a meeting-place of mathematicians working in applied mathematics and specialists from various disciplines of science and technics as well as the management people. A number of students in mathematics have got at this seminar their first experience in research and applications. In the Institute, Stefan Zubrzycki led also some other research seminars, mainly in mathematical statistics, and read several courses in mathematics. He also read courses in mathematical statistics, analysis of variance, and experimental design at Wrocław University. His lectures and seminars were always precise and clear. From the teaching experience originated his book *Lectures in the theory of probability and mathematical statistics* [34], edited first as lecture-notes [28]. This was the first really modern book in probability in the Polish literature. Its author was rewarded a prize of the Minister of Education and now the book is going to be edited in English translation. From the lectures read on the Post-Graduate Study in Operations Research, Stefan Zubrzycki published lecture-notes in the theory of games and a chapter in the book [36].

Stefan Zubrzycki rendered good services to the Polish mathematics by his editorial work. He translated several mathematical books into Polish and was the editor of many others. For many years he worked as the secretary to the editorial board of Colloquium Mathematicum where he put much of his energy into management of the editorial office. Due to his profound knowledge in mathematics he could not only give editorial refinement to many papers published in this journal but also to enrich their contents by essential comments and suplements. He was also a member of the editorial board of Applicationes Mathematicae and the editor-in-chief of Biometrical Letters (Listy Biometryczne) — a journal of the Polish Biometrical Society. At the latter post he continued the work initiated by Julian Perkal in popularization of mathematical methods in biological research [32].

Stefan Zubrzycki was a right honest man and a good colleague. His unexpected death was shock and grief to all the mathematical community in Wrocław.

He passed away at the top of his scientific activity.

BIBLIOGRAPHY OF STEFAN ZUBRZYCKI

[1] (with K. Florek, J. Łukaszewicz, J. Perkal and H. Steinhaus) *Taksonomia wrocławska* [*On Wrocław taxonomy*], Przegląd Antropologiczny 17 (1951), p. 193-211.

[2] (with K. Florek, J. Łukaszewicz, J. Perkal and H. Steinhaus) *Sur la liaison et la division des points d'un ensemble fini*, Colloquium Mathematicum 2 (1951), p. 282-285.

[3] *O łańcuszkach gwiezdnych* [*On star chains*], Zastosowania Matematyki 1 (1954), p. 197-205.

[4] *Some inequalities between the moments of equivalent random variables*, Bulletin de l'Académie Polonaise des Sciences, Cl. III, 2 (1954), p. 59-61.

[5] *Les inégalités entre les moments des variables aléatoires équivalentes*, Studia Mathematica 14 (1954), p. 232-242.

[6] (with J. Oderfeld) *O sprawdzaniu wodomierzy* [*On testing flowmeters*], Zastosowania Matematyki 1 (1954), p. 125-137.

[7] *O optymalnej metodzie odbioru wodomierzy* [*On the optimal methods of water meter acceptance*], ibidem 2 (1955), p. 199-209.

[8] *Próba matematycznego ujęcia rzeźby terenu jako czynnika erozji* [*An attempt at mathematical treatment of relief as an erosion factor*], Roczniki Nauk Rolniczych, 71-F-1 (1955), p. 45-55; also in Zastosowania Matematyki 2 (1956), p. 390-398.

[9] (with J. Aczél) *Sur un problème de la théorie des nombres liè à la distribution binomiale*, Colloquium Mathematicum 4 (1956), p. 56-67.

[10] *On the game of Banach and Mazur*, ibidem 4 (1957), p. 227-229.

[11] *O szacowaniu parametrów złóż geologicznych* [*On estimating gangue parameters*], Zastosowania Matematyki 3 (1957), p. 105-153.

[12] (with H. Steinhaus) *On the comparison of two production processes and rule of dualism*, Colloquium Mathematicum 5 (1957), p. 103-115.

[13] (with H. Steinhaus) *O porównywaniu dwóch procesów produkcyjnych i zasadzie dualizmu* [*On comparing two production processes and on the principle of dualism*], Zastosowania Matematyki 3 (1958), p. 229-257.

[14] *Remarks on random, stratified and systematic sampling in a plane*, Colloquium Mathematicum 6 (1958), p. 251-264.

[15] *Szacowanie złóż jako zagadnienie statystyczne* [*Gangue estimation as a statistical problem*], Przegląd Geologiczny 9 (1959), p. 415-419.

[16] *Concerning Yule's characteristic of style*, Zastosowania Matematyki 4 (1959), p. 328-331.

[17] *OR czyli badanie operacji* [*What is operations research*], Wiadomości Matematyczne 4 (1961), p. 271-281.

[18] *Skrypt z rachunku prawdopodobieństwa i statystyki matematycznej* [*Lecture notes on the theory of probability and mathematical statistics*], nakładem Uniwersytetu Wrocławskiego, Wrocław 1961.

[19] (with A. Łuczkiewicz-Mularczykowa and D. Schlesinger) *The linkage of genes Dd and Ee based on observation of the Polish population*, Archiwum Immunologii i Terapii Doświadczalnej 9 (1961), p. 801-808.

[20] (with T. Dalenius and J. Hajek) *On plane sampling and related geometrical problems*, Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, vol. I, Berkeley and Los Angeles 1961, p. 125-150.

[21] *Uwaga o probabilistycznym opisie złóż osadowych* [*A remark on probabilistic description of sedimentary deposits*], Zastosowania Matematyki 6 (1962), p. 199-208.

[22] *Wzór rekurencyjny dla ilości partycji ograniczonych* [*A recursive formula for the number of restricted partitions*], ibidem 6 (1962), p. 231-234.

[23] (with H. Szczotka) *Kontrola kart pomiarowych* [*Check-up of measurement cards*], *Polskie zdjęcia antropologiczne w latach 1955-1959* [*Polish antropological survey during the years 1955-1959*], Warszawa-Wrocław 1962, p. 109-118.

[24] *Zadanie o jednoczesnym trwaniu kilku zjawisk* [*A problem concerning simultaneous duration of several phenomena*], Prace Matematyczne 7 (1962), p. 7-9.

[25] *Some approximate integration formulas of statistical interest*, Colloquium Mathematicum 11 (1963), p. 123-136.

[26] *O minimaksowym szacowaniu liczności populacji* [*On minimax estimation of population size*], Zastosowania Matematyki 7 (1963), p. 183-194.

[27] *Uwagi o modelu probabilistycznym zmienności na płaszczyźnie* [*Remarks on a probabilistic model of variability on a plane*], ibidem 7 (1963), p. 219-222.

[28] (with Z. Urbaniak) *Uwagi metodyczne o oznaczaniu wilgotności nasion* [*Methodical remarks on the measurement of moisture content*

in seed], Biuletyn Instytutu Hodowli i Aklimatyzacji Roślin 3 (1964), p. 61-65.

[29] *O niektórych pracach seminarium z zastosowań matematyki* [*On some research work of the seminar for applications of mathematics*], Zastosowania Matematyki 8 (1966), p. 267-281.

[30] (with A. Trembecki) *Statystyka matematyczna w zastosowaniach do zagadnień górniczych*, część I (skrypt wydany przez Akademię Górniczo-Hutniczą w Krakowie) [*Mathematical statistics and its application to the mining problems*, part I (lecture notes edited by the Academy for Mining and Metalurgy in Kraków)], Kraków 1966.

[31] *Explicit formulas for minimax admissible estimators in some cases of restrictions imposed on the parameter*, Zastosowania Matematyki 9 (1966), p. 31-52.

[32] (obituary notice on) *Julian Perkal* (*1913-1965*), Listy Biometryczne 12-15 (1966), p. 1-2.

[33] (with W. Klonecki) *Uwagi o weryfikacji hipotez statystycznych* [*On the theory of hypotheses testing*], Przegląd Statystyczny 13 (1966), p. 319-329.

[34] *Wykłady z rachunku prawdopodobieństwa i statystyki matematycznej* [*Lectures in the theory of probability and mathematical statistics*], Państwowe Wydawnictwo Naukowe, Warszawa 1966.

[35] *Statystyka matematyczna jako metoda wnioskowania na podstawie obserwacji* (streszczenie) [*Mathematical statistics as a method for inference on the ground of observations* (summary of a lecture)], Sprawozdania Wrocławskiego Towarzystwa Naukowego, Seria B, 22 (1967), p. 65-67.

[36] *O wyborze płaszczyzny najlepszej dyskryminacji* [*On the choice of a plane of best discrimination*], Zastosowania Matematyki 9 (1968), p. 325-334.

[37] (with Z. Kowal) *O bezpieczeństwie ustrojów prętowych* [*On the safety of bar structures*], III Sesja Naukowa Wydziału Budownictwa Lądowego Politechniki Wrocławskiej 2-5 października 1968 r. p. 83-92.

[38] *Another method of estimating the parameters characterising high speed coincidence circuits*, Nukleonika 13 (1968), p. 1171-1178.

[39] *O teorii gier* [*On the theory of games*], rozdział w zbiorowej książce *Elementy nowoczesnej matematyki dla inżynierów* [a chapter in the collective book *Elements of modern mathematics for engineers*], Warszawa-Wrocław 1971, p. 185-231.

[40] *Wielopróbkowa analiza pewnego zagadnienia ekologicznego czyli pogoń za ślimakami* [*Large sample analysis of a problem in ecology or the hunting for snails*], Zastosowania Matematyki 11 (1970), p. 377-389.