COLLOQUIUM MATHEMATICUM

VOL. LVIII 1990 FASC. 2

ON SL(2)-*ACTIONS* *WITHOUT* 3-*DIMENSIONAL ORBITS*

BY

EWA DUMA (WARSZAWA)

Let us assume that SL(2) acts algebraically on an irreducible algebraic variety X. The purpose of this paper is to investigate the existence of quotients (Definition 1) of some open subsets of X by an SL(2)-action.

We deal with an action of SL(2) for which the maximal dimension of orbits equals 2. Cases of the maximal dimension of orbits equal to 0 or 1 are known: the first is trivial, the second one is described by Konarski [5] (see Section 2 of this paper).

Let G_x^0 be the component of the identity of the isotropy subgroup at a point $x \in X$. We show that there are two possibilities: either there exists an open subset $U \subset X$ composed of all points $x \in X$ such that G_x^0 is conjugate to T or there exists an open subset $U \subset X$ composed of all points $x \in X$ such that G_x^0 is conjugate to $\boldsymbol{k}^+$ (Theorem 2). In the first case we prove that there exists a geometric quotient of U by SL(2) (Theorem 5). This quotient can be complete or incomplete. We give appropriate examples. In the second case we construct an open subset $V \subset U$ such that the quotient of V by SL(2) exists and is complete (Theorem 4).

0. Assumptions and notation. All algebraic varieties, groups and morphisms are defined over an algebraically closed field $\boldsymbol{k}$ of characteristic zero.

Let an algebraic group G act on an algebraic variety X. Then X is called a *G-variety*. By X^G we denote the variety of fixed points. For $x \in X$ let G_x denote the isotropy subgroup at a point x and let G_x^0 denote the component of the identity of G_x.

For any two subgroups H_1 and H_2 of G we write $H_1 \cong H_2$ if H_1 and H_2 are conjugate in G, and $H_1 \subseteq H_2$ if there exists a subgroup H_3 such that $H_3 \subset H_2$ and $H_1 \cong H_3$.

For any connected subgroup $H \subset \mathrm{SL}(2)$ let

$$X_H = \{x \in X\colon\ G_x^0 \cong H\}.$$

Let

$$\mathrm{SL}(2) = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix}\colon\ ad-bc = 1,\ a, b, c, d \in \boldsymbol{k}\right\}$$

in its usual matrix representation. Let

$$B = \left\{\begin{pmatrix} a & b \\ 0 & 1/a \end{pmatrix} : a \in \boldsymbol{k}^*, b \in \boldsymbol{k}\right\},$$

$$T = \left\{\begin{pmatrix} a & 0 \\ 0 & 1/a \end{pmatrix} : a \in \boldsymbol{k}^*\right\},$$

$$N(T) = \left\{\begin{pmatrix} a & 0 \\ 0 & 1/a \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} a & 0 \\ 0 & 1/a \end{pmatrix} : a \in \boldsymbol{k}^*\right\},$$

and

$$N_n = \left\{\begin{pmatrix} \varepsilon & b \\ 0 & 1/\varepsilon \end{pmatrix} : \varepsilon^n = 1, b \in \boldsymbol{k}\right\},$$

$n = 1, 2, \ldots, N_1 = \boldsymbol{k}^+$.

Definition 1. Let G be an algebraic group acting on an algebraic variety X and let $f\colon X \to Y$ be a G-morphism of algebraic varieties with the trivial action of G on Y. Then (Y, f) is said to be:

(a) a *categorical quotient* if for any G-morphism $g\colon X \to Z$, where Z is a variety with the trivial action of G, there exists a unique morphism $h\colon Y \to Z$ such that $g = h \circ f$ ([11], Definition 1.4);

(b) a *quotient* if the following conditions are satisfied:

(1) f is surjective and open,

(2) for any open subset $U \subset Y$, $\boldsymbol{k}[U] \to \boldsymbol{k}[f^{-1}(U)]^G$ is an isomorphism, where $\boldsymbol{k}[U]$ denotes the ring of regular functions on U,

(3) for any $y \in Y$, $f^{-1}(y)$ is exactly one orbit of the G-action on X ([3], 6.3),

(c) a *geometric quotient* if it is a quotient and f is an affine morphism ([11], Definition 1.6).

If (Y, f) is a categorical quotient or a quotient, let us denote by X/G the variety Y.

Remark 1. It follows directly from the above definition that if (Y, f) is a categorical quotient of X by G, then f is surjective, and if (Y, f) is a quotient, then it is a categorical quotient.

1. A technical theorem.

Theorem 1. *Let X be an algebraic variety and let two algebraic groups G_1 and G_2 act on X in such a way that those actions commute. Assume that there exist categorical quotients $(X/G_1, \alpha)$, $((X/G_1)/G_2, \beta)$, $(X/G_2, \varphi)$. Then there exists a categorical quotient $((X/G_2)/G_1, \psi)$ such that*

$$(X/G_2)/G_1 = (X/G_1)/G_2.$$

Moreover, if $(X/G_1, \alpha)$ and $((X/G_1)/G_2, \beta)$ are quotients, then $((X/G_2)/G_1, \psi)$ is *also a quotient, and if $(X/G_1, \alpha)$ and $((X/G_1)/G_2, \beta)$ are geometric quotients and G_2 is a linearly reductive group, then $((X/G_2)/G_1, \psi)$ is also a geometric quotient.*

Proof. Since $\beta\circ\alpha$: $X \to (X/G_1)/G_2$ is constant on G_2-orbits, the diagram

$$\begin{array}{ccc} X & \xrightarrow{\alpha} & X/G_1 \\ \downarrow{\scriptstyle\varphi} & & \downarrow{\scriptstyle\beta} \\ X/G_2 & & (X/G_1)/G_2 \end{array}$$

can be completed by the unique ψ: $X/G_2 \to (X/G_1)/G_2$ such that $\beta\circ\alpha = \psi\circ\varphi$.

One can show that $((X/G_1)/G_2, \psi)$ is a categorical quotient of X/G_2 by G_1, i.e., ψ is a G_1-morphism with the universality property.

Now assume that $(X/G_1, \alpha)$ and $((X/G_1)/G_2, \beta)$ are quotients. Then it can be shown that ψ is an open morphism, ψ separates G_1-orbits and for any open subset $U \subset (X/G_1)/G_2$ the induced homomorphism

$$\psi^*\colon\ k[U] \to k[\psi^{-1}(U)]^{G_1}$$

is an isomorphism. Hence $((X/G_1)/G_2, \psi)$ is a quotient of X/G_2 by a G_1-action.

Let us assume that $(X/G_1, \alpha)$ and $((X/G_1)/G_2, \beta)$ are geometric quotients and G_2 is a linearly reductive group. Then ψ is an affine morphism. Indeed, let A be an affine open subset of $(X/G_1)/G_2$. Then $A_1 = \alpha^{-1}\beta^{-1}(A)$ is an affine subset of X. The k-algebra $k[A_1]^{G_2}$ is finitely generated ([7], p. 183), so the set

$$\psi^{-1}(A) = \varphi(A_1) = A_1/G_2 = \operatorname{Spec} k[A_1]^{G_2}$$

is affine.

The proof of Theorem 1 is complete.

2. SL(2)-actions; the general situation. From now on G always denotes SL(2).

THEOREM 2. *Let $G = \mathrm{SL}(2)$ act on a normal irreducible algebraic variety X. Then there exist a connected subgroup $H \subset G$ and a non-empty open subset $U \subset X$ such that $x \in U$ if and only if $G_x^0 \cong H$.*

Proof. All orbits of maximal dimension form a dense subset of X (see [10]). We have four possibilities:

(a) There exists an orbit of dimension 3. Let U be the subset composed of all 3-dimensional orbits. Then $H = \{e\}$ and U satisfy the conclusion of the theorem since

$$U = X \setminus \{x \in X\colon\ \dim G_x \geqslant 1\}$$

and, for any n, the set $\{x \in X\colon\ \dim G_x \geqslant n\}$ is closed (see [6], Part 0 in Section 3).

(b) There is no 3-dimensional orbit and there exists an orbit of dimension 2. Let V be the subset composed of all 2-dimensional orbits. Then V is dense in X. Moreover, $V = X_{k^+} \cup X_T$ since, up to conjugation, k^+ and T are the only two 1-dimensional connected subgroups of SL(2) (see [8]). For $H = k^+$ or $H = T$ we have

$$X_H = G(X^H \backslash X^B) = G(X^H) \backslash G(X^B),$$

so X_H is constructible. $X = \bar{V} = \bar{X}_{k^+} \cup \bar{X}_T$ and, by the irreducibility of X, $X = \bar{X}_{k^+}$ or $X = \bar{X}_T$.

(1) $X = \bar{X}_{k^+}$. Since $\bar{X}_{k^+} = \{x \in X\colon k^+ \lesssim G_x\}$ (see Theorem 3 in [4]), $\bar{X}_{k^+}$ is composed of all 2-dimensional orbits, i.e.,

$$\bar{X}_{k^+} = \{x \in X\colon \dim G_x \geqslant 1\} \backslash \{x \in X\colon \dim G_x \geqslant 2\}.$$

Hence $\bar{X}_{k^+}$ is locally closed, so $H = k^+$ and $U = \bar{X}_{k^+}$ satisfy the conclusion of the theorem.

(2) $X = \bar{X}_T$. Since (again by Theorem 3 in [4])

$$X_T = (\{x \in X\colon \dim G_x \geqslant 1\} \backslash \{x \in X\colon \dim G_x \geqslant 2\}) \backslash \bar{X}_{k^+},$$

X_T is locally closed. Hence $H = T$ and $U = X_T$ satisfy the conclusion of the theorem.

(c) The maximal dimension of orbits is equal to 1. Then $H = B$ and $U = X$ satisfy the conclusion of the theorem (see Theorem 5 in [5]).

(d) The maximal dimension of orbits is equal to 0. Then the action is trivial, so $H = \mathrm{SL}(2)$ and $U = X$ satisfy the theorem.

We can also show the following theorem:

THEOREM 3. *Let $G = \mathrm{SL}(2)$ act on a normal irreducible algebraic variety X without 3-dimensional orbits. Then there exist a subgroup $H \subset G$ and a non-empty open subset $U \subset X$ such that $x \in U$ if and only if $G_x \cong H$.*

Proof. Parts (c) and (d) of the proof of the previous theorem give Theorem 3 in the case of maximal dimension of orbits equal to 0 or 1.

Let us assume that the maximal dimension of orbits is equal to 2. Then X_{k^+} or X_T is open in X.

(1) Let X_{k^+} be open in X. We have

$$X_{k^+} = \bigcup_{n \in N} \{x \in X\colon G_x \cong N_n\}.$$

It can be shown that this sum is finite. Let $k_1 < k_2 < \ldots < k_s$ be all positive integers such that

$$\{x \in X\colon G_x \cong N_{k_i},\ i = 1, 2, \ldots, s\} \neq \emptyset$$

and

$$\{x \in X\colon G_x \cong N_n \text{ for } n \neq k_i,\ i = 1, 2, \ldots, s\} = \emptyset.$$

Since N_n can be deformed to N_p only if $n | p$ (see Theorem 4 in [4]), $H = N_{k_1}$ and $U = \{x \in X\colon G_x \cong N_{k_1}\}$ satisfy the conclusion of the theorem.

(2) Let X_T be open in X and let

$$Z = \{x \in X:\ G_x \cong N(T)\}.$$

Since there is no deformation of $N(T)$ to T (see Theorem 6 in [4]), Z is a closed subset of X_T. Let us take $H = N(T)$ and $U = Z$ if $X_T = Z$ or $H = T$ and $U = X_T \backslash Z$ if $X_T \backslash Z \neq \emptyset$.

Remark 2. In the case where X contains an affine non-empty G-invariant open subset it follows from [9] that there exists an open subset U such that, for any $x,\ y \in U$, $G_x \cong G_y$.

3. Existence of quotients for SL(2)-actions without 3-dimensional orbits. The goal of this paper is to investigate the existence of the quotient $U/\mathrm{SL}(2)$, where U is described by Theorem 2. In the case of $U = X_{k^+}$ it turns out that the quotient of U may not exist but there exists a quotient of an open set $V \subset U$ which is complete (see Theorem 4).

By Theorem 2 there are five possibilities for H: SL(2), B, k^+, T and $\{e\}$. The case $H = \mathrm{SL}(2)$ is trivial. In the case of $H = B$ (see Theorem 5 in [5]) all orbits are projective, isomorphic to P^1, and X is isomorphic to the product $P^1 \times X^B$. In this section we discuss the cases $H = k^+$ and $H = T$.

Let $G = \mathrm{SL}(2)$ act on an irreducible normal variety X with the maximal dimension of orbits equal to 2. For $H = k^+$ or $H = T$, the set

$$U = X_H = \{x \in X:\ G_x^0 \cong H\}$$

is non-empty and open.

Let us define actions of G and $N(H)$ on $U^H \times G/H$ in the following way:

$$g(x, [h]) = (x, [gh]) \quad \text{and} \quad n(x, [h]) = (nx, [hn^{-1}])$$

for $g \in G$, $n \in N(H)$, $x \in U^H$ and $[h] \in G/H$. These actions are well defined.

Since $H \subset N(H)$ acts trivially on $U^H \times G/H$, the action of $N(H)$ induces an action of $N(H)/H$. Easy computations show that actions of G and $N(H)/H$ commute.

LEMMA. (a) *Let α: $U^H \times G/H \to U^H$ be the projection. Then (U^H, α) is a quotient of $U^H \times G/H$ by G.*

(b) *Let φ: $U^H \times G/H \to U$ be defined by $\varphi(x, [g]) = gx$ for any $x \in U^H$ and $[g] \in G/H$. Then (U, φ) is a quotient of $U^H \times G/H$ by $N(H)/H$.*

Proof. We shall prove (b) (the proof of (a) is obvious). φ is surjective, constant on $(N(H)/H)$-orbits and separates $(N(H)/H)$-orbits. In fact, let

$$\varphi(x_1, [g_1]) = \varphi(x_2, [g_2]) \quad \text{for } x_1, x_2 \in U^H,\ [g_1], [g_2] \in G/H.$$

Then $nG_{x_1}^0 n^{-1} = G_{x_2}^0$ with $n = g_2^{-1} g_1$. Since $G_{x_i}^0 = H$ $(i = 1, 2)$, we get $n \in N(H)$ and

$$(x_2, [g_2]) = n(x_1, [g_1]).$$

Moreover, an easy computation shows that $N(H)/H$ acts on $U^H \times G/H$ with all isotropy subgroups equal to $\{e\}$. Hence any two fibres of φ have the same dimension equal to $\dim N(H)/H$. The set U is normal as an open subset of X. By A.G.18.4 in [3], φ is open and, by Theorem 6.6 in [3], (U, φ) is a quotient of $U^H \times G/H$ by $N(H)/H$.

THEOREM 4. *Let X be a smooth irreducible projective (resp., complete) variety with an algebraic action of $G = \mathrm{SL}(2)$ such that*

$$X_{k^+} = \{x \in X\colon G_x^0 \cong k^+\}$$

is dense in X. Then X_{k^+} is open in X and for some non-empty G-stable open subset $V \subset X_{k^+}$ there exists a quotient of V by G which is projective (resp., complete).

Remark 3. As V one may take $G(Y)$, where Y is the intersection of the set $(X_{k^+})^{k^+}$ with the difference of the open cell of the $(T = N(k^+)/k^+)$-action on X and X^T.

Proof. In this situation we have $N(k^+) = B$,

$$\begin{array}{ccc} U^{k^+} \times G/k^+ & \xrightarrow{\alpha} & U^{k^+} \\ \big\downarrow{\scriptstyle\varphi} & & \\ U & & \end{array}$$

where α and φ are defined in the Lemma, and B/k^+ acts on U^{k^+}. Since T normalizes k^+, U^{k^+} is T-invariant. We have Białynicki-Birula's decomposition of X determined by the induced action of T:

$$X = \bigcup_{i=1}^{r} X_i$$

with T-invariant locally closed X_i such that each X_i corresponds to exactly one connected component F_i of X^T and $F_i \subset X_i$ (see [1]). Since $U^{k^+} \subset U \subset X \backslash X^T$, we have

$$U^{k^+} = \bigcup_{i=1}^{r} U_i^{k^+},$$

where $U_i^{k^+} = U^{k^+} \cap (X_i \backslash F_i)$, $i = 1, 2, \ldots, r$. Fix $i \in \{1, \ldots, r\}$.

$U_i^{k^+}$ is a locally closed subset of U^{k^+} since it can be written as a difference of a locally closed $U^{k^+} \cap X_i$ and closed $U^{k^+} \cap F_i$. Hence $U_i^{k^+} \times G/k^+$ is a locally closed subset of $U^{k^+} \times G/k^+$. It is also T-invariant, so it can be written as a difference of two T-invariant closed subsets of $U^{k^+} \times G/k^+$. Since φ is open, $\varphi(F) \subset U$ is closed for any closed T-stable subset $F \subset U^{k^+} \times G/k^+$. Hence $\varphi(U_i^{k^+} \times G/k^+)$ is locally closed in U. Consequently, U is a finite sum of locally closed subsets:

$$U = \bigcup_{i=1}^{r} \varphi(U_i^{k^+} \times G/k^+).$$

One of these subsets, say $V = \varphi(U_1^{k^+} \times G/k^+)$, has to be open in U. We have

$$U_1^{k^+} = U^{k^+} \cap (X_1 \backslash F_1) = X^{k^+} \cap (X_1 \backslash F_1)$$

($\bar{X}_{k^+} = X$, so $k^+ \subsetneqq G_x$ for any $x \in X$, and $X^B = X^T$). $U_1^{k^+}$ is a T-stable and closed subset of $X_1 \backslash F_1$.

There exists a geometric quotient of $X_1 \backslash F_1$ by T ([12], Corollary 3). Hence there exists also a geometric quotient of $U_1^{k^+}$ by T which is closed in $(X_1 \backslash F_1)/T$ ([6], Proposition 1.9). If X is projective (resp., complete), then by the results of Section 2 and Theorem 3.1 in [2] the quotient $(X_1 \backslash F_1)/T$ is projective (resp., complete) ($X_1 \backslash F_1$ is identical to the sectional set corresponding to the section $A^+ = \{F_1\}$, $A^- = \{F_2, \ldots, F_r\}$ in the terminology of [2]). Hence $U_1^{k^+}/T$ is also projective (resp., complete). We have

$$\begin{tikzcd} U_1^{k^+} \times G/k^+ \arrow[r, "\alpha"] \arrow[d, "\varphi"] & U_1^{k^+} \arrow[d, "\beta"] \\ V = \varphi(U_1^{k^+} \times G/k^+) & U_1^{k^+}/T \end{tikzcd}$$

where $(U_1^{k^+}, \alpha)$ and (V, φ) are quotients and $(U_1^{k^+}/T, \beta)$ is the geometric quotient. By Theorem 1 there exists a quotient $(V/G, \psi)$ such that $V/G = U_1^{k^+}/T$.

PROBLEM (**P 1378**). Under the assumptions of Theorem 4 there are two canonical decompositions of X for the torus T([1], Theorem 4.3), so there are two open cells corresponding to them. Hence there exist two different open subsets V_1 and V_2 for which $V_i/\mathrm{SL}(2)$ exist and are projective (resp., complete). Does there exist another such V for which $V/\mathrm{SL}(2)$ exists and is projective (resp., complete)?

THEOREM 5. *Let X be an irreducible normal variety with an algebraic action of $G = \mathrm{SL}(2)$ such that*

$$X_T = \{x \in X: \; G_x^0 \cong T\}$$

is dense in X. Then X_T is open in X and there exists a geometric quotient of X_T by G.

Proof. We have

$$\begin{tikzcd} U^T \times G/T \arrow[r, "\alpha"] \arrow[d, "\varphi"] & U^T \\ U & \end{tikzcd}$$

where α and φ are defined in the Lemma and $N(T)/T$ acts on U^T. Since

$$T_2 = \left\{ \begin{pmatrix} \varepsilon & 0 \\ 0 & 1/\varepsilon \end{pmatrix} : \; \varepsilon^2 = 1 \right\}$$

normalizes T, U^T is T_2-invariant. There exists a geometric quotient of U^T by T_2 (a consequence of Theorem 1.3 and Proposition 0.8 in [6]). Let us denote it by $(U^T/T_2, \beta)$. Since G/T is affine, α: $U^T \times G/T \to U^T$ is an affine morphism. Hence (U^T, α) is a geometric quotient by G. We have

$$\begin{array}{ccc} U^T \times G/T & \xrightarrow{\alpha} & U^T \\ \downarrow{\scriptstyle\varphi} & & \downarrow{\scriptstyle\beta} \\ U & & U^T/T_2 \end{array}$$

By Theorem 1 there exists a geometric quotient $(U/G, \psi)$ such that $U/G = U^T/T_2$.

Remark 4. We shall show (see Examples 1 and 2) that under the assumptions of Theorem 5 nothing can be said about projectivity of $X_T/\mathrm{SL}(2)$.

We fix some notation. Let $k(\varrho_i)$ be the vector space of homogeneous polynomials of degree i in two variables x and y, with the SL(2)-action determined by the following equalities:

$$g(x) = ax+cy \quad \text{and} \quad g(y) = bx+dy \quad \text{for } g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2).$$

Let $k(\varrho_i \oplus \varrho_j)$ denote the direct sum of $k(\varrho_i)$ and $k(\varrho_j)$ with the direct sum of SL(2)-actions on $k(\varrho_i)$ and $k(\varrho_j)$. By $P(\varrho_i \oplus \varrho_j)$ we denote the projectivization of the space $k(\varrho_i \oplus \varrho_j)$ with the projectivization of SL(2)-action.

EXAMPLE 1. In $P^{11} = P(\varrho_4 \oplus \varrho_6)$ let us take two projective lines P_1 and P_2 such that

$$P_1 = \{tx^3y \oplus sx^4y^2 \colon (t, s) \neq (0, 0)\},$$

$$P_2 = \{txy^3 \oplus sx^2y^4 \colon (t, s) \neq (0, 0)\}.$$

For any $p \in P_1 \cup P_2$, $G_p = T$ and, for any $q \notin P_1 \cup P_2$, $G_q \neq T$.

Let $U = \mathrm{SL}(2)(P_1 \cup P_2)$ and $X = \bar{U}$ in P^{11}. Then $U = X_T$ and, by Theorem 5, there exists $U/\mathrm{SL}(2) = U^T/T_2$. But $U^T = P_1 \cup P_2$. There is only one element of order 2 in SL(2), namely $-e$, and it acts as e at points of $P_1 \cup P_2$. Hence $U^T/T_2 = U^T$ is projective.

EXAMPLE 2. In $P^5 = P(\varrho_0 \oplus \varrho_4)$ let us take an affine line Y such that

$$Y = \{s \oplus x^2y^2 \colon s \in k\}.$$

For any $p \in Y$, $G_p = N(T)$ and, for any $q \notin Y$, $G_q \neq N(T)$.

Let $U = \mathrm{SL}(2)(Y)$ and $X = \bar{U}$ in P^5. Then $U = X_T$. By Theorem 5, there exists $U/\mathrm{SL}(2) = U^T/T_2$. But $U^T = Y$ and $-e$ acts as e at points of Y. Hence $U^T/T_2 = U^T$ is affine.

I wish to thank Professor Andrzej Białynicki-Birula for his help in the preparation of this paper.

REFERENCES

[1] A. Białynicki-Birula, *Some theorems on actions of algebraic groups*, Ann. of Math. 98 (1973), pp. 480–497.

[2] — and J. Święcicka, *Complete quotients by algebraic torus actions*, pp. 10–22 in: *Group Actions and Vector Fields*, Lecture Notes in Math. 956, Springer, 1982.

[3] A. Borel, *Linear Algebraic Groups*, Benjamin, New York–Amsterdam 1969.

[4] E. Duma, *Algebraic* SL(2)*-actions: deformations of infinite isotropy subgroups*, this fascicle, pp. 221–231.

[5] J. Konarski, *Properties of projective orbits of actions of affine algebraic groups*, pp. 79–91 in: *Group Actions and Vector Fields*, Lecture Notes in Math. 956, Springer, 1982.

[6] D. Mumford, *Geometric Invariant Theory*, Springer, New York–Berlin–Heidelberg 1965.

[7] — and K. Suominen, *Introduction to the theory of moduli*, Proceedings of the 5-th Nordic Summer School in Oslo, 1970.

[8] V. L. Popov, *Quasi-homogeneous affine algebraic varieties of* SL(2) (in Russian), Izv. Akad. Nauk USSR 37 (1973), pp. 799–832.

[9] R. W. Richardson, *Principal orbit types for algebraic transformation spaces in characteristic zero*, Invent. Math. 16 (1972), pp. 6–14.

[10] — *Deformations of Lie subgroups and the variation of isotropy subgroups*, Acta Math. 129 (1972), pp. 35–73.

[11] C. S. Seshadri, *Quotient spaces modulo reductive algebraic groups*, Ann. of Math. 95 (1972), pp. 511–556.

[12] H. Sumihiro, *Equivariant completions*, J. Math. Kyoto Univ. 14 (1974), pp. 1–28.

INSTITUTE OF MATHEMATICS
WARSAW UNIVERSITY
00-901 WARSZAWA
PKiN IX p., POLAND

Reçu par la Rédaction le 10.5.1984;
en version définitive le 6.6.1988