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A REMARK TO A PROBLEM OF J. MYCIELSKI ON ARITHMETIC SEQUENCES

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Let $a(n)$ denote the arithmetic sequence

$$\ldots, a-2n, a-n, a, a+n, a+2n, \ldots$$

Definition 1. A system of arithmetic sequences

$$a_i(n_i), \quad i = 1, 2, \ldots, k, \tag{1}$$

will be called *covering* if every integer belongs to at least one of them.

Definition 2. System (1) will be called a *disjoint covering* if every integer belongs to exactly one sequence.

In [1] a conjecture was made which we state as

THEOREM 1. *If* (1) *is a disjoint covering and if for some* i_0 *we have*

$$n_{i_0} = \prod_{t=1}^{r} p_t^{\lambda_t}, \tag{*}$$

then

$$k \geqslant 1 + \sum_{t=1}^{r} \lambda_t(p_t - 1). \tag{**}$$

In [2] we proved this conjecture. Now we show that (**) is true under weaker conditions too; namely we have

THEOREM 2. *Let* (1) *be a covering system; let the sequence* $a_{i_0}(n_{i_0})$ *be disjoint with all other sequences of* (1); *then* (*) *implies* (**).

Proof. We shall consider the numbers

$$a_{i_0} + c_t q_t p_t^{\alpha_t}, \tag{2}$$

where $t = 1, 2, \ldots, r$; $c_t = 1, 2, \ldots, p_t - 1$; $q_t = n_{i_0}/p_t^{\lambda_t}$; $\alpha_t = 0, 1, \ldots \ldots, \lambda_t - 1$.

Obviously none of numbers (2) belongs to the sequence $a_{i_0}(n_{i_0})$. Now we prove the following assertion:

If the number $a_{i_0}+c_t q_t p_t^{a_t}$ *belongs to the j-th sequence of* (1) (*i.e. if we have*

$$a_{i_0}+c_t q_t p_t^{a_t} = a_j + h n_j), \tag{3}$$

then

$$p_t^{a_t+1} \mid n_j. \tag{4}$$

From (3) it follows that the number $(n_j, c_t q_t p_t^{a_t})$ divides $a_{i_0}-a_j$. Because the sequence $a_{i_0}(n_{i_0})$ is disjoint with any other sequence, the number (n_j, n_i) does not divide $a_{i_0}-a_j$. Thus we have $(n_j, n_i) > (n_j, c_t q_t p_t^{a_t})$ hence $p_t^{a_t+1} \mid n_j$.

Further we can proceed in the same way as in the proof of theorem 1 in [2].

Remark. In theorem 2, system (1) need not be a disjoint covering, therefore theorem 2 is a generalization of theorem 1.

REFERENCES

[1] J. Mycielski et W. Sierpiński, *Sur une propriété des ensembles linéaires*, Fundamenta Mathematicae 58 (1966), p. 143-147.

[2] Š. Znám, *On Mycielski's problem on systems of arithmetic sequences*, Colloquium Mathematicum 15 (1966), p. 201-204.

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