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HARDY SPACES ON SU(2)

BY

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1. Preliminaries. Every point $z = (z_1, z_2) \in C^2$ can be associated with a (2×2)-matrix

$$u = u(z) = -i\begin{pmatrix} -\bar{z}_2 & z_1 \\ \bar{z}_1 & z_2 \end{pmatrix}. \tag{1.1}$$

The set of all such matrices satisfying the condition $\det u = |z_1|^2 + |z_2|^2 = 1$ constitutes the *special unitary group* SU(2). The group SU(2) acts on C^2 preserving the ball

$$B = \{z = (z_1, z_2) \in C^2 \colon |z_1|^2 + |z_2|^2 < 1\}$$

and its boundary – the unit sphere S. The sphere S can be identified as above with SU(2), and then the normalized Lebesgue measure on S corresponds to the Haar measure on SU(2).

The irreducible representations of SU(2) can be realized on the spaces $\mathscr{P}^l$ of homogeneous polynomials of degree $2l$ in $w = (w_1, w_2) \in C^2$, where $l = 0, 1/2, 1, 3/2, \ldots$ The space $\mathscr{P}^l$ is a unitary space with an appropriate inner product for which the polynomials

$$p_j(w) = \frac{1}{\sqrt{(l-j)!\,(l+j)!}} w_1^{l-j} w_2^{l+j}, \tag{1.2}$$

$j = -l, -l+1, \ldots, l$, form an orthonormal basis. The representation T is given by the mapping

$$(T_u p)(w) = p(u' w), \qquad p \in \mathscr{P}^l, \tag{1.3}$$

where u' is the transpose of u. The transformation T_u is unitary with respect to the inner product in $\mathscr{P}^l$.

For every complex measure μ on SU(2) let

$$\|\mu\| = \int_{SU(2)} d|\mu|(u)$$

denote the *total variation* of μ. The space of all such measures with the total variation norm will be denoted by $M(\mathrm{SU}(2))$.

The *Fourier transform* $\hat{\mu}$ of a measure μ is a sequence of $((2l+1)\times(2l+1))$-matrices such that

$$\hat{\mu}(l) = \int_{SU(2)} T^l_{u^{-1}}\, d\mu(u). \tag{1.4}$$

This for $f \in L^1(SU(2))$ yields

$$\hat{f}(l) = \int_{SU(2)} f(u)\, T^l_{u^{-1}}\, du.$$

The matrix entries of T^l_u in the basis (1.2) will be denoted by $t^l_{jk}(u)$, $-l \leqslant k, j \leqslant l$, and the space of linear combinations of these functions by $T(SU(2))$. The functions t^l_{jk} form an orthogonal basis for $L^2(SU(2))$ with $\|t^l_{jk}\|_{L^2} = (2l+1)^{-1/2}$. The relation

$$(-i)^{2l}\frac{(-\bar{z}_2 w_1 + \bar{z}_1 w_2)^{l-j}(z_1 w_1 + z_2 w_2)^{l+j}}{\sqrt{(l-j)!\,(l+j)!}} = \sum_{k=-l}^{l} t^l_{kj}(u)\frac{w_1^{l-k} w_2^{l+k}}{\sqrt{(l-k)!\,(l+k)!}}$$

implies that the functions $t^l_{kj}(u(z))$ are homogeneous polynomials of degree $2l$.

The representation T can be extended on C^2 by comparing (1.1) and (1.3) which for t^l_{kj} yield

$$t^l_{jk}(z) = |z|^{2l}\, t^l_{jk}\left(u\left(\frac{z}{|z|}\right)\right), \quad z \in C^2.$$

This gives a natural extension of the polynomials t^l_{jk} on C^2. Moreover, by an elementary calculation we obtain

$$t^l_{jk}(\zeta z) = \zeta^{l+k}\bar{\zeta}^{l-k}\, t^l_{jk}(z), \qquad \zeta \in C^2. \tag{1.5}$$

In other words, $t^l_{jk}(z)$ are homogeneous polynomials in z_1, z_2 of degree $l+k$ and of degree $l-k$ in $\bar{z}_1, \bar{z}_2$.

Note that the functions $t^l_{jk}(z)$ are harmonic with respect to the Laplacian

$$\frac{\partial}{\partial z_1}\frac{\partial}{\partial \bar{z}_1} + \frac{\partial}{\partial z_2}\frac{\partial}{\partial \bar{z}_2}$$

(cf. [1], p. 34).

We define the *Poisson kernel* on B by

$$P(z) = \sum_{l=0}^{\infty} (2l+1)\,\chi^l(z), \tag{1.6}$$

where

$$\chi^l(z) = \operatorname{tr} T_z = \sum_{j=-l}^{l} t^l_{jj}(z)$$

and we write $P_r(u) = P(ru)$, $u \in SU(2)$.

The sequence (P_r) is an approximative identity if $r \to 1$.

For a measure μ in $M(\mathrm{SU}(2))$ let us denote by $P[\mu]$ the function on the ball B given by

$$P[\mu](z) = (P * \mu)(z),$$

where the *convolution* $*$ *of a function f on B and a measure μ* is defined by

$$(f * \mu)(z) = \int_{\mathrm{SU}(2)} f(v^{-1} z)\, d\mu(v).$$

LEMMA 1.1. *The function $P[\mu]$ is harmonic in B and the following expansion holds*:

$$P[\mu](z) = \sum_{l=0}^{\infty} (2l+1)(\chi^l * \mu)(z).$$

Proof. Since $|\chi^l(z)| \leqslant (2l+1)|z|^{2l}$, the series in (1.6) is almost uniformly convergent in B. Let z be in a compact subset K of B; then

$$\begin{aligned} P[\mu](z) &= \int_{\mathrm{SU}(2)} \sum_{l=0}^{\infty} (2l+1)\chi^l(v^{-1} z)\, d\mu(v) \\ &= \sum_{l=0}^{\infty} (2l+1) \int_{\mathrm{SU}(2)} \chi^l(v^{-1} z)\, d\mu(v) \\ &= \sum_{l=0}^{\infty} (2l+1)(\chi^l * \mu)(z) \end{aligned}$$

and, by the inequality $|(\chi^l * \mu)(z)| \leqslant (2l+1)\|\mu\|\, |z|^{2l}$, the series is uniformly convergent in K.

We see that

$$\begin{aligned} (\chi^l * \mu)(z) &= \sum_{k=-l}^{l} \int_{\mathrm{SU}(2)} t^l_{kk}(v^{-1} z)\, d\mu(v) \\ &= \sum_{k=-l}^{l} \sum_{j=-l}^{l} \int_{\mathrm{SU}(2)} t^l_{kj}(v^{-1})\, d\mu(v)\, t^l_{jk}(z) \\ &= \mathrm{tr}\left(\hat{\mu}(l)\, T^l_z\right) \end{aligned}$$

is a linear combination of harmonic functions $t^l_{jk}(z)$.

It follows that $P[\mu]$ is a harmonic function.

The next fact is a transformation of the corresponding classical theorem proved in [5] (cf. Theorem 11.19).

THEOREM 1.1. *The mapping $\mu \to P[\mu]$ is a linear one-to-one correspondence between $M(\mathrm{SU}(2))$ and the space of all harmonic functions h in B which satisfy the condition*

$$(*) \qquad \sup_{r<1} \int_{\mathrm{SU}(2)} |h(ru)|\, du \leqslant C < \infty .$$

2. Hardy spaces $H^1_n(\mathrm{SU}(2))$.

DEFINITION 2.1. A measure μ in $M(\mathrm{SU}(2))$ is said to be of *analytic type* n if its Fourier transform $\hat{\mu}$ is of the following form:

$\hat{\mu}(l) = 0$ (zero matrix) for $l < n$,

$\hat{\mu}(l)$ is supported by the n-th row from below (0-th is the last one) for $l \geqslant n$.

By Theorem 1.1 we see that there exists a space of harmonic functions in B which corresponds to the space of all measures of analytic type n. Using Lemma 1.1 we can see that the space may be defined as follows:

DEFINITION 2.2. A function h harmonic in B belongs to the *Hardy space* $H^1_n(\mathrm{SU}(2))$ if

$h * \chi^l = 0$ for $l < n$,

$h * \chi^l$ is a linear combination of functions t^l_{kj} ($j = l-n$, $-l \leqslant k \leqslant l$) for $l \geqslant n$, and

$$\|h\|_{H^1} = \sup_{r<1} \int_{\mathrm{SU}(2)} |h(ru)|\,du < \infty.$$

In particular, $H^1_0(\mathrm{SU}(2))$ is the usual Hardy space defined by analytic functions in B (cf. [1] and [6]).

It turns out that most of the properties of the usual Hardy space hold in the spaces $H^1_n(\mathrm{SU}(2))$.

For a function f on the ball B we define slice functions

$$f^u(\zeta) = f(\zeta z_1, \zeta z_2), \qquad u = u(z_1, z_2) \in \mathrm{SU}(2),$$

where ζ is in the disc $U = \{\zeta \in C\colon |\zeta| < 1\}$. By invariance of the Haar measure and Fubini's theorem we have

$$\int_{\mathrm{SU}(2)} f(ru)\,du = \int_{\mathrm{SU}(2)} \frac{1}{2\pi} \int_0^{2\pi} f^u(re^{it})\,dt\,du. \tag{2.1}$$

If $f \in H^1_n(\mathrm{SU}(2))$, then, as follows from Theorem 1.1 and Definition 2.1, there exists a measure μ such that $f = P[\mu]$ and μ is of analytic type n.

LEMMA 2.1. *Let $f \in H^1_n(\mathrm{SU}(2))$ and μ be a measure of analytic type n such that $f = P[\mu]$. Then every function $g^u(\zeta) = |\zeta|^{-2n} f^u(\zeta)$ is an analytic function in U with the Taylor expansion*

$$g^u(\zeta) = \sum_{l=0}^{\infty} f_l(u)\,\zeta^{2(l-n)},$$

where $f_l(u) = (2l+1)(\chi^l * \mu)(u)$. Moreover, for almost all $u \in \mathrm{SU}(2)$ the functions g^u belong to the Hardy space $H^1(T)$ and (see also [6])

$$\|f\|_{H^1} = \int_{\mathrm{SU}(2)} \|g^u\|_{H^1}\,du.$$

Proof. It follows from Lemma 1.1 that f has the expansion

$$f(z) = \sum_{l=0}^{\infty} (2l+1)(\chi^l * \mu)(z) = \sum_{l=0}^{\infty} (2l+1) \sum_{k=-l}^{l} c_k^l t_{k,l-n}^l(z),$$

where $c_k^l = (\hat{\mu}(l))_{l-n,k}$. Consequently, by (1.5),

$$\begin{aligned} f(\zeta z) &= |\zeta|^{2n} \sum_{l=0}^{\infty} (2l+1) \sum_{k=-l}^{l} c_k^l t_{k,l-n}^l(z) \zeta^{2(l-n)} \\ &= |\zeta|^{2n} \sum_{l=0}^{\infty} (2l+1)(\chi^l * \mu)(z) \zeta^{2(l-n)}. \end{aligned}$$

Hence

$$f^u(\zeta) = |\zeta|^{2n} g^u(\zeta) \quad \text{and} \quad g^u(\zeta) = \sum_{l=0}^{\infty} f_l(u) \zeta^{2(l-n)}.$$

Since $|f_l(u)| \leqslant (2l+1)^2 \|\mu\|$, the series which defines g^u is almost uniformly convergent in U. This means that g^u is an analytic function.

For every function h harmonic in U,

$$m(r; h) = \frac{1}{2\pi} \int_0^{2\pi} |h(re^{it})| \, dt$$

is an increasing function of r (cf. [5]). The Beppo–Levi theorem and (2.1) yield

$$\begin{aligned} \int_{SU(2)} \|g^u\|_{H^1} du &= \int_{SU(2)} \lim_{r\to 1} m(r; g^u) du \\ &= \int_{SU(2)} \lim_{r\to 1} m(r; f^u) du \\ &= \lim_{r\to 1} \int_{SU(2)} m(r; f^u) du = \|f\|_{H^1}, \end{aligned}$$

which completes the proof.

For a function f on the ball B we write $f_r(u) = f(ru)$, where $u \in SU(2)$ and $0 \leqslant r < 1$.

THEOREM 2.1. *If* $f \in H_n(SU(2))$ *and* $f^*(u) = \lim_{r\to 1} f_r(u)$, *then*

$$\lim_{r\to 1} \|f^* - f_r\|_{L^1} = 0.$$

Proof. Let g^u be defined as in Lemma 2.1 and let

$$g_r^u(e^{it}) = g^u(re^{it}).$$

The limit

$$(g^u)^*(e^{it}) = \lim_{r\to 1} g_r^u(e^{it})$$

exists a.e. on T and (cf. [5])

$$\lim_{r\to 1} \|(g^u)^* - g_r^u\|_{L^1} = 0.$$

We see that $f^*(e^{it}u) = (g^u)^*(e^{it})$, so the limit f^* exists a.e. on SU(2). By (2.1) we obtain

$$\begin{aligned}\|f^* - f_r\|_{L^1} &= \int_{SU(2)} \frac{1}{2\pi}\int_0^{2\pi} |(f^u)^*(e^{it}) - f_r^u(e^{it})|\,dt\,du \\ &= \int_{SU(2)} \|(f^u)^* - f_r^u\|_{L^1}\,du \\ &= \int_{SU(2)} \|(g^u)^* - r^{2n} g_r^u\|_{L^1}\,du.\end{aligned}$$

From Lemma 2.1 we have

$$\|(g^u)^* - r^{2n} g_r^u\|_{L^1} \leqslant 2\|g^u\|_{H^1} \quad \text{and} \quad \int_{SU(2)} \|g^u\|_{H^1}\,du = \|f\|_{H^1},$$

so the Lebesgue bounded convergence theorem implies the result.

The next theorem is a version of the F. and M. Riesz theorem.

THEOREM 2.2. *A measure μ of analytic type n is absolutely continuous with respect to the Haar measure on* SU(2).

Proof. The function $f = P[\mu]$ is in $H_n^1(SU(2))$. Moreover, for every $r < 1$ the function $f(rz)$ is continuous in the ball $B_{1/r}$ and it can be represented as a convolution with the Poisson kernel. Hence, by Theorem 2.1,

$$f(z) = \lim_{r\to 1} f(rz) = \lim_{r\to 1}(P * f_r)(z) = (P * f^*)(z).$$

Consequently, $P[\mu] = P[f^*]$ and, by Theorem 1.1, $d\mu = f du$; the proof is completed.

We remark that for $f \in H_n^1(SU(2))$ and its radial limit f^* we have $\|f\|_{H^1} = \|f^*\|_{L^1}$.

Using Theorem 2.1 and Lemma 1.1 we can find a sequence of linear combinations of the functions $t_{k,l-n}^l$, $l \geqslant n$, which is convergent to f^* in L^1-norm. This implies that the mapping $f \to f^*$ is an isometry of $H_n^1(SU(2))$ onto the closure in L^1-norm of all linear combinations of the functions $t_{k,l-n}^l$, $l \geqslant n$.

We have shown

COROLLARY 2.1. *The space $H_n^1(SU(2))$ can be identified with a closed subspace of $L^1(SU(2))$ under the mapping $f \to f^*$.*

It follows that $H_n^1(\mathrm{SU}(2))$ are Banach spaces.

From now on we identify f and f^*.

3. The Paley inequality and multipliers on $H_n^1(\mathrm{SU}(2))$.

DEFINITION 3.1. A sequence $\{l_k\}$ of half-integers is called *lacunary of type n* if $l_1 > n$ and

$$\frac{l_{k+1}-n}{l_k-n} \geqslant \lambda > 1 \quad \text{for } k = 1, 2, 3, \ldots$$

LEMMA 3.1. *If $f \in H_n^1(\mathrm{SU}(2))$ and $\{l_k\}$ is a lacunary sequence of type n, then there exists a constant $K = K(\lambda)$ such that*

$$\Big(\sum_{k=1}^{\infty} \|f_{l_k}\|_{L^1}^2\Big)^{1/2} \leqslant K \|f\|_{H^1},$$

where $f_l(u) = (2l+1)(\chi^l * f)(u)$.

Proof. By Lemma 2.1 the function f can be expressed as

$$f(\zeta u) = |\zeta|^{2n} \sum_{l=n}^{\infty} f_l(u)\zeta^{2(l-n)},$$

and for a.e. $u \in \mathrm{SU}(2)$ the functions

$$g^u(\zeta) = \sum_{l=n}^{\infty} f_l(u)\zeta^{2(l-n)}$$

belong to $H^1(T)$. We are going to use the classical Paley inequality for g^u.

For a function $h \in H^1(T)$ with the Taylor expansion

$$h(\zeta) = \sum_{n=0}^{\infty} c_n \zeta^n$$

and a lacunary sequence $\{n_k\}$ such that $n_{k+1}/n_k \geqslant \lambda > 1$, there exists a constant $K = K(\lambda)$ such that

$$\Big(\sum_{k=1}^{\infty} |c_{n_k}|^2\Big)^{1/2} \leqslant K \|h\|_{H^1}$$

(cf. [7], Theorem 7.8). Hence

$$\Big(\sum_{k=1}^{\infty} |f_{l_k}(u)|^2\Big)^{1/2} \leqslant K \|g^u\|_{H^1}.$$

Let $\{a_k\}$ be a sequence in $l^2(N)$ with $\|a_k\|_{l^2} = 1$. Then, by the Schwartz inequality, we can write

$$\sum_{k=1}^{\infty} |a_k|\,|f_{l_k}(u)| \leqslant K \|g^u\|_{H^1}.$$

We integrate both sides of the last inequality over SU(2) and from Lemma 2.1 we obtain

$$\sum_{k=1}^{\infty} |a_k| \, \|f_{l_k}\|_{L^1} \leqslant K \|f\|_{H^1}.$$

Now we take the supremum over all the sequences $\{a_k\}$ with $\|a_k\|_{l^2} = 1$ of the left-hand side, and this completes the proof.

The following can be called the *Paley inequality* for $H_n^1(\mathrm{SU}(2))$.

THEOREM 3.1.

$$\Big(\sum_{k=1}^{\infty} |||\hat{f}(l_k)|||^2\Big)^{1/2} \leqslant K \|f\|_{H^1},$$

where $|||X||| = \operatorname{tr}\sqrt{XX^*}$ *is the Hilbert–Schmidt norm of* X.

Proof. It is sufficient to show that $|||\hat{f}(l_k)||| \leqslant \|f_{l_k}\|_{L^1}$ and use Lemma 3.1.

Since every matrix $\hat{f}(l_k)$ has only one non-zero row, we see that $\sqrt{\hat{f}(l_k)\hat{f}(l_k)^*}$ has at most one proper value. Hence $|||\hat{f}(l_k)||| = \|\hat{f}(l_k)\|$, where $\|X\|$ is the maximum of proper values of $\sqrt{XX^*}$. It is well known that $\|\hat{f}(l_k)\| \leqslant \|f_{l_k}\|_{L^1}$, and this completes the proof.

An operator M on $H_n^1(\mathrm{SU}(2))$ which commutes with right translations on SU(2) is completely described by a sequence $\{\hat{M}(l)\}$ of $((2l+1)\times(2l+1))$-matrices (cf. [1] and [2]). In our notation we have

$$Mf(u) = \sum_{l=0}^{\infty} \operatorname{tr} \hat{f}(l)\hat{M}(l)\,T_u^l = \sum_{l=0}^{\infty} (f_l * M_l)(u),$$

where $M_l(u) = \operatorname{tr} \hat{M}(l)\,T_u^l$. We see that every matrix $\widehat{Mf}(l)$ is supported by the n-th row from below; thus, if the operator M is bounded in L^p-norm, then M maps $H_n^1(\mathrm{SU}(2))$ into

$$H_n^p(\mathrm{SU}(2)) = (H_n^1 \cap L^p)(\mathrm{SU}(2)), \qquad p \geqslant 1.$$

DEFINITION 3.2. We call a bounded operator $M\colon H_n^1 \to H_n^p$ which commutes with right translations an *$(H_n^1 - H_n^p)$-multiplier.*

Denote the space of all $(H_n^1 - H_n^p)$-multipliers by $\mathcal{M}(H_n^1, H_n^p)$.

PROPOSITION 3.1. *If for every natural number k a sequence of matrices $\{\hat{M}(l)\}$ satisfies the condition*

$$\sum_{l=k}^{2k} (2l+1)\|\hat{M}(l)\|^2 = A^2 < \infty,$$

then the operator M defined by the sequence is an $(H_n^1 - H_n^2)$-multiplier.

Proof. First observe that

$$|||\hat{f}(l)\hat{M}(l)||| \leqslant \|\hat{M}(l)\| \, |||\hat{f}(l)|||.$$

Let now $k = 0, 1, 2, 4, 8, \ldots$ Using the Parseval equality and Theorem 3.1 we get

$$\|Mf\|_{L^2}^2 = \sum_{l=0}^{\infty} (2l+1) \big|\big\|\hat{f}(l)\,\hat{M}(l)\big\|\big|^2$$

$$= \sum_{k=0}^{\infty} \sum_{l=k}^{2k} (2l+1) \|\hat{M}(l)\|^2 \big|\big\|\hat{f}(l)\big\|\big|^2$$

$$\leqslant \sum_{k=0}^{\infty} \Big(\sum_{l=k}^{2k} (2l+1) \|\hat{M}(l)\|^2\Big) \max_{k \leqslant l \leqslant 2k} \big|\big\|\hat{f}(l)\big\|\big|^2$$

$$\leqslant \sum_{k=0}^{\infty} A^2 \big|\big\|\hat{f}(l_k)\big\|\big|^2 \leqslant A^2 K^2 \|f\|_{H^1}^2$$

for every $f \in H_n^1(\mathrm{SU}(2))$. The proof is completed.

PROPOSITION 3.2. *There exist $(H_n^1 - H_n^2)$-multipliers which are not of the form $Mf(u) = (f * \mu)(u)$ for any measure μ on* SU(2).

Proof. Let $\{l_k\}$ be a lacunary sequence of type n and let $\hat{M}$ be defined as follows: $\hat{M}(l) = 0$ for l not appearing in $\{l_k\}$ and $(2l_k+1)^{1/2}\hat{M}(l_k) = J_k$ is a diagonal $((2l_k+1) \times (2l_k+1))$-matrix. Assume that the sequence $(2l_k+1)^{-1/2}\operatorname{tr} J_k$ is not bounded and $\|J_k\| \leqslant A$ for $k = 1, 2, \ldots$ (e.g., J_k can be identity matrices). It follows from Proposition 3.1 that the operator M defined in this way is an $(H_n^1 - H_n^2)$-multiplier.

We will show that M cannot be represented as a convolution with a measure.

Suppose to the contrary that there is $\mu \in M(\mathrm{SU}(2))$ such that $\hat{\mu} = \hat{M}$. Then the linear functional L on $T(\mathrm{SU}(2))$ defined by

$$L(p) = \int_{\mathrm{SU}(2)} p\,d\mu = \sum_{l=0}^{\infty} (2l+1)\operatorname{tr}\hat{p}(l)\,\hat{M}(l)$$

is bounded in L^∞-norm.

Since the sequence $\operatorname{tr}\hat{M}(l_k)$ is not bounded, there exists a sequence $\{a_k\}$ in $l^1(N)$ such that

$$\sum_{k=1}^{\infty} a_k \operatorname{tr}\hat{M}(l_k) = \infty.$$

The functions

$$p_m(u) = \sum_{k=1}^{m} a_k\big(\chi^{l_k}(u) - \chi^{l_k-1}(u)\big)$$

are central functions, so it is sufficient to calculate their values on elements of SU(2) of the form

$$e(\theta) = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix}, \quad 0 \leqslant \theta < 2\pi.$$

But the following formula holds:

$$\chi^{l_k}(e(\theta)) - \chi^{l_k-1}(e(\theta)) = 2\cos((2l_k+2)\theta)$$

(cf. [1], p. 32). Hence

$$\|p_m\|_\infty = \sup p_m(e(\theta)) \leqslant \sup \sum_{k=1}^{m} |a_k|\, 2|\cos(2l_k+2)\theta|$$

$$\leqslant 2 \sum_{k=1}^{\infty} |a_k|.$$

On the other hand, $(2l+1)\hat{\chi}^l = I$ (identity matrix). Hence

$$L(p_m) = \sum_{k=1}^{m} a_k \operatorname{tr} \hat{M}(l_k),$$

and so $L(p_m) \to \infty$ as $m \to \infty$. This gives a contradiction.

The proposition yields the following

COROLLARY 3.1. $M(\mathrm{SU}(2)) \subsetneqq \mathcal{M}(H_n^1, H_n^1)$.

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