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TOPOLOGICAL ZERO-ONE LAWS

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In [1] a topological zero-one law was obtained for sets with the property of Baire which are invariant under a group of homeomorphisms. In the present paper we generalize the result of [1] to sets with the property of Baire which are invariant under an equivalence relation. We also obtain a product theorem for topological zero-one laws from which Oxtoby's (analogue of Kolmogoroff's) zero-one law follows. Finally, we give an example to show that Oxtoby's zero-one law which Oxtoby proved for topological spaces with countable pseudobases is not true in general.

1. Definitions and notation. A topological space X is said to be a *Baire space* if no non-empty open subset of X is of first category. A subset D of a topological space X is said to have the *property of Baire* if we can write D as $E\Delta P$, where E is an open subset of X, and P is a set of first category in X. For properties of sets with the property of Baire and sets of first category see [6] and [8]. A family $\boldsymbol{B}$ of non-empty open sets in a topological space X is called a *pseudobase* if every non-empty open subset of X contains a set from $\boldsymbol{B}$ (see [7]). For any set A, $\bar{A}$ stands for the closure of A. If $A \subset X \times Y$ and $y \in Y$, A^y stands for the set $\{x: (x, y) \in A\}$. For any set A, A^c denotes the complement of A.

Let $(X, \boldsymbol{T})$ be a topological space and let $\sim$ be an equivalence relation on X. For $x \in X$ we write $[x]$ for the equivalence class containing x. For $A \subset X$ we write A^* for $\bigcup_{x \in A} [x]$, the saturation of A. We say that a set $A \subset X$ is *invariant* if $A = A^*$ (that is, A is saturated in the sense of Bourbaki [2]).

For any sort of study of an equivalence relation on a topological space we should have some relation between the topology and the equivalence relation. In view of this we need the following definitions.

Definition 1. We say that $(X, \boldsymbol{T}, \sim)$ is a *triple* if $(X, \boldsymbol{T})$ is a topological space, $\sim$ is an equivalence relation on X and, for every open set A, A^* is open (that is, the decomposition induced by $\sim$ is a lower semicontinuous decomposition in the sense of Kuratowski [6] or $\sim$ is an open relation in the sense of Bourbaki [2]).

Definition 2. If for every invariant set B with the property of Baire in a triple $(X, T, \sim)$ either B or B^c is of first category, then we call $(X, T, \sim)$ a *topological zero-one triple.*

2. A topological zero-one law.

THEOREM 1. *For a triple* $(X, T, \sim)$,

(i) $(X, T, \sim)$ *is a topological zero-one triple*

if and only if

(ii) (a) *for any two open sets* U *and* V *of second category,* $U^* \cap V \neq \emptyset$,

(ii) (b) *for every invariant set* A *with the property of Baire and every open set* U *of second category at every point, whenever* $A \cap U$ *is of first category,* $A \cap U^*$ *is also of first category.*

Proof. (i)$\Rightarrow$(ii)(a). If U is an open set of second category, U^* is also open, and so it is an invariant set with the property of Baire and is of second category. But then $U^{*c} \not\supset V$, if V is of second category. Hence $U^* \cap V \neq \emptyset$.

(i)$\Rightarrow$(ii)(b). If Z is an invariant set with the property of Baire and if Z is of second category, by (i), Z^c is of first category. This implies that $Z \cap U$ cannot be of first category for any open set U of second category.

(ii)$\Rightarrow$(i). Let Z be an invariant set with the property of Baire. Write $Z = U \Delta P$, where U is open and P is of first category. Suppose that Z is not of first category. So we can assume that U is of second category at every point (see [6], Section 11). Now $Z^c \cap U \subset P$ and hence $Z^c \cap U$ is of first category. By (ii)(b), $Z^c \cap U^*$ is of first category.

Let U_0 be the union of all open sets of first category. By the Banach category theorem, U_0 is of first category. By (ii)(a), $U^* \cup U_0$ is dense in (X, T), and so $(U^* \cup U_0)^c$ is nowhere dense. Now,

$$Z^c \subset (Z^c \cap U_0) \cup (Z^c \cap U^*) \cup (U^* \cup U_0)^c.$$

Hence Z^c is of first category.

The proof is completed.

In general, for a given triple, condition (ii)(a) is easy to verify, but condition (ii)(b) is not.

Definition 3. A triple $(X, T, \sim)$ is called a **-triple* if (ii)(b) is satisfied for $(X, T, \sim)$. In this case we call $\sim$ a **-relation* on (X, T).

Example 1. Let (X, T) be a topological space and let $\sim$ be defined on X by $x \sim y$ for every $x, y \in X$. Then $(X, T, \sim)$ is a *-triple.

Example 2. Let (X, T) be a topological space and let G be a group of homeomorphisms on (X, T). Define $\sim_G$ on X as follows: $x \sim_G y$ if there is a $g \in G$ such that $g(x) = y$. Then $(X, \boldsymbol{T}, \sim_G)$ is a *-triple. To see this one has to use the Banach category theorem.

Remark 1. It is not difficult to find relations between condition (ii)(a) and other similar conditions for a *-triple. We state a few.

Consider the following:

(iii) *Every equivalence class is dense.*

(iv) *Almost every equivalence class is dense (i.e., there is a first category set A such that $x \notin A$ implies that $[x]$ is dense).*

(v) *There is a first category set A such that, whenever $x \notin A$ and U is an open set of second category, $[x] \cap U \neq \emptyset$.*

(vi) *There is an x such that, whenever U is an open set of second category, $[x] \cap U \neq \emptyset$.*

(vii) *Complement of every invariant open set of second category is of first category.*

Then (iii) $\Rightarrow$ (iv) $\Rightarrow$ (v) $\Rightarrow$ (vi) $\Rightarrow$ (vii) $\Leftrightarrow$ (ii)(a) for any *-triple.

(ii)(a) $\Rightarrow$ (v) for any *-triple the topology of which has a countable pseudobase (observe that if $U_1, U_2, \ldots$ is a countable pseudobase and if U is any set, then $\bar{U} - [\bigcup_{U_i \subset U} U_i]$ is nowhere dense). So, if U is an open set of second category, then there exists $U_{i_0} \subset U$ such that U_{i_0} is of second category. Hence

$$\{x\colon [x] \cap U = \emptyset \text{ for some open set } U \text{ of second category}\}$$
$$= \{x\colon [x] \cap U_i = \emptyset \text{ for some } U_i \text{ of second category}\}$$

and this set is of first category if (ii)(a) is assumed.

(ii)(a) $\Rightarrow$ (iv) for any *-triple $(X, \boldsymbol{T}, \sim)$, where $(X, \boldsymbol{T})$ is a Baire space with a countable pseudobase.

Because of Example 2 and Remark 1 we have the following strengthened form of the theorem of [1]. Note that the theorem of [5] is also included here.

THEOREM 2. *Let X be a topological space and let G be a group of homeomorphisms on X. Consider the following conditions:*

(1) *The orbit of some point is dense in X.*

(2) *Any invariant open set of second category is dense in X.*

(3) *For any two open sets U and V of second category, there is a g in G such that $g(U) \cap V = \emptyset$.*

(4) *For any invariant set B with the property of Baire either B or B^c is of first category.*

Then (1) $\Rightarrow$ (2) $\Rightarrow$ (3) $\Leftrightarrow$ (4), *and if X is a Baire space with a countable pseudobase, then also* (4) $\Rightarrow$ (1).

3. A product theorem for topological zero-one laws. Let $(X_i, T_i, \sim_i)_{i\in I}$ be an indexed set of triples. Let $(\bigotimes_{i\in I} X_i, \bigotimes_{i\in I} T_i)$ be the product of topological spaces. Define $\bigotimes_{i\in I} \sim_i$ as follows: for $x, y \in \bigotimes_{i\in I} X_i$,

$$x \bigotimes_{i\in I} \sim_i y \quad \text{if } x_i \sim_i y_i$$

for finitely many i's and

$$x_i = y_i$$

for the rest of the i's. Clearly,

$$(\bigotimes_{i\in I} X_i, \bigotimes_{i\in I} T_i, \bigotimes_{i\in I} \sim_i) \tag{$*$}$$

is a triple.

THEOREM 3 (product theorem for topological zero-one laws). *If $(X_i, T_i, \sim_i)_{i\in I}$ is a set of topological zero-one triples and if $(*)$ is a *-triple, then $(*)$ is a topological zero-one triple.*

Proof. Since each $(X_i, T_i, \sim_i)$ is a topological zero-one triple, by Theorem 1, (ii)(a) is satisfied for each $(X_i, T_i, \sim_i)$. The proof will be completed if we show that (ii)(a) is satisfied for $(*)$.

By the Banach category theorem it suffices to verify condition (ii)(a) for basic open sets of second category. But this is clear from the assertion that if a product set is of second category, then none of the coordinate sets is of first category.

Theorem 3 applied to groups of homeomorphisms on topological spaces yields

THEOREM 4. *Let G_i be a group of homeomorphisms on (X_i, T_i) for $i \in I$. Let*

$$\bigotimes_{i\in I} G_i = \{\{g_i\}_{i\in I} : g_i \in G_i \text{ and } g_i \text{ is the identity for all but finitely many } i\text{'s}\}.$$

If each $(X_i, T_i, \sim_{G_i})$ is a topological zero-one triple, then so is

$$(\bigotimes_{i\in I} X_i, \bigotimes_{i\in I} T_i, \bigotimes_{i\in I} \sim_{G_i}).$$

Proof. Clearly,

$$\bigotimes_{i\in I} \sim_{G_i} = \sim_{\bigotimes_{i\in I} G_i}.$$

Now the result follows from Theorem 3 and Example 2.

Remark 2. Theorem 4 applied to products of category analogues of the Hewitt-Savage zero-one laws gives the category analogue of the extended Hewitt-Savage zero-one law for measures proved by Horn and Schach [4].

Now the question arises, in view of Theorem 3, whether a product of $*$-triples is a $*$-triple. As we shall observe in Section 4, this is not true in general. However, for topological spaces with countable pseudobases, the product of $*$-triples is a $*$-triple.

PROPOSITION 1. *Let* $(X, T, \sim_1)$ *and* $(Y, S, \sim_2)$ *be* $*$*-triples, where* (X, T) *and* (Y, S) *have countable pseudobases. Then*

$$(X \times Y, T \otimes S, \sim_1 \otimes \sim_2)$$

is also a $*$*-triple.*

Proof. Let Z be an invariant set in $X \times Y$ with the property of Baire.

First we show that if $U \times V$ is an open set in $X \times Y$ such that $Z \cap (U \times V)$ is of first category, and U (respectively, V) is of second category at every point, then $Z \cap (U^* \times V)$ (respectively, $Z \cap (U \times V^*)$) is of first category.

Evidently, the symmetry allows us to consider only the first case. Let $A = Z \cap (U \times V)$. Since Z has the property of Baire and since X has a countable pseudobase, by Theorem 15.2 of [8], Z^y has the property of Baire for all y except a set N_1 of first category. Since A is of first category, by the Kuratowski-Ulam theorem [8] A^y is of first category for all y except a set N_2 of first category. So, if $y \notin N_1 \cup N_2$, Z^y is a set with the property of Baire and $Z^y \cap U$ is of first category. Since $(X, T, \sim_1)$ is a $*$-triple, and since Z^y is $\sim_1$-invariant, $Z^y \cap U^*$ is of first category for $y \in V \setminus (N_1 \cup N_2)$. It follows from Theorem 15.4 of [8] that the set $Z \cap (U^* \times V)$ is of first category.

Let $U \times V$ be an open set in $X \times Y$ of second category at every point (thus both U and V are of second category at every point) and assume that $Z \cap (U \times V)$ is of first category. Using the initial observation we infer that $Z \cap (U^* \times V)$ is of first category; using this observation once again we conclude that

$$Z \cap (U^* \times V^*) = Z \cap (U \times V)^*$$

is of first category.

Finally, given an open set U in $X \times Y$ which is of second category at every point, we have

$$U = \bigcup_{i \in I} U_i \times V_i,$$

where $U_i \times V_i$ are open in $X \times Y$ (and hence of second category at every point). Assume that $Z \cap U$ is of first category. We write

$$Z \cap U^* = \bigcup_{i \in I} [Z \cap (U_i \times V_i)^*]$$

and, by the case considered above, every set $Z \cap (U_i \times V_i)^*$ is of first category. From the Banach category theorem we conclude (note that $(U_i \times V_i)^*$ is open) that $Z \cap U^*$ is of first category. The proof is completed.

Dr. E. Grzegorek has pointed out that the statement above fails to be true if the sets U of second category at every point in the definition of *-triples are replaced by the sets U of second category (which yields a more general notion than *-triples); in Grzegorek's counter-example both $(X, \boldsymbol{T})$ and $(Y, \boldsymbol{S})$ are subspaces of the real line.

PROPOSITION 2. *Let Ω be a set of equivalence relations on a space $(X, \boldsymbol{T})$. For every subfamily $\Gamma \subset \Omega$, define the equivalence relation $\sim_\Gamma$ by letting $x \sim_\Gamma y$ if there exist $x_1, x_2, \ldots, x_{n-1}$ and $\sim_1, \sim_2, \ldots, \sim_n$ from Γ such that*

$$x \sim_1 x_1, \quad x_1 \sim_2 x_2, \quad \ldots, \quad x_{n-1} \sim_n y.$$

*If $(X, \boldsymbol{T}, \sim_\Gamma)$ is a *-triple for every finite $\Gamma \subset \Omega$, then $(X, \boldsymbol{T}, \sim_\Omega)$ is also a *-triple.*

The proposition follows immediately from the equality

$$A^{*(\sim_\Omega)} = \bigcup \{A^{*(\sim_\Gamma)}\colon \Gamma \subset \Omega \text{ is finite}\}$$

and from the Banach category theorem.

THEOREM 5. *If $(X_i, \boldsymbol{T}_i, \sim_i)_{i\in I}$ is a set of *-triples, where each $(X_i, \boldsymbol{T}_i)$ has a countable pseudobase, then $(*)$ is also a *-triple.*

This theorem follows from Propositions 1 and 2.

In view of Theorems 3 and 5 we have

THEOREM 6. *If $(X_i, \boldsymbol{T}_i, \sim_i)$ is a set of topological zero-one triples and if each $(X_i, \boldsymbol{T}_i)$ has a countable pseudobase, then $(*)$ is also a topological zero-one triple.*

A special case of Theorem 6 is

THEOREM 7 (Oxtoby's zero-one law). *If X is the Cartesian product of a family of topological spaces each of which has a countable pseudobase, and if E is a tail set with the property of Baire in X, then either E or $X-E$ is of first category in X.*

Proof. Specialize Theorem 6 to the case where each topological space is as in Example 1.

4. Oxtoby's zero-one law is not true in general. In this section we give an example to show that Oxtoby's zero-one law need not be true for general topological spaces and we prove a restricted version of Oxtoby's zero-one law for general topological spaces. Our example exploits heavily a recent construction of Fleissner [3] of two Baire spaces whose product is not a Baire space.

We refer to the equivalence relation which induces the tail sets as Kolmogoroff's equivalence relation and denote it by $\sim$ throughout this section.

Let ω_1 denote the set of all countable ordinals and equip it with the discrete topology. Let ω_1^N be the product of countably many copies of ω_1 equipped with the product of discrete topologies, say $\boldsymbol{T}$.

PROPOSITION 3. *There is a tail set $F \subset \omega_1^N$ such that*

(i) *$F \times F$ and $F^c \times F^c$ are of second category at every point,*

(ii) *$F \times F^c$ and $F^c \times F$ are of first category in $\omega_1^N \times \omega_1^N$.*

Let us first see how this proposition helps us in getting the example.

The following example shows that *Oxtoby's zero-one law is not true in general.*

Let $X_0 = \omega_1^N$ be equipped with the topology generated by F, F^c, and $\boldsymbol{T}$. Then $T = X_0 \times F$ is a tail set in $\bigotimes_{i=0}^{\infty} X_i$, where $X_1 = X_2 = \ldots = \omega_1$, with the discrete topology. Then

(a) $F^c \times F$ and $F \times F^c$ are of first category because of Lemma 1 below.

(b) $F \times F$ and $F^c \times F^c$ are of second category because of Lemma 1 below.

(c) T has the property of Baire, since

$$T \Delta (F \times \bigotimes_{i=1}^{\infty} X_i) \subset F^c \times F \cup F \times F^c$$

and $F \times \bigotimes_{i=1}^{\infty} X_i$ is open.

(d) T and T^c are both of second category, since $T \supset F \times F$ and $T^c \supset F^c \times F^c$.

LEMMA 1. *Let $(X, \boldsymbol{T})$ be a topological space and let $A \subset X$ be a dense set such that A^c is also dense. Let $\boldsymbol{T}_0$ be the topology generated by A, A^c, and $\boldsymbol{T}$. Then a set B contained in A or in A^c is of first category in $(X, \boldsymbol{T}_0)$ if and only if B is of first category in $(X, \boldsymbol{T})$.*

Remark 3. In the example above we have essentially shown that the Kolmogoroff's equivalence relation is not a *-relation. This incidentally shows that countable products of *-triples need not be *-triples. Even a finite product of *-triples need not be a *-triple, as can also be easily shown from Proposition 3.

Proof of Proposition 3. First we establish some notation.

For $x \in \omega_1^N$, let

$$x^* = \sup_n x_n.$$

Let $C = \{x \in \omega_1^N\colon x^* = x_n \text{ for some } n\}$. Let $\{A, B\}$ be a partition of the limit ordinals in ω_1 into stationary sets (a set $S \subset \omega_1$ is said to be *stationary* if S intersects every unbounded closed subset of ω_1). For $D \subset \omega_1$, write $M(D) = \{x \in \omega_1^N\colon x^* \in D\}$.

With the above notation we have

(i) C is of first category:

$$C = \bigcup_i \bigcup_a \{x \in \omega_1^N : x_i = a \text{ and } x^* = x_i\}.$$

Now

$$\{x \in \omega_1^N : x_i = a \text{ and } x^* = x_i\} = [0, a] \times [0, a] \times \ldots \times \underset{i}{[a]} \times [0, a] \times \ldots$$

is a nowhere dense set. But for a fixed i and a variable, the sets $\{x \in \omega_1^N : x_i = a \text{ and } x^* = x_i\}$ form a family of nowhere dense sets all of which are at a fixed distance from each other (in, for example, the first difference metric). Hence their union is nowhere dense. Hence C is of first category.

(ii) $M(A) \cup M(B) \cup C = \omega_1^N$ and $M(A) \cap M(B) = \emptyset$ (easy to prove).

(iii) $M(A) \times M(B)$ and $M(B) \times M(A)$ are of first category, and $M(A) \times M(A)$ and $M(B) \times M(B)$ are of second category at every point (a result of Fleissner [3]).

(iv) If $x \sim y$, then either $x \in C$ or $y \in C$ or $x^* = y^*$.

Indeed, if $x^* < y^*$ and $y \notin C$, then there are infinitely many y_n's greater than x^*. Since $x \sim y$, infinitely many x_n's are greater than x^*, which is not possible. So $x^* < y^*$ implies that $y \in C$. Similarly, $y^* < x^*$ implies that $x \in C$.

(v) Let

$$F = \{x \in \omega_1^N : \text{there exists } y \in M(A) - C \text{ such that } x \sim y\}.$$

Then

$$F \Delta M(A) \subset C \quad \text{and} \quad F^c \Delta M(B) \subset C.$$

$F \Delta M(A) \subset C$ since $M(A) - C \subset F$ because of the definition of F, and $F \subset M(A) \cup C$ because of (iv). $F^c \Delta M(B) \subset C$ because of (ii) and the inclusion $F \Delta M(A) \subset C$.

(vi) Each of the sets

$$(F \times F) \Delta \big(M(A) \times M(A)\big), \quad (F^c \times F^c) \Delta \big(M(B) \times M(B)\big),$$

$$(F \times F^c) \Delta \big(M(A) \times M(B)\big), \quad (F^c \times F) \Delta \big(M(B) \times M(A)\big)$$

is contained in $C \times \omega_1^N \cup \omega_1^N \times C$ and this last set is of first category.

(vii) Combining (iii) and (vi) we have the proposition.

The reason for the success of our example is that, though the set $X_0 \times F$ defined in the example has the property of Baire, the set F has it not. This is strengthened by the following theorem:

THEOREM 8 (generalized Oxtoby's zero-one law). *Let X be the Cartesian product of a family of topological spaces $\{X_i: i \in I\}$. For every finite set $J \subset I$, let*

$$B(J) = \{\bigotimes_{i\in J} X_i \times B: B \text{ has the property of Baire in } \bigotimes_{i\in I-J} X_i\}.$$

If

$$E \in B_0 = \bigcap_{J\subset I} B(J),$$

then either E or E^c is of first category.

Proof. Suppose that E^c is of second category. We shall show that E should be of first category.

Let U be open and let P be of first category such that $E^c = U\Delta P$. Then U is of second category, and hence there is a non-empty basic open set $V \subset U$ such that every non-empty open subset of V is of second category.

Let $V = V(J_0)\times X(J_0)$, where J_0 is a finite subset of I, $V(J_0)$ is an open subset of $\bigotimes_{i\in J} X_i$ and

$$X(J_0) = \bigotimes_{i\in I-J_0} X_i.$$

Since $E \in B(J_0)$, we can write

$$E = \bigotimes_{i\in J_0} X_i \times B$$

for some set B with the property of Baire in $X(J_0)$.

Now, $E\cap V = V(J_0)\times B$ is of first category, since $E\cap V \subset P$.

Let $B = W\Delta Q$, where W is open and Q is of first category. If B is not of first category, then $W \neq \emptyset$ and $\big(V(J_0)\times W\big)\Delta\big(V(J_0)\times B\big) = V(J_0)\times Q$ is of first category. Hence $V(J_0)\times W$, a non-empty open subset of V, is of first category and this is a contradiction to the choice of V.

The important point in the proof of Theorem 8 is the following lemma which seems to be interesting in itself.

LEMMA 2. *Let Y and Z be two topological spaces such that the product $Y\times Z$ is a Baire space. If $A \subset Y$ and $B \subset Z$ are two sets with the property of Baire and if $A\times B$ is of first category, then either A or B is of first category.*

We named Theorem 8 the generalized Oxtoby's zero-one law, since it actually generalizes Oxtoby's zero-one law. This is because of the following theorem:

THEOREM 9. *Let $\{X_i\colon i \in I\}$ be a family of Baire spaces with countable pseudobases. Then*

$$A \subset \bigotimes_{i\in I} X_i$$

is a tail set with the property of Baire if and only if $A \in \boldsymbol{B}_0$ of Theorem 8.

We give only the crucial lemma needed to prove this theorem.

LEMMA 3. *Let Y be the Cartesian product of a family of topological spaces $\{Y_i\}_{i\in I}$ each of which has a countable pseudobase. Let X be a topological space also with a countable pseudobase. If $E \subset X \times Y$ is nowhere dense, then E_x is nowhere dense for all x except for a set of first category.*

Proof. If Y is a countable product, the result follows from 2.5 of [7] and from 15.1 of [8].

If Y is an uncountable product, let $G = X \times Y - E$. Take a maximal family of pairwise disjoint basic open sets $G_1, G_2, \ldots$ contained in G. Since $X \times Y$ satisfies the countable chain condition (see [7], p. 161), this family is countable. Since $G_1, G_2, \ldots$ are all basic open sets, we can find countably many coordinates $i_1, i_2, \ldots$ such that

$$G_n = H_n \times \bigotimes_{i \neq i_1, i_2, \ldots} Y_i,$$

where H_n is open and

$$H_n \subset X \times \bigotimes_{k=1}^{\infty} Y_{i_k}.$$

Let

$$Z = \bigotimes_{k=1}^{\infty} Y_{i_k} \quad \text{and} \quad H = \bigcup_{n=1}^{\infty} H_n.$$

Since E is nowhere dense, H is an open dense set in $X \times Z$. So H_x^c is nowhere dense in Z for all x except for a set of first category. But

$$E \subset H^c \times \bigotimes_{i \neq i_1, i_2, \ldots} Y_i.$$

Hence E_x is nowhere dense for all x except for a set of first category of points in X.

Added in proof. The results of preprint [3] are included in a joint paper of W. G. Fleissner and K. Kunen *Barely Baire spaces*, Fundamenta Mathematicae (1978) (to appear); for some results related to Section 4 we refer also to the second-named author's paper *Note on category in Cartesian products of metrizable spaces*, Fundamenta Mathematicae (1978) (to appear).

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